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POLARIZABILITY CONTRIBUTIONS TO THE NEUTRON-LEPTON AMPLITUDE AT THRESHOLD

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A B S T R A C T

Motivated by recent interest in neutron-electron scattering amplitude at threshold, a detailed investigation of the two-photon exchange contribution, commonly known as polarizability correction, to this amplitude is made, for general lepton mass.

The contribution is related to the amplitudes describing forward virtual Compton scattering on neutrons. To calculate it, we write dispersion relations for the Compton amplitudes and make use of the present knowledge on the neutron structure functions as well as the scaling hypothesis. The correction is much larger for muons than for electrons.

Further, we discuss the region of validity of the extreme rclativistic and the classical approximations treated in the literature by giving the relevant parameter which leads naturally from one case to the other.

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1. INTRODUCTION

The neutron-electron spin averaged forward scattering amplitude at threshold a is measured from scattering of thermal neutrons on atoms. The most accurate determinations of a, using different experimental techniques, give the results $(-1.34 \pm 0.03) \times 10^{-3}$ fm [1] and $(-1.56 \pm 0.04) \times 10^{-3}$ fm [2].

In second order in quantum electrodynamics this amplitude, $a^{(1\gamma)}$, is given by the slope of the electric form factor of the neutron at zero momentum transfer [*)]

$$a^{(1\gamma)} = \alpha \, 2M \left. \frac{d G_{En}(q^2)}{d q^2} \right|_{q^2=0} \tag{1}$$

where α is the fine structure constant ($\alpha \simeq 1/137$). M is the mass of the neutron and $G_{En}(q^2)$ is the electric form factor of the neutron. In terms of the Dirac and Pauli form factors

$$G_{En}(q^2) = F_{1n}(q^2) + \frac{q^2}{4M^2} F_{2n}(q^2) \tag{2}$$

with $F_{1n}(0) = 0$ and $F_{2n} = \mu_n$, where $\mu_n = -1.913$ is the anomalous magnetic moment of the neutron. The contribution coming from the magnetic moment (the Foldy term [3]) gives the complete answer to $a^{(1\gamma)}$ if we put $F'_{1n}(0) = 0$. Then $G'_{En}(0) = \mu_n/4M^2$ and

$$a^{(1\gamma)} = \alpha \frac{\mu_n}{2M} = -1.466 \times 10^{-3} \text{ fm} \tag{3}$$

Recent careful analyses [4] of elastic electron scattering on deuterium are in accord with the above value for $G'_{En}(0)$. Furthermore, the quasi-elastic electron deuterium scattering data [5] are not inconsistent with the parametrization

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*) In our metric $q^2 = q_0^2 - \vec{q}^{\,2}$.

$$G_{E_n}(q^2) = \mu_n \frac{q^2}{4M^2} G_{E_p}(q^2) \qquad (4)$$

which corresponds to the choice $F_{1_n}(q^2) = 0$ and the scaling law for the magnetic form factor of the neutron.

In Ref. 6) a simple estimate of the fourth order electromagnetic contribution to a was given from polarizability contributions by iterating the Coulomb potential. This picture reproduces the classical limit of a r^{-4} potential if the electric field is considered to be static. Soon afterwards, it was pointed out [7] that, due to the smallness of the electron mass, the relativistic aspects of the electron propagator must be very important. In estimating the two-photon-exchange contribution, the authors in Ref. 7) noted that if the neutron excitation is not taken into account the relevant Feynman diagrams diverge logarithmically but can be made finite by introducing a cut-off Λ. The correction thus obtained is proportional to $m \ln \Lambda/m$, m being the electron mass, and is for Λ of the order of the nucleon mass much smaller than in the classical treatment [6].

In this paper we are interested in a detailed study of this fourth order contribution for electrons and muons. This study, in terms of the charged lepton mass, allows us to understand clearly the relationship between the classical and the relativistic cases and also gives us the relevant parameter which naturally leads from the logarithmic dependence on the cut-off (in the relativistic case) to the linear one present in the classical case. Further, the meaning and value of the cut-off Λ introduced in Ref. 7) is somewhat obscure. We find that a general formulation of the two-photon exchange contribution can be given without resorting to a cut-off. If one chooses to introduce such a cut-off, from our considerations, its meaning and value are well defined.

The plan of the paper is as follows. In Section 2 we give the general formula for the two-photon exchange contribution (Fig. 1). The major quantity entering here is a tensor describing forward virtual Compton scattering on neutron. The gauge invariant decomposition of this tensor includes two structure functions $T_i(\nu, q^2)$, $i = 1,2$ where ν is the energy of the virtual photon and q^2 is the square of its mass.

From the diagrams shown in Fig. 2 we calculate the Born contribution to the two-photon exchange contribution and discuss our results in terms of the lepton mass. In Section 3, starting with our general formula, we perform a Wick rotation in the ν complex plane, for $|\vec{q}| = (\nu^2 - q^2)^{\frac{1}{2}}$ fixed, and write dispersion relations for the amplitudes T_i at q^2 fixed. The problem of the subtractions is extensively discussed. Each dispersion integral is made up of two parts, namely, the pole and the continuum contributions. In Section 4 the information from inelastic electron neutron scattering data is used to calculate the continuum contribution to the dispersion integrals. More specifically, we use the scaling hypothesis [8] (so far in agreement with experiment) to find the large q^2 dependence of the structure functions and we compare the behaviour of the fourth order contribution as a function of q^2 for electrons and muons. Finally, in Section 5, we present our results and conclusions.

2. TWO-PHOTON EXCHANGE AMPLITUDE AND THE BORN CONTRIBUTION

The neutron-electron scattering amplitude in question, in our normalization convention, is given by *)

$$a = \frac{1}{8\pi m} T \tag{5}$$

where m is the electron mass and T the invariant T matrix element. For the two-photon exchange contribution this quantity is given by

$$i T^{(2\gamma)} = 2 \frac{\alpha^2}{\pi^2} \int \frac{d^4 q}{(q^2)^2} \frac{1}{q^2 - 2m\nu} T_{\mu\rho} L^{\mu\rho} \tag{6}$$

Here $\nu = q^0$ and $L^{\mu\rho}$ is the leptonic tensor given by

$$L^{\mu\rho} = p^\mu (p^\rho - q^\rho) + p^\rho (p^\mu - q^\mu) + g^{\mu\rho} m\nu \tag{7}$$

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*) All our formulae apply just as well for muons in which case m is the muon mass.

p^{μ} being the four-momentum of the electron $p^{\mu} = (m,\vec{0})$. $T_{\mu\rho}$ is the hadronic tensor describing the spin averaged forward virtual Compton scattering, given by

$$T_{\mu\rho} = i \int d^4x \; e^{iqx} \, \theta(x^0) \langle P | [J_\mu(x), J_\rho(0)] | P \rangle \tag{8}$$

This tensor has the gauge invariant decomposition *)

$$T_{\mu\rho} = -\left(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2}\right) T_1(\nu, q^2) + \frac{1}{M^2}\left(P_\mu - \frac{M\nu}{q^2} q_\mu\right)\left(P_\rho - \frac{M\nu}{q^2} q_\rho\right) T_2(\nu, q^2) \tag{9}$$

where $P^{\mu} = (M,\vec{0})$, M being the neutron mass. From the crossing property of $T_{\mu\rho}$ we find that $T_i(\nu,q^2)$ are even functions of ν.

Contracting the hadronic tensor with the leptonic one and using the property $T_i(\nu,q^2) = T_i(-\nu,q^2)$ we find for the two-photon exchange contribution the general formula

$$i\, T^{(2\gamma)} = 2m^2 \frac{\alpha^2}{\pi^2} \int \frac{d^4q}{(q^2)^2} \left[\frac{1}{q^2 - 2m\nu} + \frac{1}{q^2 + 2m\nu}\right] \left[-\left(1 + \frac{2\nu^2}{q^2}\right) T_1(\nu,q^2) + \left(1 - \frac{\nu^2}{q^2}\right) T_2(\nu,q^2)\right] \tag{10}$$

The diagrams giving the fourth order Born contribution are shown in Fig. 2. From these diagrams this contribution can be calculated. According to the discussion above we take for the neutron $F_1(q^2) = 0$ and find the Born contribution to the functions $T_i(\nu,q^2)$ to be given by

$$T_1^B(\nu,q^2) = T_2^B(\nu,q^2) = [F_2(q^2)]^2 \frac{2\,(q^2)^2}{(q^2)^2 - 4M^2\nu^2} \tag{11}$$

We substitute the T_i^B in our general formula, Eq. (10), and use for the $F_2(q^2)$ the dipole fit $F_2(q^2) = \mu_n(1-q^2/\Lambda_B^2)^{-2}$, $\Lambda_B^2 = 0.71\ \mathrm{GeV}^2$, which, as discussed before, is consistent with experiment. Calculating the resulting Feynman integral by standard methods [9] we find

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*) We shall use the same letter $T_i(\nu,q^2)$ for the complex functions in Eq. (9) and for their real parts, which appear in the amplitude $a \propto \mathrm{Re}\, T$ [Eq. (5)].

$$a^{(2\gamma)}(\text{Born}) = \frac{\alpha^2}{4\pi}\,\frac{\mu_n^2}{M^2-m^2}\,3m\,f(\Lambda_B\,;M,m) \tag{12}$$

For $4m^2 < \Lambda_B^2 < 4M^2$, which is the physically interesting case for electrons and muons, we have

$$\begin{aligned} f(\Lambda_B,M,m) = &-\left(\frac{\Lambda_B}{2M}\right)\left(1-\frac{\Lambda_B^2}{4M^2}\right)^{-5/2}\left[\frac{5}{4}\left(1-\frac{3}{5}\frac{\Lambda_B^2}{M^2}+\frac{1}{10}\frac{\Lambda_B^4}{M^4}\right)\tan^{-1} 2\frac{M}{\Lambda_B}\sqrt{1-\frac{\Lambda_B^2}{4M^2}}+\frac{7}{12}\frac{\Lambda_B}{2M}\left(1-\frac{5}{14}\frac{\Lambda_B^2}{M^2}\right)\sqrt{1-\frac{\Lambda_B^2}{4M^2}}\right] \\ &-\left(1-\frac{4m^2}{\Lambda_B^2}\right)^{-5/2}\left[\left(1-6\frac{m^2}{\Lambda_B^2}+10\frac{m^4}{\Lambda_B^4}\right)\ln\frac{1-\sqrt{1-\frac{4m^2}{\Lambda_B^2}}}{1+\sqrt{1-\frac{4m^2}{\Lambda_B^2}}}+\frac{5}{6}\left(1-\frac{14}{5}\frac{m^2}{\Lambda_B^2}\right)\sqrt{1-\frac{4m^2}{\Lambda_B^2}}\right] \end{aligned} \tag{13}$$

The behaviour of $f(\Lambda,M,m)$ as a function of Λ, for the cases of electron and muon, is shown in Figs. 3 and 4, respectively. For the physical value of Λ, $\Lambda^2 = \Lambda_B^2 = 0.71\ \text{GeV}^2$, we find $f(\Lambda_B) = 13.2$ for electrons and $f(\Lambda_B) = 2.73$ for muons.

