Ref.TH.1752-CERN

CHARGE AND CURRENT DISTRIBUTIONS IN ELASTIC ELECTRON SCATTERING BY 1p SHELL NUCLEI

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A B S T R A C T

We study the charge and magnetic form factors appearing in elastic electron scattering by 1p shell nuclei. The question that the form factors may be obtained from simple nuclear models by simply introducing a scaling factor has been examined using the j-j coupling, the L-S coupling and the intermediate one of Cohen-Kurath resulting from effective interactions. Results for ^{6}Li, ^{7}Li, ^{9}Be and ^{13}C are given and the q^2 dependences of their form factors are compared in the three models and with experiment. The C-K scheme gives similar results to the L-S coupling for ^{6}Li and ^{7}Li, in agreement with experiment, whereas it is intermediate between j-j and L-S couplings in ^{9}Be and ^{13}C.

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Ref.TH.1752-CERN
25 September 1973

1. - INTRODUCTION

The fact that electrons interact with nuclei through a known and relatively weak interaction, together with the space-like character of the four momentum transfer q^2 (in contrast to the light-like q^2 in processes with real photons), has been extensively used in the past as a powerful tool for studying nuclear structure [1].

In particular, one aspect of these processes, which has been repeatedly pointed out and has recently been given considerable attention both from the experimental and theoretical points of view, is the elastic magnetic scattering which provides a picture of the nuclear ground state current and magnetization distributions [see Ref. 2) for a lucid exposition of the problem].

Early backward experiments on 1p shell nuclei [3] were interpreted by Griffy and Yu [4] in terms of a scaling of the form factors of highest multipolarity. This fact has been recently explained by Donnelly and Walecka [2] in their study of magnetic multipoles. The question that the form factors may be obtained from (the ones calculated with) simple nuclear models by simply introducing a scaling factor, has gained new interest for the unified description of nuclear electromagnetic and semi-leptonic weak processes. It seems then important to have some more detailed investigation of the scaling properties of the form factors.

In a previous paper [5] this study was carried out in the case of ^{11}B using the shell model wave function with configuration mixing of Cohen and Kurath [6]. It would be interesting to extend this kind of analysis to other nuclei in the 1p shell in order to make a systematic comparison of the configuration mixing effects on the different form factors. Experiments on these nuclei are being performed systematically at IKO, where data on ^{7}Li have been recently published [7], new data from ^{9}Be are already obtained [8], and furthermore they plan to measure ^{6}Li and ^{13}C, with improved accuracy.

In this paper such an analysis is performed for these nuclei, and the results of the intermediate coupling scheme are compared with the extreme cases of j-j and L-S couplings. In Section 2, general formulae are given in terms of invariant form factors [9], a formalism which is well suited for the unified study of electromagnetic and weak nuclear processes without any assumption on the effective pseudopotential or the nuclear physics problem. The single impulse approximation is used to express the form factors in terms of reduced nuclear matrix elements, which are calculated with the effective interaction results in the 1p shell. In Section 3, results are given for ^{6}Li, ^{7}Li, ^{9}Be and ^{13}C, and the effect of the different coupling schemes on the form factors is emphasized in order to show the possible scaling from one case to the other. In Section 4, some conclusions are drawn.

2. - THEORY

2.1. General formulae

In the one-photon exchange approach, the nuclear vertex (matrix element of the hadronic electromagnetic current) can be expanded in terms of invariant nuclear form factors [9]. The differential cross-section for elastic electron scattering from a nucleus of spin J and mass M is given by

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_M}{8M^2(2J+1)} \left[2\, A(q^2) + \frac{1+\sin^2 \theta/2}{\cos^2 \frac{\theta}{2}}\, F(q^2) \right] \qquad (1)$$

where σ_M is the Mott cross-section and $A(q^2)$, $F(q^2)$ stand for the following combinations of the invariant form factors $A^{(L)}(q^2)$ and $F^{(\ell,L)}(q^2)$

$$A(q^2) = \sum_L \left| A^{(L)}(q^2) \right|^2$$
$$F(q^2) = \sum_{\ell,L} \left| F^{(\ell,L)}(q^2) \right|^2 \qquad (2)$$

In Eq. (2) the sums run over values of L $(0 \leq L \leq 2J)$ and ℓ $(|L-1| \leq \ell \leq L+1)$ compatible with the appropriate selection rules, which, in our case are : in $A(q^2)$ L must be even and in $F(q^2)$ $\ell = L$ must be odd.