It is particularly illuminating to examine the behaviour of the function $f(\Lambda,M,m)$ in the two extreme limits $\Lambda/2m \gg 1$ and $\Lambda/2m \ll 1$.

In the limit $\Lambda/2m \gg 1$ (valid in nature for electrons $\Lambda_B/2m_e \sim 840$, while $\Lambda_B/2m_\mu \sim 4$) we find

$$f(\Lambda,M,m) \sim \ln\frac{\Lambda^2}{m^2} - \frac{5}{6} - \frac{5}{8}\,\pi\,\frac{\Lambda}{2M} \sim \ln\frac{\Lambda^2}{m^2} \tag{14}$$

if $\Lambda/2M$ is relatively small, as it is the case physically. This approximation is good for electrons but not for the muons. For electrons we find in this limit $f(\Lambda_B) \sim 14.8$ as compared to the exact value 13.2. For muons these two numbers become 4.17 and 2.73 respectively.

It is interesting to note that in the limit of point-like hadron, i.e., $\Lambda \to \infty$, this Born contribution is convergent, that is f is of the form

$$f \underset{\Lambda \to \infty}{\sim} \ln \frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2}{M^2} = \ln \frac{M^2}{m^2} \tag{15}$$

This finite result is due to the extra parameter M introduced in the calculation when the intermediate particle in the diagram is specified.

We now consider the second extreme limit, namely $\Lambda/2m \ll 1$, for which we expect to be able to treat the lepton as a source of a Coulomb field. In this limit in fact we find

$$f \sim \frac{5}{8} \pi \left(\frac{\Lambda}{2m} - \frac{\Lambda}{2M} \right) = \frac{5}{8} \pi \frac{M-m}{M} \frac{\Lambda}{2m} \tag{16}$$

and the linear dependence on the parameter Λ, as expected from a classical treatment, explicitly emerges. From a comparison of the classical and exact results, we conclude that the classical approximation for electrons over-estimates the contribution by two orders of magnitude while for the muon the classical result is only about twice as large as the exact result.

3. GENERAL FORMULATION IN TERMS OF NEUTRON STRUCTURE FUNCTIONS

Starting with our general formula for the two-photon exchange contribution, Eq. (10), we perform a Wick rotation in the $\nu = q^0$ plane, with $|\vec{q}|$ fixed. This procedure is similar to the one used in quantum electrodynamics when calculating Feynman integrals [9]. It has also been applied to the problems of nucleon self-masses [10] and polarizability corrections to the hyperfine splitting in hydrogen [11]. After this rotation the variable ν is integrated from $-i\infty$ to $i\infty$, By change of variable $\nu = iK_0$ our general formula can be written in the form

$$\begin{aligned} iT^{(2\gamma)} &= \int d^4q \, \mathcal{F}(\nu, q^2) = 4\pi i \int_{-\infty}^{0} dq^2 \int_0^{\sqrt{-q^2}} dK_0 \sqrt{-q^2 - K_0^2} \, \mathcal{F}(iK_0, q^2) \\ \mathcal{F}(\nu, q^2) &= 2m^2 \frac{\alpha^2}{\pi^2} \frac{2}{q^2} \frac{1}{(q^2)^2 - 4m^2\nu^2} \left[-\left(1 + 2\frac{\nu^2}{q^2}\right) T_1(\nu, q^2) + \left(1 - \frac{\nu^2}{q^2}\right) T_2(\nu, q^2) \right] \end{aligned} \tag{17}$$

where we have used that $\mathcal{F}$ is an even function of ν. In this way it is seen clearly that we need to know the function $\mathcal{F}$ only for space-like photons.

We may now write dispersion relations for the amplitudes $T_i(\nu, q^2)$. From the analysis of Harari [12] it is known that $T_2(\nu, q^2)$ satisfies an unsubtracted dispersion relation, whereas $T_1(\nu, q^2)$ needs a subtraction. Introducing $W_i(\nu, q^2) = 1/2\pi \, \mathrm{Im}\, T_i(\nu, q^2)$, and using the property $W_i(\nu, q^2) = -W_i(-\nu, q^2)$, we write for fixed mass squared q^2

$$T_1(\nu, q^2) = T_1(0, q^2) + 4\nu^2 \int_0^\infty \frac{d\nu'}{\nu'} \frac{W_1(\nu', q^2)}{\nu'^2 - \nu^2}$$

$$T_2(\nu, q^2) = 4 \int_0^\infty d\nu' \frac{\nu' W_2(\nu', q^2)}{\nu'^2 - \nu^2} \tag{18}$$

The absorptive parts $W_i(\nu', q^2)$ are the so-called neutron structure functions measured in electron neutron scattering. The subtraction function $T_1(0, q^2)$ is not directly accessible to experiments. We shall return to the question of its determination later on.

We begin by separating the "pole" contribution in the dispersion integrals. We use this terminology to distinguish this contribution from the Born contribution as calculated in Section 2, directly from the diagrams in Fig. 2. The elastic electron neutron scattering is described by the functions

$$W_1(\nu, q^2) = W_2(\nu, q^2) = \frac{-q^2}{4M} \left[F_{2n}(q^2)\right]^2 \delta\left(\nu + \frac{q^2}{2M}\right) \tag{19}$$

if we put $F_{1n}(q^2) = 0$ as discussed in the Introduction. Thus, the pole contribution to $T_i(\nu, q^2)$ [denoted by $T_i^p(\nu, q^2)$] is given by

$$T_1^p(\nu, q^2) = \left[F_{2n}(q^2)\right]^2 \frac{8M^2\nu^2}{(q^2)^2 - 4M^2\nu^2}$$

$$T_2^p(\nu, q^2) = \left[F_{2n}(q^2)\right]^2 \frac{2(q^2)^2}{(q^2)^2 - 4M^2\nu^2} \tag{20}$$

Comparing these relations with (11) we conclude

$$T_1^P(\nu, q^2) = T_1^B(\nu, q^2) - 2\,[F_{2n}(q^2)]^2 \qquad (21)$$

$$T_2^P(\nu, q^2) = T_2^B(\nu, q^2)$$

The non-equality of the Born and pole terms for $T_1(\nu, q^2)$ is due to the fact that this amplitude satisfies a subtracted dispersion relation.

The dispersion relations, Eq. (18), can be written then in the form

$$T_1(\nu, q^2) = T_1^B(\nu, q^2) + \left\{T_1(0, q^2) - 2\mu_n^2 G_{E_p}^2(q^2)\right\} + 4\nu^2 \int_{\nu'_{th}(q^2)}^{\infty} \frac{d\nu'}{\nu'} \frac{W_1(\nu', q^2)}{\nu'^2 - \nu^2} \qquad (22a)$$

$$T_2(\nu, q^2) = T_2^B(\nu, q^2) + 4 \int_{\nu'_{th}(q^2)}^{\infty} d\nu' \frac{\nu' W_2(\nu', q^2)}{\nu'^2 - \nu^2} \qquad (22b)$$

where $\nu'_{th}(q^2) = \mu + (\mu^2 - q^2)/2M$ is the threshold value for one-pion production and μ is the pion mass. Substituting Eqs. (22) into the right-hand side of Eq. (17), the K_0 integration can be done explicitly, and we decompose the $T^{(2\gamma)}$ matrix element into three parts, namely, the Born, the subtraction and the dispersion *) contributions. We write

$$T^{(2\gamma)} = T(\text{Born}) + T(\text{sub}) + T(\text{disp})$$

$$T(\text{Born}) = \alpha^2\, 3\, \frac{\mu_n^2}{M^2 - m^2} \int_{-\infty}^{0} dq^2 \left\{\sqrt{1 - \frac{4m^2}{q^2}} - 1 - \frac{m^2}{M^2}\left[\sqrt{1 - \frac{4M^2}{q^2}} - 1\right]\right\} G_{E_p}^2(q^2) \qquad (23a)$$

$$T(\text{sub}) = -\alpha^2\, 2 \int_{-\infty}^{0} \frac{dq^2}{q^2} \left\{\sqrt{1 - \frac{4m^2}{q^2}} + \frac{q^2}{2m^2}\left[\sqrt{1 - \frac{4m^2}{q^2}} - 1\right]\right\} \left[T_1(0, q^2) - 2\mu_n^2 G_{E_p}^2(q^2)\right] \qquad (23b)$$

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*) We note that T(disp) need not be identical to the continuum contribution, because a priori $T_1(0, q^2)$ may get contributions from the continuum.

$$T(disp) = \alpha^3 8 \int_{-\infty}^{0} \frac{dq^2}{q^2} \int_{\nu_{th}(q^2)}^{\infty} \frac{d\nu}{\nu} W_2(\nu, q^2) \Bigg\{ \sqrt{1-\frac{4m^2}{q^2}} - 1$$

$$- \frac{1}{1+R(\nu,q^2)} \left(1 - \frac{\nu^2}{q^2}\right) \left[\frac{R(\nu,q^2)}{(q^2)^2 - 4m^2\nu^2} \left((q^2)^2 \left[\sqrt{1-\frac{4m^2}{q^2}} - 1\right] - 4m^2\nu^2 \left[\sqrt{1-\frac{q^2}{\nu^2}} - 1\right] \right) \right. \tag{23c}$$

$$\left. + 1 + \frac{1}{2m^2q^2((q^2)^2 - 4m^2\nu^2)} \left((q^2)^4 \left[\sqrt{1-\frac{4m^2}{q^2}} - 1\right] - 16m^4\nu^4 \left[\sqrt{1-\frac{q^2}{\nu^2}} - 1\right] \right) \right] \Bigg\}$$

Here the function $R(\nu, q^2)$ denotes the ratio of the total cross-sections of longitudinal and transverse photons on the neutron

$$R(\nu,q^2) = \frac{W_L(\nu,q^2)}{W_1(\nu,q^2)} \quad , \qquad W_L(\nu,q^2) \equiv \left(1 - \frac{\nu^2}{q^2}\right) W_2(\nu,q^2) - W_1(\nu,q^2) \tag{24}$$

We check easily that the Born contribution thus obtained agrees with our previous result, Eqs. (12), (13). The subtraction term, at the first glance, seems to be infra-red divergent, but actually it is convergent. In fact, $T_1(0, q^2=0)$ is determined from low energy theorems in real Compton scattering. From the decomposition of the tensor $T_{\mu\rho}$, Eq. (9), we know that

$$T_1(\nu, q^2=0) = \lim_{-q^2\to 0} \nu^2 \frac{T_2(\nu,q^2)}{-q^2} \xrightarrow[\nu\to 0]{} -2 F_1^2(0) \tag{25}$$

which reproduces the Thomson limit, $F_{1_n}(o) = 0$. From Eq. (22a) *)

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*) Equation (26) is not inconsistent, as it seems at the first glance. On the left-hand side we take $\lim_{\nu\to 0} T_1(\nu, q^2=0)$ as in Eq. (25), whereas the meaning of $T_1(0,0)$ in all our expressions is $\lim_{-q^2\to 0} T_1(0,q^2)$.

$$T_1(\nu, q^2=0) = T_1(0,0) - 2\mu_n^2 + O(\nu^2) \tag{26}$$

and then $T_1(0,0) = 2\mu_n^2$. In this way the apparent divergence in Eq. (23b) for $-q^2 \to 0$ is cancelled.

The determination of the two-photon exchange contribution, as seen from Eqs. (23), requires the knowledge of the three quantities $T_1(0,q^2)$, $W_2(\nu,q^2)$ and $R(\nu,q^2)$. The last two quantities are measurable in inelastic electron neutron scattering. However, the data being insufficient, for their determination (see next Section) we rely on "reasonable" or popular assumptions which are so far in agreement with the experimental findings. The subtraction function $T_1(0,q^2)$, as mentioned before, is not directly measured. However, it can be reasonably determined as follows. We construct the amplitude

$$T_R(\nu,q^2) = \left(1 - \frac{\nu^2}{q^2}\right) T_2(\nu,q^2) - \left(1 + R_\infty(q^2)\right) T_1(\nu,q^2) \tag{27}$$

$$R_\infty(q^2) \equiv \lim_{\nu\to\infty} R(\nu,q^2)$$

In the limit $\nu \to \infty$, the absorptive part of $T_R(\nu,q^2)$ tends to zero. We assume, as discussed in Refs. 13), 14) and 15), that $T_R(\nu,q^2)$ satisfies an unsubtracted dispersion relation *). With this assumption the subtraction function $T_1(0,q^2)$ is found to be

$$T_1(0,q^2) - 2\mu_n^2 G_{Ep}^2(q^2) = \frac{2q^2}{1+R_\infty(q^2)} \frac{\mu_n^2}{4M^2} G_{Ep}^2(q^2) + 4\int_{\nu_{th}(q^2)}^{\infty} \frac{d\nu}{\nu} W_2(\nu,q^2) \frac{1}{1+R(\nu,q^2)} \left\{1 - \frac{\nu^2}{q^2} \frac{R_\infty(q^2) - R(\nu,q^2)}{1+R_\infty(q^2)}\right\} \tag{28}$$

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*) The assumption here is that the possible presence of fixed poles (in our case for the neutron) does not alter this picture. In real Compton scattering, the fixed pole seems to be given 16) by the Thompson limit which is zero for the neutron. See also, however, Ref. 17), where the dependence of the result on the cut used in the finite energy sum rule is indicated.

Using the properties $W_2(\nu,q^2) \sim O(q^2)$ and $R(\nu,q^2) \sim O(q^2)$ as q^2 tends to zero we find again $T_1(0,0) = 2\mu_n^2$.

The expression for $T_1(0,q^2)$, Eq. (28), together with Eqs. (23) constitute our main results from which we can compute the two-photon exchange amplitude in terms of the quantities $W_2(\nu,q^2)$ and $R(\nu,q^2)$ measured in inelastic electron neutron scattering.

4. INELASTIC ELECTRON NEUTRON FUNCTIONS

In this section we discuss how the functions $W_2(\nu,q^2)$ and $R(\nu,q^2)$ can be reasonably determined. We begin by noting that precisely the same two amplitudes considered by us [$T_i(\nu q,^2)$, $i = 1,2$] also enter in the problem of nucleon self masses. For a recent review of this problem the reader may consult the excellent article by Zee [18]. Here we only mention that in general the expression for the electromagnetic neutron-proton mass difference diverges quadratically. One introduces the longitudinal amplitude

$$T_L(\nu,q^2) = \left(1 - \frac{\nu^2}{q^2}\right) T_2(\nu,q^2) - T_1(\nu,q^2) \qquad (29)$$

which is the special case of the amplitude $T_R(\nu,q^2)$, Eq. (27), corresponding to $R_\infty(q^2) = 0$. By introducing the assumptions that $R_\infty(q^2) = 0$ and $T_L(\nu,q^2)$ satisfies an unsubtracted dispersion relation *), the quadratic divergence disappears [19] leaving a logarithmically divergent mass shift. Further, the logarithmic divergence is rendered finite [21] by the extra assumption $R \underset{-q^2\to\infty}{\sim} \frac{1}{2}(-q^2/\nu^2)$. It is interesting to examine the implications of these assumptions for our problem.

According to these discussions we will consider the following two sets of assumptions: i) $T_L(\nu,q^2)$ satisfies an unsubtracted dispersion relation and $(\sigma - \nu^2/q^2)W_2(\nu,q^2) - W_1(\nu,q^2) = 0$ which corresponds to

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*) There are models for which the fixed poles in $T_1(\nu,q^2)$ and $T_2(\nu,q^2)$ exactly cancel in the combination (29) [see Ref. 20].

$$R(\nu, q^2) = \frac{(1-\sigma)(-q^2)}{\sigma(-q^2) + \nu^2} \xrightarrow[-q^2\to\infty]{} (1-\sigma)\frac{-q^2}{\nu^2} \qquad (30)$$

This assumption is consistent with the quark light cone model where $\nu W_L(\nu, q^2)$ scales [22] and R is of the form $(-q^2/\nu^2)f(\omega)$, $\omega = 2M\nu/-q^2$ being the scaling variable. With σ taken to be a constant, the positivity requirement for R gives $-0.32 \leq \sigma \leq 1$. The upper limit for σ is rigorous and corresponds to $R = 0$. The lower limit is given by the maximum value of ν^2/q^2 in the integration region. The choice $\sigma = \frac{1}{2}$ gives finite mass shift for nucleons [21]. The second possibility which we consider is ii) $R_\infty(q^2) \neq 0$, as suggested from a recent analysis by Sakurai [23] using ρ electroproduction data at $-q^2 \simeq 1\ \mathrm{GeV}^2$ where $R(\nu, q^2)$ is found to be independent of ν within the experimental errors. In this case we shall assume that $R(\nu, q^2)$ is a slowly increasing function of $-q^2$ [24] given by the simple parametrization [25]

$$R(\nu, q^2) \equiv R(q^2) = \frac{-q^2}{M^2 - 5q^2} \qquad (31)$$

For the first case i) we find from Eqs. (28), (30)

$$T_1(0, q^2) - 2\mu_n^2 G_{E_p}^2(q^2) = \frac{q^2}{2M^2}\mu_n^2 G_{E_p}^2(q^2) + 4\sigma\int_{\nu_{th}(q^2)}^{\infty}\frac{d\nu}{\nu} W_2(\nu, q^2) \qquad (32)$$

and we write the amplitude, up to the two-photon exchange contribution, in the form

$$a = a^{(1\gamma)}\left[1 + \delta_B + \delta_E + \delta_c\right] \qquad (33)$$

Here δ_B corresponds to the Born term [Eq. (23a)], δ_E comes from the first term in the right-hand side of Eq. (32) and δ_c is the sum of all continuum contributions. It turns out that, in terms of σ, we can write

$$\delta_c(\sigma) = \sigma\, \delta_c(\sigma = 1) + (1-\sigma)\, \delta_c(\sigma = 0) \tag{34}$$

For the case ii) we obtain

$$T_1(0,q^2) - 2\mu_n^2 G_{E_p}^2(q^2) = \frac{1}{1+R(q^2)} \left\{ \frac{q^2}{2M^2} \mu_n^2 G_{E_p}^2(q^2) + 4 \int_{\nu_{th}(q^2)}^{\infty} \frac{d\nu}{\nu} W_2(\nu,q^2) \right\} \tag{35}$$

and our amplitude is of the form

$$a = a^{(1\gamma)} \left[1 + \delta_B + \delta'_E + \delta'_c \right] \tag{36}$$

where the meaning of the different terms are analogous to the previous case.