In the backward direction, Eq. (1) leads to

$$\left.\frac{d\sigma}{d\Omega}\right|_{back} = \frac{\alpha^2}{4M^2(2J+1)} \, \frac{F(q^2)}{-q^2} \qquad (3)$$

and only magnetic form factor contributions appear.

In the impulse approximation the form factors are expressed in terms of nuclear matrix elements. As we are going to use non-relativistic wave functions for the composite system we keep only terms of first order in the internal momentum distribution, compared to the nucleon mass m. Then we obtain

$$|A^{(L)}(q^2)| = 2M\sqrt{\pi}\; G(q^2)\; |[Y_L]|$$

$$|F^{(L,L)}(q^2)| = 2M\sqrt{\pi}\; G(q^2)\,\frac{|\vec{q}|}{2m}\left|\sqrt{\frac{L+1}{2L+1}}\,[\mathcal{Y}_L^{(L-1)}] - \sqrt{\frac{L}{2L+1}}\,[\mathcal{Y}_L^{(L+1)}] - 2[\mathcal{D}_L^{(L)}]\right| \tag{4}$$

where $|\vec{q}|^2 \approx -q^2$ and, for nuclei with isospin T and $T_3 = -T$ in the ground state $|a>$,

$$[Y_L] = \frac{1}{\sqrt{2T+1}}\left\{\langle a\|\sum_{i=1}^{A} j_L(|\vec{q}|r_i)\,Y_L(\hat{x}_i)\|a\rangle - \sqrt{\frac{T}{T+1}}\,\langle a\|\sum_{i=1}^{A}\tau_i\, j_L(|\vec{q}|r_i)\,Y_L(\hat{x}_i)\|a\rangle\right\}$$

$$[\mathcal{Y}_L^{(\ell)}] = \frac{1}{\sqrt{2T+1}}\left\{(1+\mu_p+\mu_n)\langle a\|\sum_{i=1}^{A} j_\ell(|\vec{q}|r_i)\,\mathcal{Y}_L^{(\ell)}(\hat{x}_i)\|a\rangle - \sqrt{\frac{T}{T+1}}\,(1+\mu_p-\mu_n)\langle a\|\sum_{i=1}^{A}\tau_i\, j_\ell(|\vec{q}|r_i)\,\mathcal{Y}_L^{(\ell)}(\hat{x}_i)\|a\rangle\right\} \tag{5}$$

$$[\mathcal{D}_L^{(\ell)}] = \frac{1}{\sqrt{2T+1}}\,\frac{1}{|\vec{q}|}\left\{\langle a\|\sum_{i=1}^{A} j_\ell(|\vec{q}|r_i)\,\mathcal{D}_L^{(\ell)}(\hat{x}_i)\|a\rangle - \sqrt{\frac{T}{T+1}}\,\langle a\|\sum_{i=1}^{A}\tau_i\, j_\ell(|\vec{q}|r_i)\,\mathcal{D}_L^{(\ell)}(\hat{x}_i)\|a\rangle\right\}$$

In Eqs. (4) and (5) the scaling law has been used for the magnetic form factors of proton and neutron

$$G_M^{(p)}(q^2) = (1+\mu_p)\; G(q^2)$$
$$G_M^{(n)}(q^2) = \mu_n\; G(q^2) \tag{6}$$

where $G(q^2)$ is the electric form factor of the proton, for which we shall take the dipole fit.

The irreducible tensors appearing in Eq. (5) are defined as

$$\mathcal{Y}^{(\ell)}_{LM} = \sum_m C(\ell\, 1\, L \mid M-m, m)\, Y_\ell^{M-m}\, \sigma_m$$
$$\mathcal{D}^{(\ell)}_{LM} = \sum_m C(\ell\, 1\, L \mid M-m, m)\, Y_\ell^{M-m}\, \nabla_m \qquad (7)$$

and Y_ℓ are ordinary spherical harmonics, σ the spin matrices and ∇ the gradient operators. Their reduced matrix elements are taken with respect to the direct product $O(3) \times SU(2)_T$.

From these expressions it becomes clear that the three terms in Eq. (5) correspond, respectively, to the multipole expansion of the charge density, spin and convection contributions of the nuclear current density.