The δ's in Eqs. (33) and (36) are of the form

$$\delta_B = \frac{\alpha}{\pi}\, 3\, \frac{\mu_n}{2M}\, \frac{m M^2}{M^2 - m^2} \int_{-\infty}^{0} dq^2\, F_B(q^2) \tag{37a}$$

$$\begin{Bmatrix} \delta_E \\ \delta'_E \end{Bmatrix} = -\frac{\alpha}{\pi}\, 3\, \frac{\mu_n}{4M}\, m \int_{-\infty}^{0} dq^2 \begin{Bmatrix} F_E(q^2) \\ F'_E(q^2) \end{Bmatrix} \tag{37b}$$

$$\begin{Bmatrix} \delta_c \\ \delta'_c \end{Bmatrix} = -\frac{\alpha_n}{\pi}\, \frac{2M}{\mu_n}\, \frac{5}{4}\, M m \int_{-\infty}^{0} dq^2 \begin{Bmatrix} F_c(q^2) \\ F'_c(q^2) \end{Bmatrix} \tag{37c}$$

The F's can be easily found from Eqs. (23), (32) and (35). The factor α_n in Eq. (37c), which is related to the behaviour of the integrand as $-q^2 \to 0$, is defined by

$$\alpha_n = \frac{1}{2\pi^2} \int_{\nu_{th}(0)}^{\infty} d\nu\, \frac{\sigma_t(\nu)}{\nu^2} \tag{38}$$

and has been introduced for convenience. This quantity corresponds to the sum of electric and magnetic polarizabilities in real Compton scattering: $\sigma_t(\nu)$ is the total photoabsorption cross-section at photon laboratory energy ν. The value of α_n has been calculated [6] from the analysis of deuterium data [16] to be $\alpha_n = 1.4 \times 10^{-3} \mathrm{fm}^3$.

For $\delta_c(\delta_c')$, where an explicit knowledge of $W_2(\nu, q^2)$ is needed, it is interesting to discuss the behaviour of $F_c(q^2)$ $[F_c'(q^2)]$ in three different regions of q^2: $-q^2 \ll 4m^2$; $4m^2 \ll -q^2 \ll \mu^2$; $-q^2 \gg \mu^2$. To this end we use real photoproduction results in the first and second regions, and the scaling hypothesis [8], confirmed by experimental results [26], in the last one. This is presented in Table I, where the parameter

$$\Lambda_c^4 = 4M \frac{\alpha}{\alpha_n} \int_1^\infty \frac{d\omega}{\omega^2} F_2(\omega) \tag{39}$$

has been introduced. From the experimental values of this integral [26),27)] we obtain $\Lambda_c^2 = 0.14 \text{ GeV}^2$. It is clear that the second region has no sense in the muon case, and it should be omitted. Precisely this is the zone which is expected to dominate the integral (37c) in the electron case. But from Table I we make two observations:

1) the electron mass cannot be neglected because otherwise a logarithmic infra-red divergence is introduced, and

2) we may not neglect the structure effects for $(-q^2) \to \infty$, i.e., take $\Lambda_c \to \infty$, since in this case an ultraviolet divergence appears.

Further, Table I shows that our results are sensitive to the specific assumptions i) and ii). In the second case $[1 + R(q^2)]^{-1}$ in Eq. (35) is always very near to 1, and a cancellation mechanism between the continuum contribution coming from the subtraction function and the leading behaviour of T(disp) [Eq. (23c)] is operating. This cancellation is not effective in the case i) if $\sigma \sim 0$ [Eq. (32)]. The reason for this marked difference between the two cases is that the determination of the subtraction function given by Eq. (28) depends crucially on the behaviour of $R(\nu, q^2)$ in the Regge region. Clearly this behaviour is quite different for the two cases considered here. For $R = 0$ ($\sigma = 1$) both results coincide.

As the inelastic excitations for virtual photons must be taken into account to get convergent results, we need the knowledge of the neutron structure function $W_2(\nu, q^2)$ between the zones of real photoproduction and scaling. We are going to use the duality results of Rittenberg and Rubinstein [Refs. 28), 29), 30)] who extend scaling to all values of q^2 in a suitable variable $\omega_W = (2M\nu + B^2)/(-q^2 + A^2)$, A and B being constants. In terms of the scaling function $F_2(\omega_W)$ we write

$$W_2(\nu, q^2) = \frac{-q^2}{2\nu^2}\, \omega_W\, F_2(\omega_W) \tag{40}$$

which has explicitly the correct kinematical behaviour as $-q^2 \to 0$.

For the neutron case we take the simple parametrization compatible with present results [26)]

$$F_2^{(n)}(x) = \left(1 - \frac{x}{2}\right)^2 F_2^{(p)}(x) \tag{41}$$

with $x = \omega_W^{-1}$ and

$$F_2^{(p)}(x) = \sum_{n=3}^{7} c_n (1-x)^n \tag{42}$$

where the c_n values and parameters A^2 and B^2 in the variable ω_W are taken from Ref. 29).

We have checked that this parametrization gives the correct behaviour for large $-q^2$ and the value of Λ_c^4 in Eq. (39) is numerically reproduced. However, the zone of real photoproduction, and the value of α_n [Eq. (38)], is overestimated by 20%. This last "anomaly" will be taken into account in the presentation of numerical results, given in the next Section.

In order to see in each case the important region of integration of Eqs. (37) we have studied the integrands in terms of the variable $v = \sqrt{-q^2}/(\gamma + \sqrt{-q^2})$ $(0 \leq v \leq 1)$, by putting [*)]

---

*) Note that the functional form of F in the two sides of Eq. (43) is not the same.

$$\int_{-\infty}^{0} dq^2\, F(q^2) = \int_{0}^{1} dv\, F(v) \qquad (43)$$

and choosing γ^2 in order to have significant numbers for $F(v)$ in the entire region of integration. We have found that a good choice corresponds to $\gamma_e^2 = 10^{-4}$ GeV^2 for electrons and $\gamma_\mu^2 = m_\mu^2$ for muons, indicating the zone of $-q^2$ dominating the integration.

5. RESULTS AND DISCUSSION

We begin by discussing the behaviour of the functions $F(v)$ in Eq. (43) as function of $v = \sqrt{-q^2}/(\gamma + \sqrt{-q^2})$. Figure 5 shows our results for the Born contribution, $F_B(v)$, and $F_E(v)$ [case i) of Section 4] and $F'_E(v)$ [case ii)] for electrons. The leading behaviour around $v \sim 0,5$ $(-q^2 \sim \gamma_e^2)$ is always of the form $F(v) \sim 1/v(1-v)$. Clearly, the parameters m_e^2 and Λ_B^2, in the vicinity of $v \sim 0$ and $v \sim 1$ respectively, cut off the logarithmic divergence and render convergent results. Thus, the dependence $\ln(\Lambda_B^2/m_e^2)$, as discussed in Section 2 for $m \to 0$, appears naturally in this context. However, in the case of muons, Fig. 6, no trace of this logarithmic behaviour is present. Instead, $F(v) \sim (1-v)^{-2}$ for $-q^2 \ll \Lambda_B^2$, which is the expected behaviour for a large mass m. It is precisely in this limit that one reproduces the linear dependence of Eq. (16). The integration of $F(v)$ allows us to calculate δ_B, δ_E, δ'_E from Eqs. (37a), (37b), and the results are given on the first three lines of Table II.

The continuum contribution for the case i) is exhibited in Fig. 7 for electrons and in Fig. 8 for muons. Figures 9 and 10 give the corresponding contributions for the case ii). From these results, the discussion given in the last paragraph can now be repeated for $F_c(v)$. The major difference is that the relevant parameter is now Λ_c^2, Eq. (39) (apart from the mass) instead of Λ_B^2. $F_{c1}(v)$, which corresponds to $R = 0$ $(\sigma = 1)$, gives a connection between the cases i) and ii) and its value is the rigorous lower bound of the continuum contribution. From Eq. (37c) we obtain the values of δ_c presented on the last three lines of Table II. We see that with R given by Eq. (31) [in the case ii)] the result is very close to the lower bound. The parametrization (30) in the case i) gives a positive δ_c if $\sigma < 0.85$; with $\sigma \sim 0$, δ_c is of the order of several percent for muons. For any σ, Eq. (34) gives the result easily. In particular, we find the upper bound on δ_c (corresponding to σ attaining its minimum value) to be $\delta_c = 6.2 \times 10^{-4}$ for electrons and $\delta_c = 2.6 \times 10^{-2}$ for muons.

The transition to the classical treatment for the continuum contribution δ_c is governed by the condition $\Lambda_c/2m < 1$, completely similar to the discussion given for the Born contribution (see Section 2). For muons we are precisely in the transition zone. The classical description of the polarizability contributions is given in terms of the potential r^{-4} for static electric fields. In terms of the "electric" polarizability α_E, the contribution to δ_c can be written as $\delta_c^{(E)} = -\alpha_E(2M/\mu n)M R_{eff}^{-1}$ where R_{eff} is the effective cut-off for the r^{-4} potential. From comparison with our expressions, we identify $R_{eff} = 2\Lambda_c^{-1} \sim 1$ fm, and the simple description is valid for $m R_{eff} > 1$. It is interesting to point out that our "total" polarizability corrections δ_c become also positive in the classical region. With the case i), $\delta_c = -\alpha_n(2M/\mu n)(1-\sigma)R_{eff}^{-1}$, which is only a factor 1.6 larger than the "exact" result given in Table II for muons. In the case ii) the value of δ_c depends on the specific parametrization of $R(\nu, q^2) \equiv R(q^2)$. Equation (31) replaces $(1-\sigma)$ in the result given for the case i) by $(\Lambda_c/M + \sqrt{6}\,\Lambda_c)^2 \sim 0.04$, which is very small. For the rigorous lower bound, corresponding to $R(q^2) = 0$ $(\sigma = 1)$, in the classical limit we obtain $\delta_c = 0$ as expected, because classically only longitudinal photons contribute to the polarizability correction.

We conclude by giving a short summary of our results.

1) The polarizability contribution to the neutron-lepton amplitude at threshold is, with the present experimental accuracies, negligible for electrons, in agreement with the estimate given in Ref. 7). It can be of the order of several percent and positive for muons (see Table II). However, for finite lepton mass, a small negative contribution is obtained if the cross-section for longitudinal photons is practically zero for all values of q^2 and ν, mass squared and energy of the virtual photons.

2) The zone of low $-q^2$ gives the dominant contribution to the polarizability correction. This zone becomes larger as the lepton mass increases (compare Figs. 5-10 for electrons and muons).

3) The effective cut-off Λ_c describing the virtual inelastic excitations is given by the behaviour of the neutron structure function in Eq. (39) and its value is $\Lambda_c^2 = 0.14$ GeV^2, less than the elastic one, $\Lambda_B^2 = 0.71$ GeV^2, appearing in the Born contribution.

4) The transition from the extreme relativistic limit, with its characteristic logarithmic dependence $m \ln(\Lambda_c/m)$, to the classical treatment, for which the corresponding quantity is linear in Λ_c and independent of m, is governed by the value of $\Lambda_c/2m$.

5) The rigorous lower bound [given by the condition $R(\nu, q^2) = 0$] to the δ_c contribution, which is negative for finite lepton mass, goes to zero in the classical region, as expected from the fact that classically only longitudinal photons contribute. For muons (unlike electrons) the exact results approach the classical result where $\delta_c \geq 0$. In the case i) the two results differ only by a factor ~ 1.5.

6) The effective cut-off of the r^{-4} potential describing electric polarizability contributions when the field is considered to be static is given by $R_{eff} = 2\Lambda_c^{-1} \sim 1$ fm.

Finally, it is interesting to extend our considerations to the cases of bound systems of μ^- and proton or light nuclei, where the polarizability corrections, with the present experimental precision, may be significant. Such a study is now in progress.

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	$-q^2 \ll 4m^2$	$4m^2 \ll -q^2 \ll \mu^2$	$-q^2 \gg \mu^2$
i) F_c	$\frac{4(1-\sigma)}{5m\sqrt{-q^2}^{\,2}} - \frac{3}{10m^2}$	$\frac{1-6\sigma/5}{-q^2}$	$(1-6\sigma/5)\,\frac{\Lambda_c^4}{(-q^2)^3}$
ii) F'_c	$\frac{-3}{10m^2} + \frac{4R(q^2)}{5m\sqrt{-q^2}^{\,2}}$	$\frac{1}{5q^2}\,\frac{1-3R(q^2)}{1+R(q^2)}$	$\frac{\Lambda_c^4}{5(q^2)^3}\,\frac{1-3R(q^2)}{1+R(q^2)}$

TABLE I - The behaviour of the functions $F_c(q^2)$ and $F'_c(q^2)$ in Eq. (37c) under different assumptions (see the text).

		Electrons	Muons
	δ_B	-4.8×10^{-5}	-2.1×10^{-3}
i)	δ_E	$+2.4 \times 10^{-5}$	$+1.1 \times 10^{-3}$
ii)	δ'_E	$+2.4 \times 10^{-5}$	$+1.1 \times 10^{-3}$
i)	$\delta_c \, [\sigma = 0]$	$+ 4.5 \times 10^{-4}$	$+1.9 \times 10^{-2}$
	$R = 0 \; [\sigma = 1]$	-8.3×10^{-5}	-2.9×10^{-3}
ii)	δ'_c	-8.1×10^{-5}	-2.3×10^{-3}

TABLE II - Two-photon exchange corrections to the neutron-lepton amplitude at threshold. The cases i) and ii) are discussed in Section 4 of the text.

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FIGURE CAPTIONS

Figure 1 :

General two-photon exchange diagram contributing to the forward neutron-lepton scattering amplitude.

Figure 2 :

Two-photon exchange Born contribution.

Figure 3 :

The function $f(\Lambda, M, m)$ [Eq. (12)], as function of Λ, for electrons. The line $\Lambda = \Lambda_B = 0.84$ GeV corresponds to the physical result.

Figure 4 :

Same as Fig. 3, but for muons.

Figure 5 :

The function $F(v)$ (see text) for electrons. The corresponding values of $(-q^2)$ are given at the top. ——— Born contribution F_B; – – – – F_E, in the case i); –·–·– F'_E, in the case ii).

Figure 6 :

Same as Fig. 5, but for muons.

Figure 7 :

Continuum contribution $F_c(v)$ in the case i) for electrons. The line F_{c_2} corresponds to $\sigma = 0$ in Eq. (30); F_{c_1} with negative sign, is for $\sigma = 1$ [$R = 0$].

Figure 8 :

Same as Fig. 7, but for muons.

Figure 9 :

Continuum contribution $F_c(v)$ in the case ii) for electrons.
——— F'_c, corresponding to R as given by Eq. (31);
– – – F_{c_1} is for $R = 0$.

Figure 10 :

Same as Fig. 9, but for muons.