2.2. Nuclear matrix elements

We are going to study 1p shell nuclei in which we have $n = A-4$ nucleons outside a core of four nucleons with spin-isospin zero. If we denote by $f(k^J, k^T)$ any of the operators which enter the expression of the form factors the quantities we are interested in are

$$f_A^{(J,T)}(k^J, k^T) = f_c + f_n^{(J,T)}(k^J, k^T) \qquad (8)$$

where f_c stands for the core contribution

$$f_c = \delta_{k^J,0}\, \delta_{k^T,0}\, \frac{2}{\sqrt{\pi}} \sqrt{(2J+1)(2T+1)}\, \langle j_0 \rangle_s \qquad (9)$$

with $\langle j_0 \rangle_s = e^{-1/4(b|\vec{q}|)^2}$ if harmonic oscillator radial wave functions are used.

The quantity

$$f_n^{(J,T)}(k^J, k^T) \equiv \langle a(J,T) \| \sum_{i=1}^{n} f_i(k^J, k^T) \| a(J,T) \rangle$$

can be expressed as a function of single nucleon matrix elements, by using the coefficients of fractional parentage $< JT \{| J_o T_o \alpha_o , j >$ for separating a nucleon with angular momentum j

$$f_n^{(J,T)}(k^J, k^T) = n\,(2J+1)(2T+1) \sum_{j,j'=1/2}^{3/2} S_{jj'}^{(J,T)}(k^J, k^T) \langle j \tfrac{1}{2} \| f(k^J, k^T) \| j' \tfrac{1}{2} \rangle \tag{10}$$

where

$$S_{jj'}^{(J,T)}(k^J, k^T) = \sum_{J_o T_o} \sum_{\alpha_o} \langle JT\{| J_o T_o \alpha_o , j \rangle \langle JT \{| J_o T_o \alpha_o , j' \rangle$$
$$\times\, W(j' k^J J_o J ; j J)\, W(\tfrac{1}{2}\, k^T T_o T ; \tfrac{1}{2} T) \tag{11}$$

It is easily verified that

$$S_{jj'}^{(J,T)}(k^J, k^T) = (-1)^{j-j'} S_{j'j}^{(J,T)}(k^J, k^T)$$

and the contribution of the sum in Eq. (10) is symmetric in j, j'.

The only model dependent quantities which enter these expressions are the fractional parentage coefficients, for which we shall take the values in the L-S coupling [10], j-j coupling [11] and the intermediate coupling scheme of Cohen and Kurath [12] using effective interactions in the 1p shell [6]. The single particle matrix elements appearing in Eq. (10) have been repeatedly given in the literature and they will not be reproduced here. For the radial wave functions we shall use the harmonic oscillator ones, and then the centre-of-mass correlation is simply included by multiplying the form factors by $\exp \left\{ 1/4A(b|\vec{q}|)^2 \right\}$.

We would like to point out that from Eqs. (4) and (10) we already discover the following facts. As in the form factors $A^{(L)}(q^2)$ only one matrix element is effective, and we use the same radial wave function for both $1p_{\frac{3}{2}}$ and $1p_{\frac{1}{2}}$ nucleons, the conjectured scaling will be always satisfied in this framework. Furthermore, for $L = 0$ the result will be completely independent of the nuclear structure details.

The situation is different for the magnetic form factors. In general, the values of $F^{(L,L)}(q^2)$ depend on three matrix elements with different q^2 dependence and the results will not scale when we go from one model to the other. However, in the 1p shell $F^{(3,3)}(q^2)$, if it exists, only gets contribution from the matrix element $[\mathcal{Y}^{(2)}{}_3]$ of the spin current and then scaling is restored. So, in our applications, only the form factor $F^{(1,1)}(q^2)$ can have different q^2 dependence when the model is changed. Of course, the quantity accessible experimentally in magnetic scattering is, from Eq. (3),

$$\mathcal{F}(q^2) = F(q^2) \;/\; F(0) \tag{12}$$

where

$$F(q^2) \xrightarrow[q^2 \to 0]{} 4 M^2 (2J+1) \frac{-q^2}{4m^2} \, 2 \, \frac{J+1}{3J} \, \mu^2$$

and μ is the static magnetic moment. $\mathcal{F}(q^2)$ will be compared in the three models we are going to use.