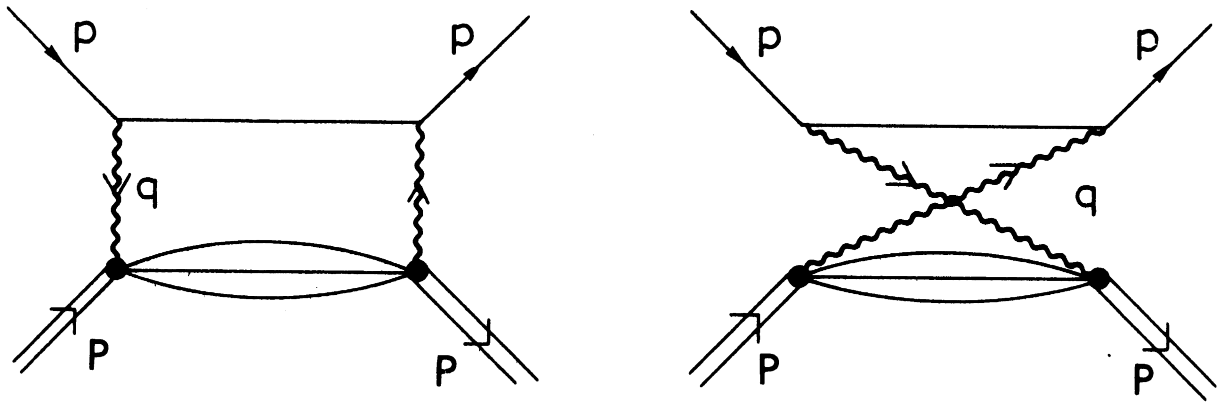


FIG. 1

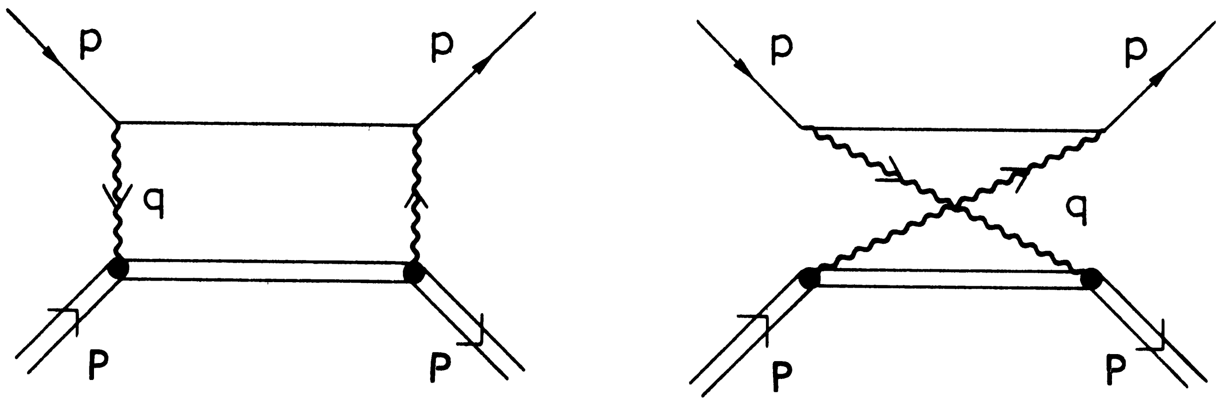


FIG. 2

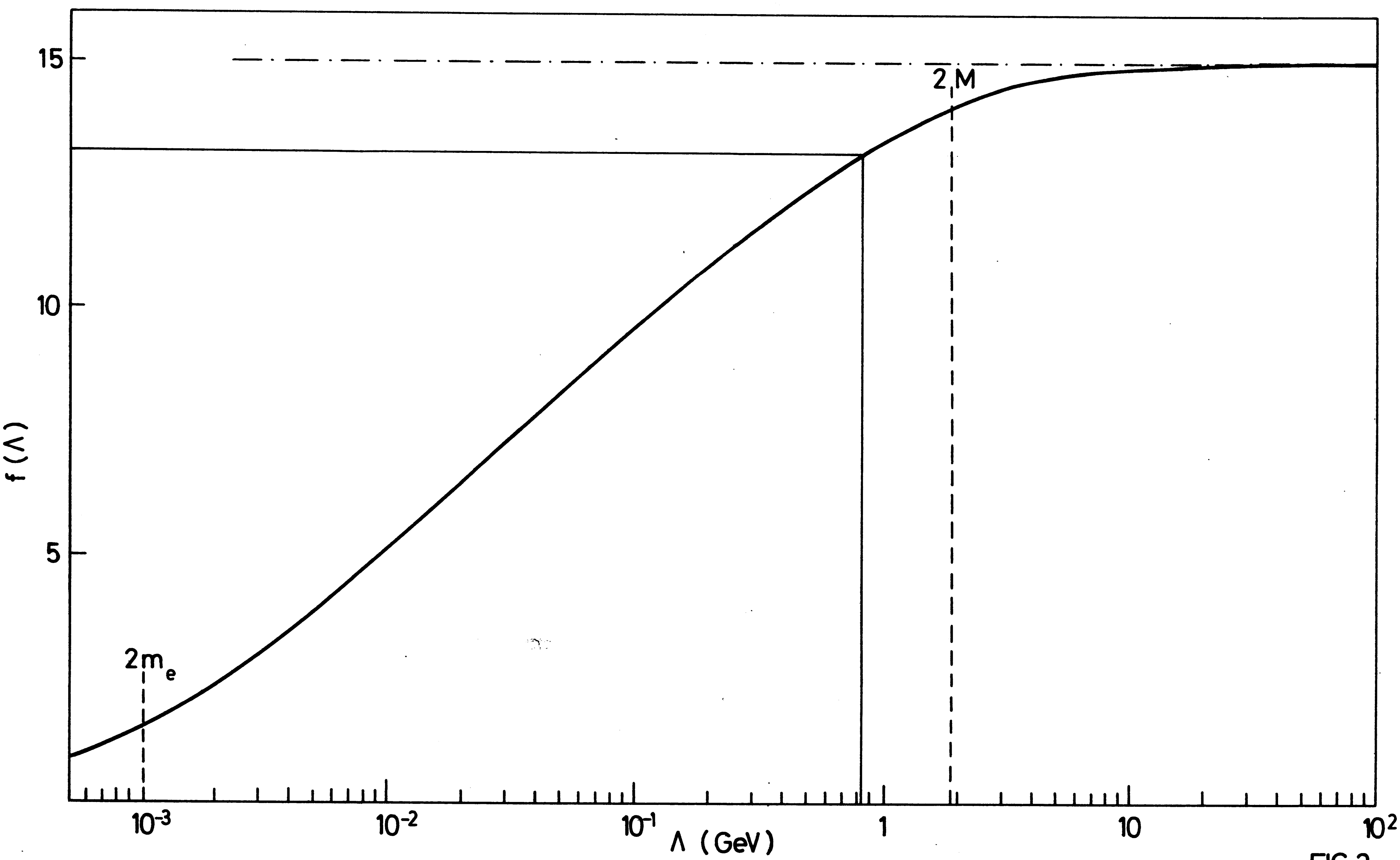


FIG.3

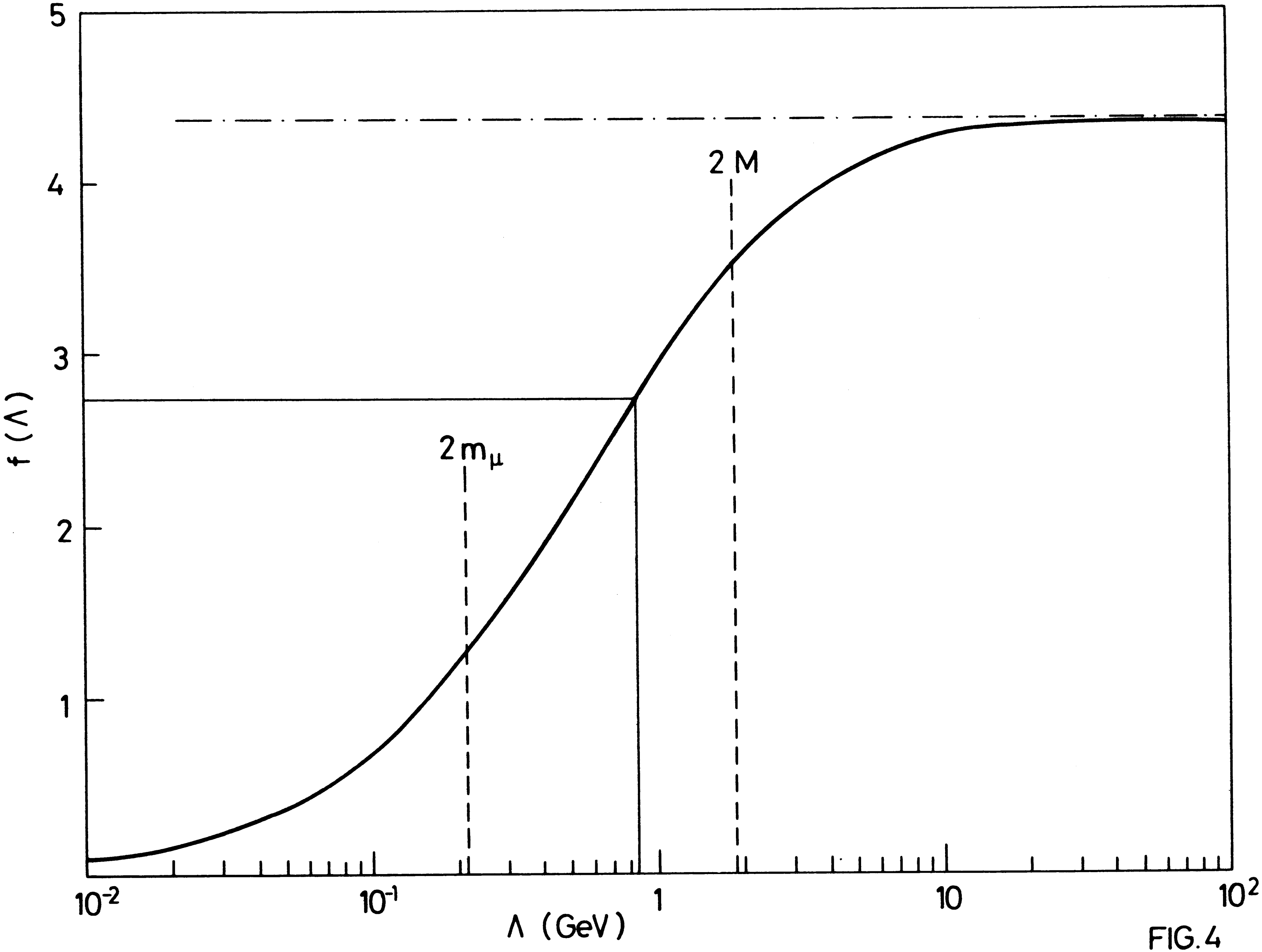
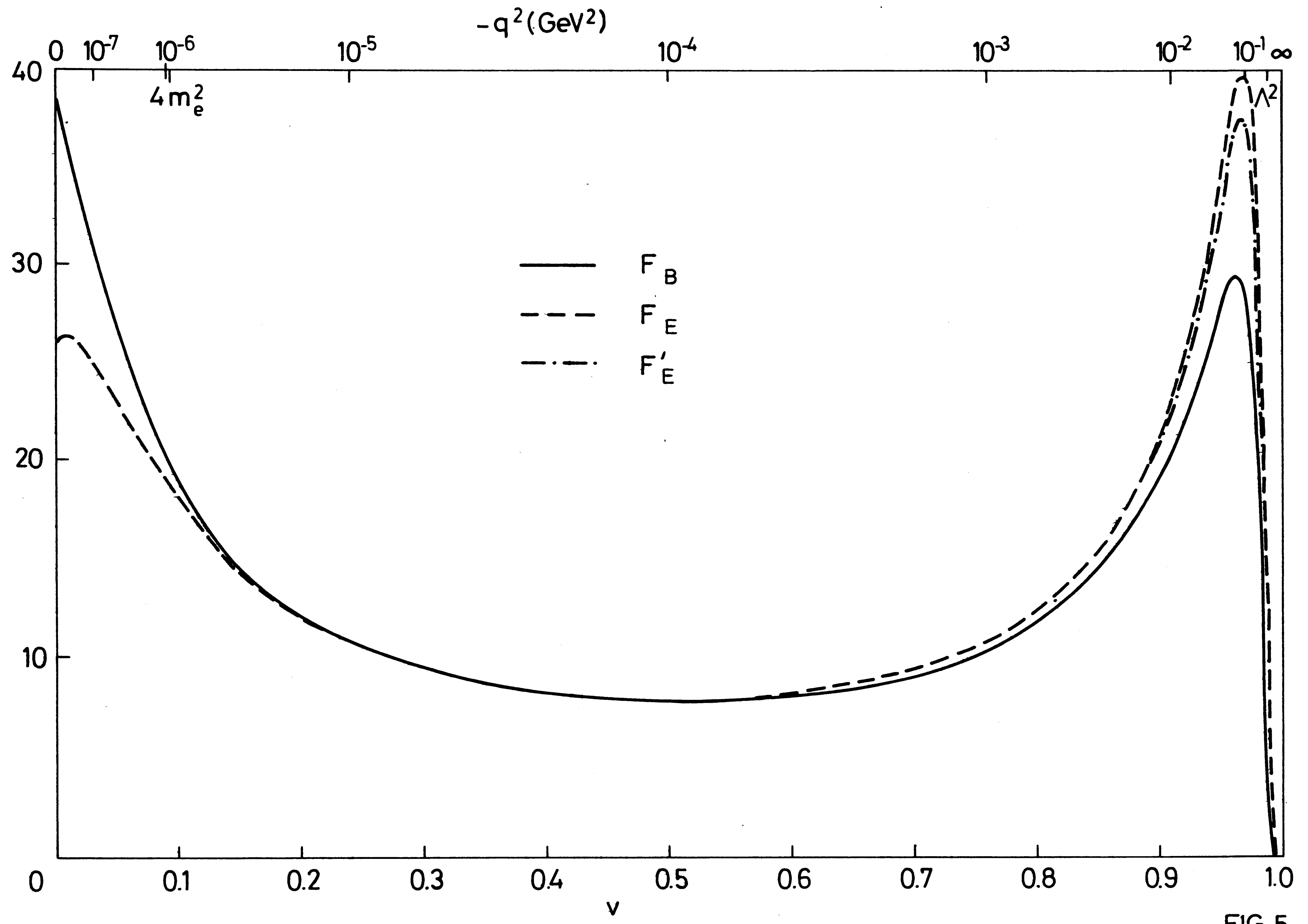
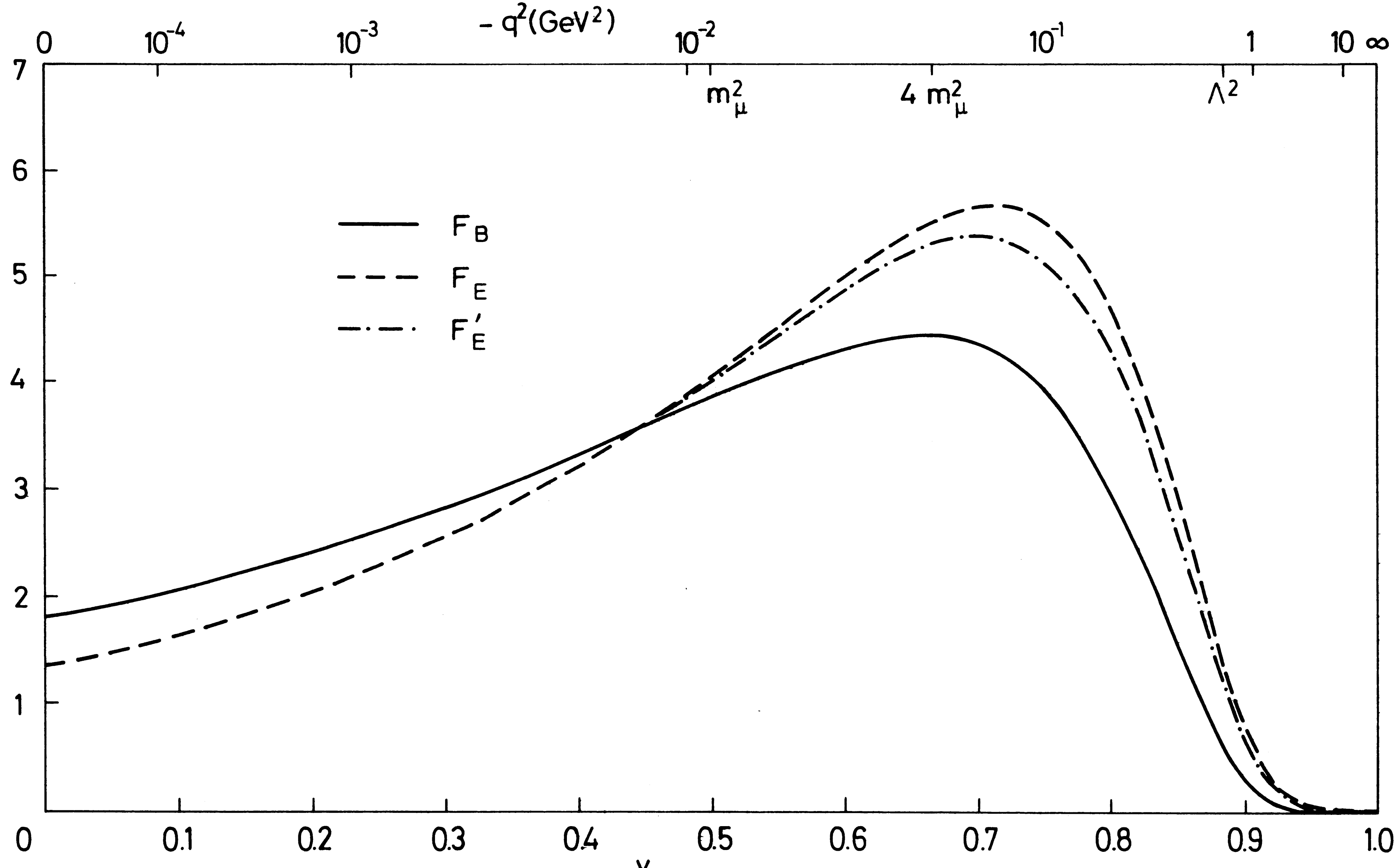