3. - APPLICATIONS

We apply the above formalism to the cases of ^6Li, ^7Li, ^9Be and ^{13}C. In Fig. 1 the charge form factor is given, independent of the model, for the following values of the oscillator parameter : $b(^6Li) = 1.98$ fm [13], $b(^7Li) = 1.90$ fm [7], $b(^9Be) = 1.76$ fm [14], $b(^{13}C) = 1.50$ fm [15]. In the range of q^2 we have studied, $|A^{(0)}|$ presents a minimum at $4.6\ fm^{-2}$ for ^6Li, $3.9\ fm^{-2}$ for ^9Be and $4\ fm^{-2}$ for ^{13}C. We indicate that for this and other form factors the plotted quantities are related to the ones entering Eq. (1) by

$$\text{Form factor} \equiv 2M\sqrt{2J+1}\,\{\text{Form factor}\}_{\text{plotted}}$$

With respect to the other form factors we are going to discuss each nucleus separately.

3.1. ^{6}Li

For this nucleus, $J^{\pi} = 1^{+}$, we have a form factor $|A^{(2)}(q^2)|$ different from zero in the j-j and intermediate coupling schemes. In Fig. 2 the result in this last case is given and the scale in the other models is j-j : C-K : L-S = 1.53 : 1 : 0.

The magnetic form factor $|F^{(1,1)}(q^2)|$ is, as we can see in Fig, 2, very similar for the Cohen-Kurath scheme (continuous line) and the L-S coupling (dashed-dotted line), whereas the q^2 dependence in the j-j coupling (broken line) is different. It is interesting to notice that the minimum will be seen experimentally, because in this nucleus $F^{(3,3)}$ does not exist. The theoretical zero is located at $(-q^2) \approx 1.6\ fm^{-2}$ in the C-K and L-S results and at $(-q^2) \approx 2.7\ fm^{-2}$ in the j-j coupling.

3.2. ^{7}Li

In that case, $J^{\pi} = \frac{3}{2}^{-}$, we have the form factor $A^{(2)}(q^2)$ and two magnetic form factors, $F^{(1,1)}(q^2)$ and $F^{(3,3)}(q^2)$. In Fig. 3, we show the quadrupole form factor $|A^{(2)}(q^2)|$ in the Cohen-Kurath scheme, and the scale in the other models is j-j : C-K : L-S = 1.19 : 1 : 1.38.

The dipole form factor $|F^{(1,1)}(q^2)|$ is not very different in its q^2 dependence from one case to the other. The magnitude is similar for the L-S and Cohen-Kurath schemes. The position of the zero is moved from 1.7 fm^{-2} to 2 fm^{-2} in the extreme L-S and j-j couplings, respectively. Unlike ^{6}Li, this zero is not seen experimentally because $|F^{(3,3)}(q^2)|$ dominates in this region of momentum transfers. In the same Figure, the value of this last form factor is given for the Cohen-Kurath wave function. The scale is, in this case, given by j-j : C-K : L-S = = 1.66 : 1 : 0.98.

3.3. ^{9}Be

With quantum numbers $J^{\pi} = \frac{3}{2}^{-}$ we have here the same form factors as in ^{7}Li. $|A^{(2)}(q^2)|$ is given in Fig. 4 for the Cohen-Kurath scheme. The scale in the other models is j-j : C-K : L-S = 1.08 : 1 : 0.88.

The q^2 dependence in the magnetic form factor $|F^{(1,1)}(q^2)|$ is not very different in the three models, if we are not near the zero position, around $(-q^2) \approx 1.3\ fm^{-2}$. In $|F^{(3,3)}(q^2)|$ the scale among the schemes is j-j : C-K : L-S = 0.91 : 1 : 1.08.

3.4. ^{13}C

Here $J^{\pi} = \frac{1}{2}^-$ and we have to discuss only the dipole form factor $|F^{(1,1)}(q^2)|$. As we can see in Fig. 5, there is no approximately fixed scale among the q^2 dependence predicted in the three models. Instead, the position of the minimum is very different : from 0.9 fm^{-2} in the j-j model to 2.1 fm^{-2} in the L-S model. Like ^6Li this form factor is measured directly in magnetic scattering.

The gross features of that comparison can be summarized by noting that the intermediate coupling of Cohen-Kurath gives very similar results to the L-S model in ^6Li and ^7Li [with the exception of the form factor $|A^{(2)}(q^2)|$ in ^6Li] whereas in ^9Be and ^{13}C the configuration mixing results are intermediate between the extreme models. With respect to the conjectured scaling in magnetic scattering, it does not appear in ^6Li and ^{13}C, which measure directly the form factor $|F^{(1,1)}(q^2)|$. The planned experiments at IKO will be very interesting for this particular aspect of the problem.