FIG. 4



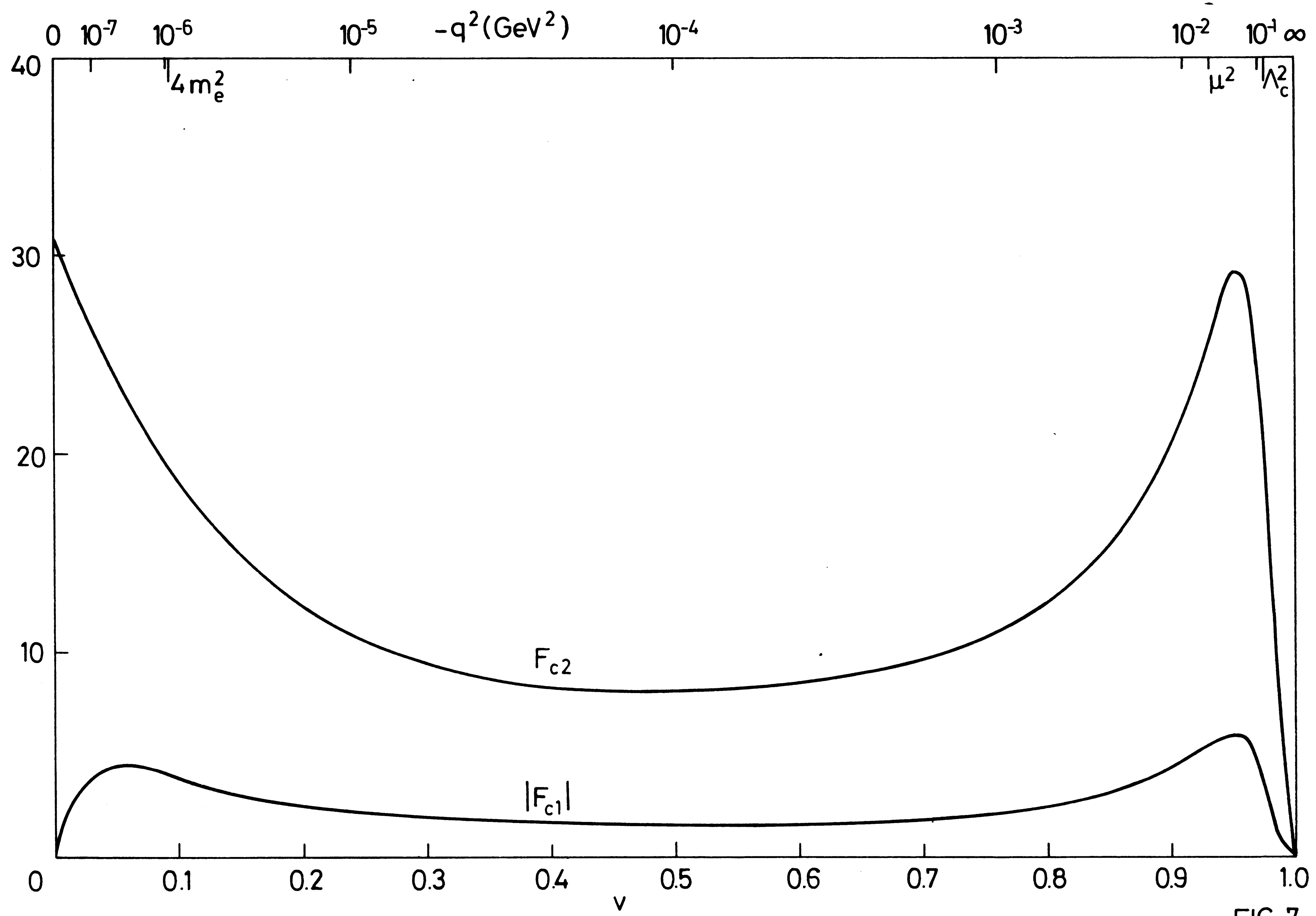
$-q^2$ (GeV^2)
0
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
∞
$4m_e^2$
Λ^2
40
30
20
10
0
F_B
F_E
F'_E
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG. 5



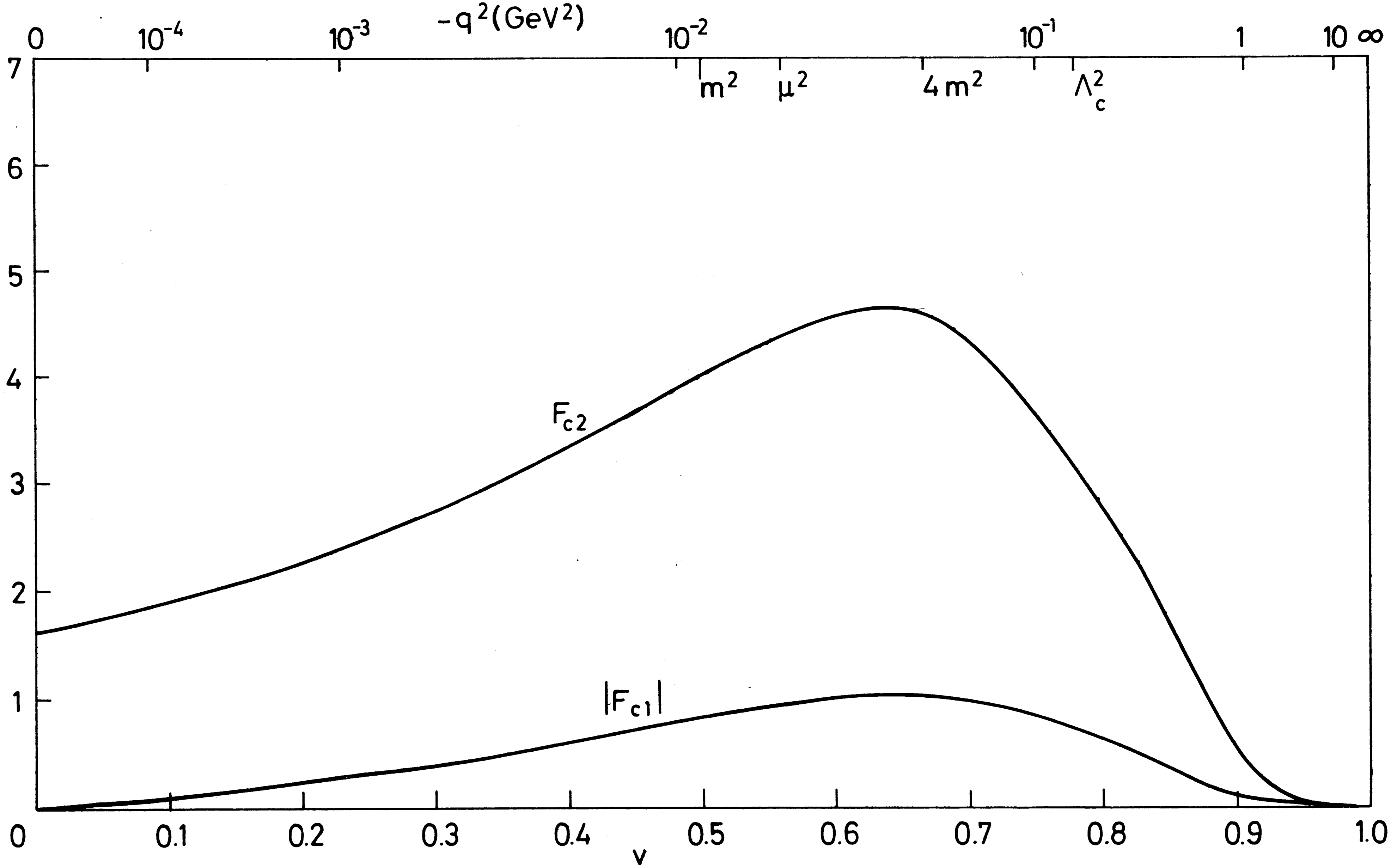
- q^2 (GeV^2)
0
10^-4
10^-3
10^-2
10^-1
1
10 ∞
m_μ^2
4 m_μ^2
Λ^2
F_B
F_E
F'_E
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG.6



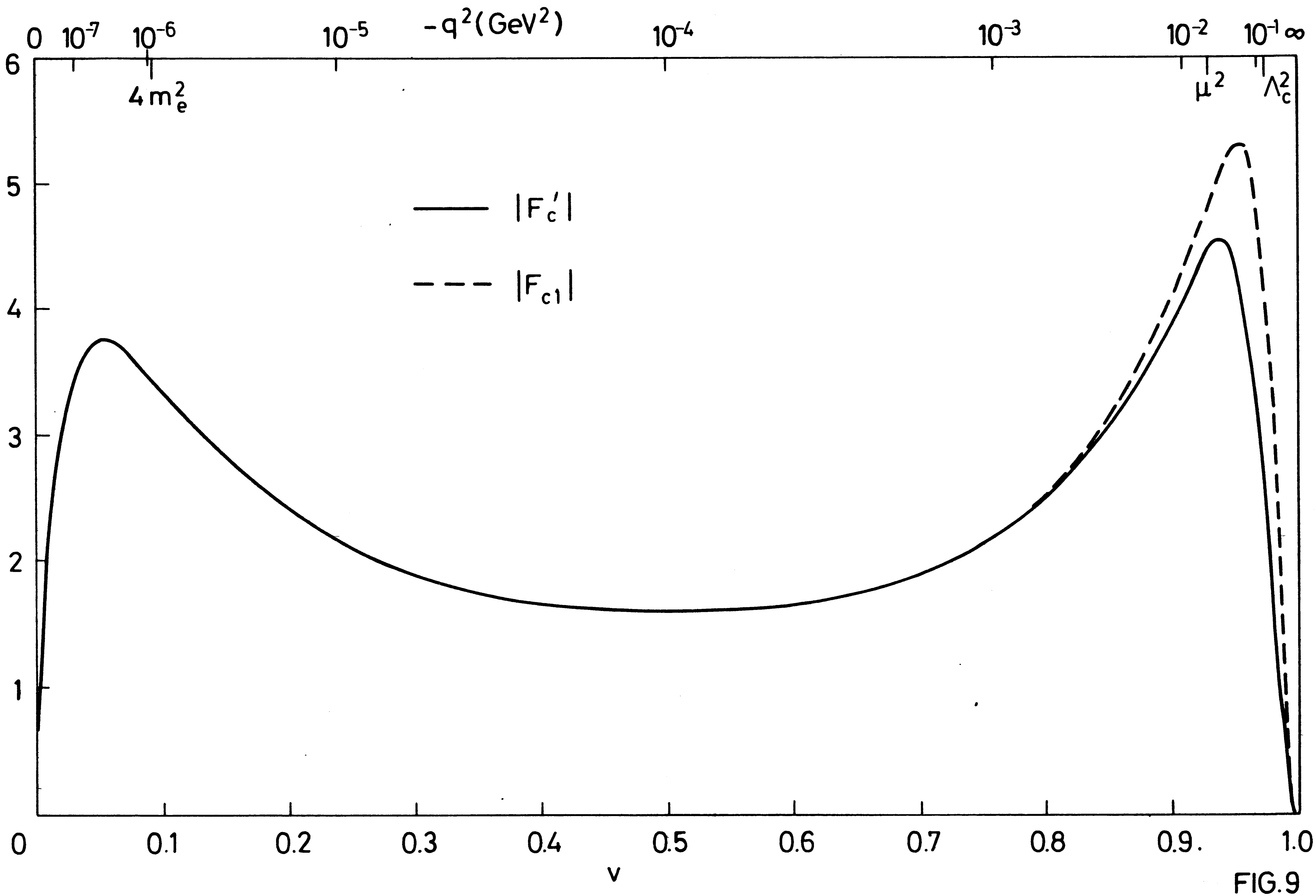
0
10-7
10-6
10-5
-q2 (GeV2)
10-4
10-3
10-2
10-1
∞
4 m2e
μ2
Λ2c
40
30
20
10
0
Fc2
|Fc1|
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG. 7



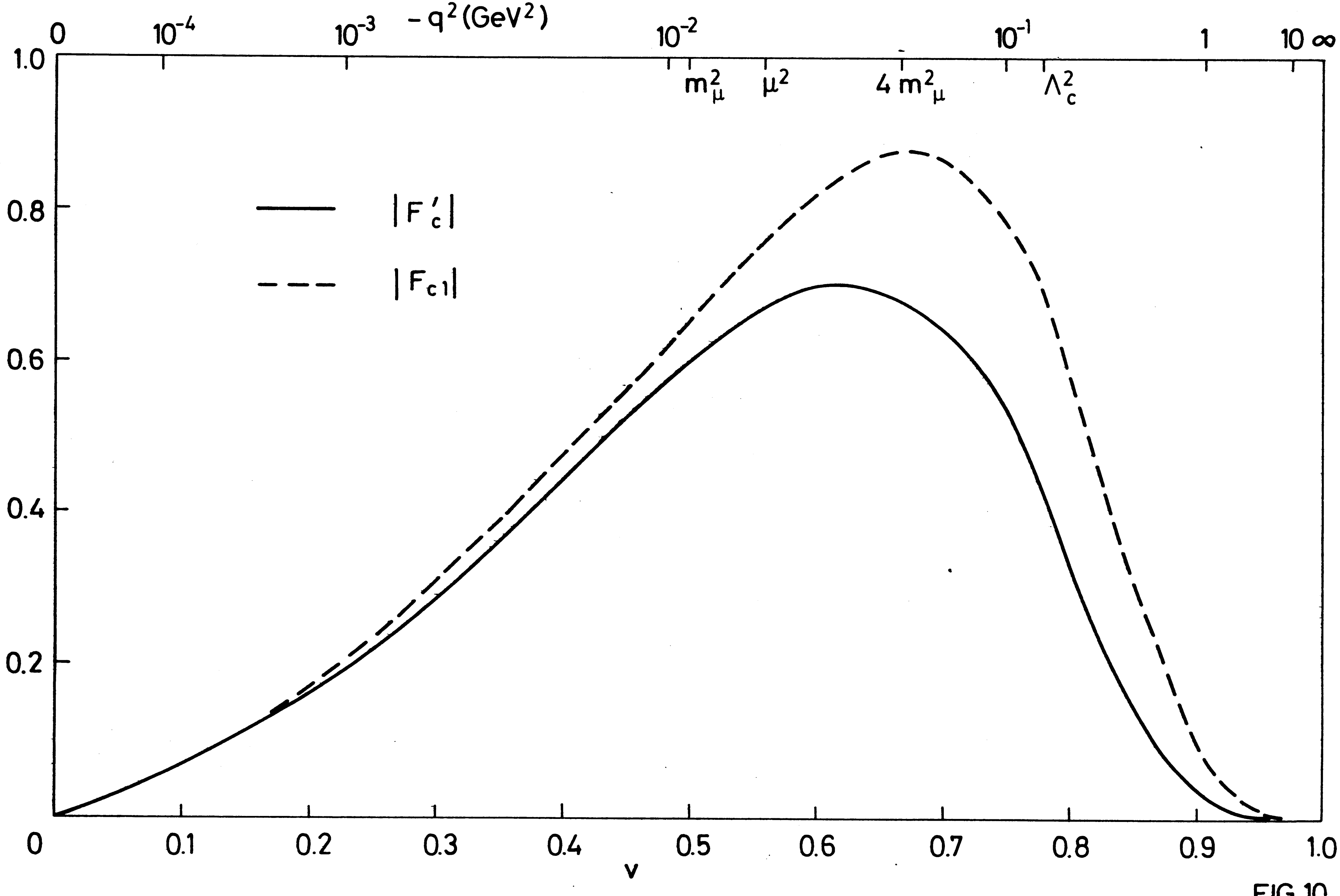
-q 2(GeV2)
0
10-4
10-3
10-2
10-1
1
10 ∞
m2
μ2
4 m2
Λ2c
7
6
5
4
3
2
1
0
Fc2
|Fc1|
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG. 8



0
10⁻⁷
10⁻⁶
10⁻⁵
$-q^2$ (GeV²)
10⁻⁴
10⁻³
10⁻²
10⁻¹
∞
$4m_e^2$
μ^2
Λ_c^2
$|F_c'|$
$|F_{c1}|$
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG.9



$-q^2$ (GeV^2)
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
∞
m_μ^2
μ^2
$4\,m_\mu^2$
Λ_c^2
1.0
0.8
0.6
0.4
0.2
$|F'_c|$
$|F_{c1}|$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v

FIG.10