We compare now the normalized square form factor $\mathfrak{F}(q^2)$, Eq. (12), for the different models with experiment in the cases of ^7Li [7] and ^9Be [3),8)]. As we see in Fig. 6, for ^7Li there are no significant differences between the C-K and L-S models in its q^2 dependence. The agreement with experiment is very good in the region of q^2 measured. However, the j-j model does not scale with the others and it is excluded for this nucleus. In Fig. 7 we give the results for $\mathfrak{F}(q^2)$ in the case of ^9Be, together with the experimental points from Lapikas et al. [8] and Rand et al. [3]. The theoretical line corresponding to the Cohen-Kurath coupling is intermediate between the j-j and L-S models. There are no important differences for small values of q^2 and the slope agrees with experiment. In the intermediate region of $-q^2$, from 0.8 fm^{-2}, the comparison shows that the L-S coupling is favourized, although all theoretical results are above the experimental points of Rand et al. [3]. This is precisely the zone in which $|F^{(3,3)}(q^2)|$ dominates (see Fig. 4), so we conclude that this last form factor is the responsible for the disagreement ; the magnetic octupole moment of ^9Be is less than the predicted values *).

4. - CONCLUSION

From this study of elastic form factors it has been shown that the behaviour of the q^2 dependence depends essentially on the number of independent nuclear matrix elements which enter the expression for the corresponding form factor. This sentence assumes that one is choosing a framework for the effective pseudopotential (see the discussion in Section 2). Here the impulse approximation and the use of non-relativistic wave functions have been taken. Consequently, only first order terms in the internal momentum distribution of the nucleons have been maintained. In that case, the only form factor which will present different q^2 dependence when the nuclear model is changed is the magnetic dipole one $|F^{(1,1)}(q^2)|$. Three nuclear models, the j-j coupling, the L-S coupling and the intermediate one resulting from effective interactions in the 1p shell, have been worked out.

Our results for ^6Li, ^7Li, ^9Be and ^{13}C are discussed in Section 3. The expected general features are confirmed, and it is emphasized that precise experimental results on magnetic scattering in ^6Li and ^{13}C will discriminate directly among different models. In particular, the position of the minimum of $|F^{(1,1)}(q^2)|$ is very sensitive to the coupling used. To this end, experiments up to values of $-q^2$ of the order of 2 fm^{-2} will be interesting. Our results for ^7Li and ^9Be, where an important contribution from the form factor $|F^{(3,3)}(q^2)|$ is also present, are compared with experimental measurements of Ref. 7) and Refs. 8), 3) respectively. The j-j model is excluded for ^7Li as we can see in Fig. 6. In ^9Be, the slope of the theoretical distribution agrees with experiment for $(-q^2) \lesssim 0.7$ fm^{-2}. However, the predicted octupole form factor $|F^{(3,3)}(q^2)|$, when normalized to the dipole one, seems to be too large.

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*) This must be understood in the sense of normalized to the corresponding dipole moment.

ACKNOWLEDGEMENTS

This work was supported by the G.I.F.T. The authors wish to thank the CERN Theoretical Study Division (J.B.) and the Department of Theoretical Physics, Oxford (J.R.) for hospitality. We are grateful to L. Lapikas for an interesting correspondence and for sending us the results of ^{9}Be prior to publication, and to T.E.O. Ericson for very useful remarks. One of us (J.B.) thanks J.D. Walecka for a discussion about the subject of this work.

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FIGURE CAPTIONS

Figure 1 Charge form factor $|A^{(0)}(q^2)|$ for the nuclei ^{6}Li, ^{7}Li, ^{9}Be and ^{13}C. The oscillator parameters b are 1.98, 1.90, 1.76 and 1.50 fm, respectively.

Figure 2 Form factors for ^{6}Li :
----- j-j coupling ;
——— C-K model ;
.-.-. L-S coupling.
The value of $|A^{(2)}(q^2)|$ is scaled by j-j : C-K : L-S = = 1.53 : 1 : 0.

Figure 3 Same as Fig. 2 for ^{7}Li. For $|A^{(2)}(q^2)|$, j-j : C-K : L-S = = 1.19 : 1 : 1.38. For $|F^{(3,3)}(q^2)|$, the corresponding values are 1.66 : 1 : 0.98.

Figure 4 Same as in Fig. 2 for ^{9}Be. $|A^{(2)}(q^2)|$ gives 1.08 : 1 : 0.88, whereas $|F^{(3,3)}(q^2)|$ gives 0.91 : 1 : 1.08.

Figure 5 Same as Fig. 2 for ^{13}C.

Figure 6 Normalized square magnetic form factor for ^{7}Li. Experimental points from Ref. 7). There are no important differences between C-K and L-S couplings.

Figure 7 Same as Fig. 6 for ^{9}Be. Experimental points :
● from Lapikas [8] ;
▲ from Rand [3].

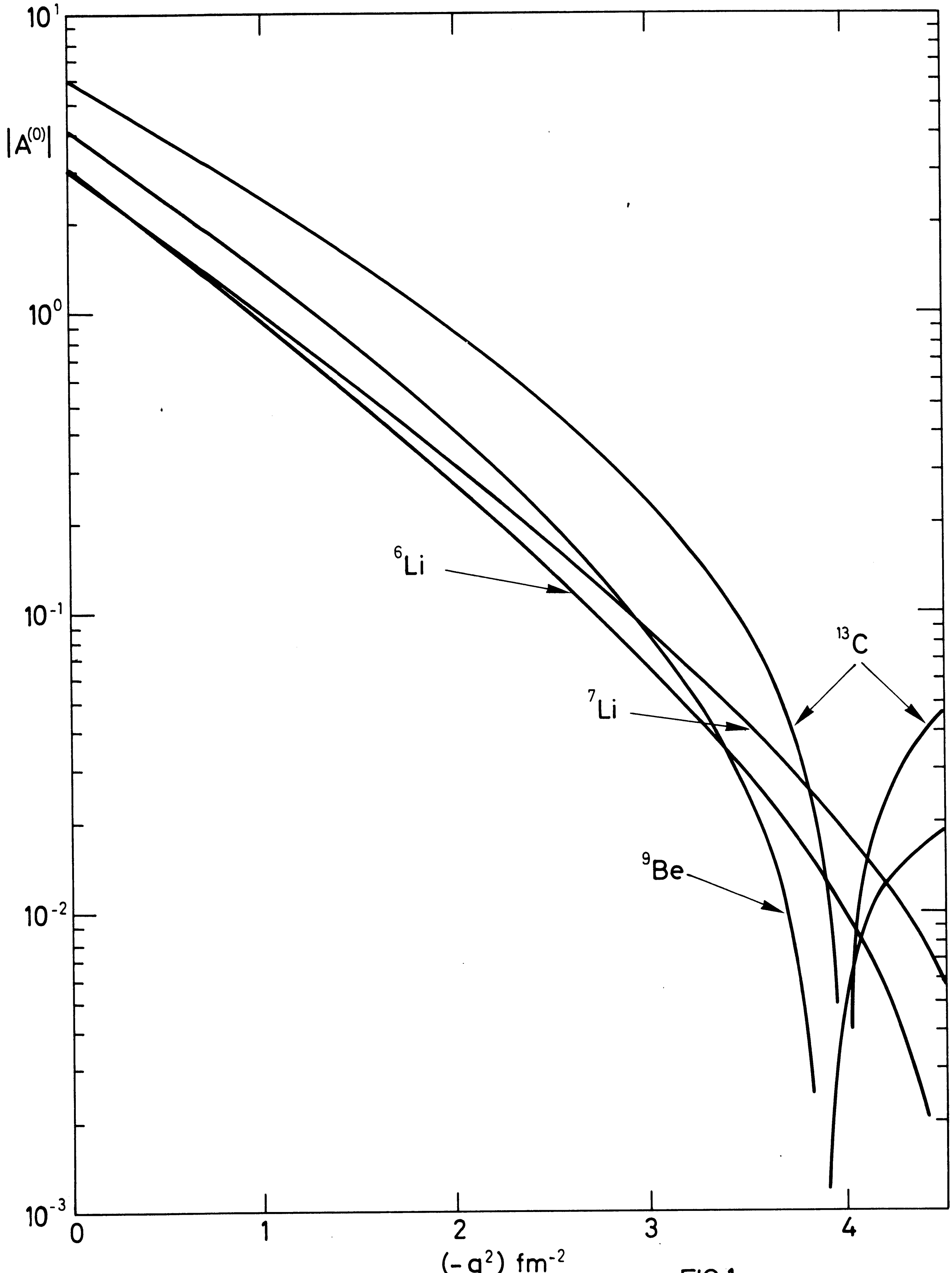


FIG.1

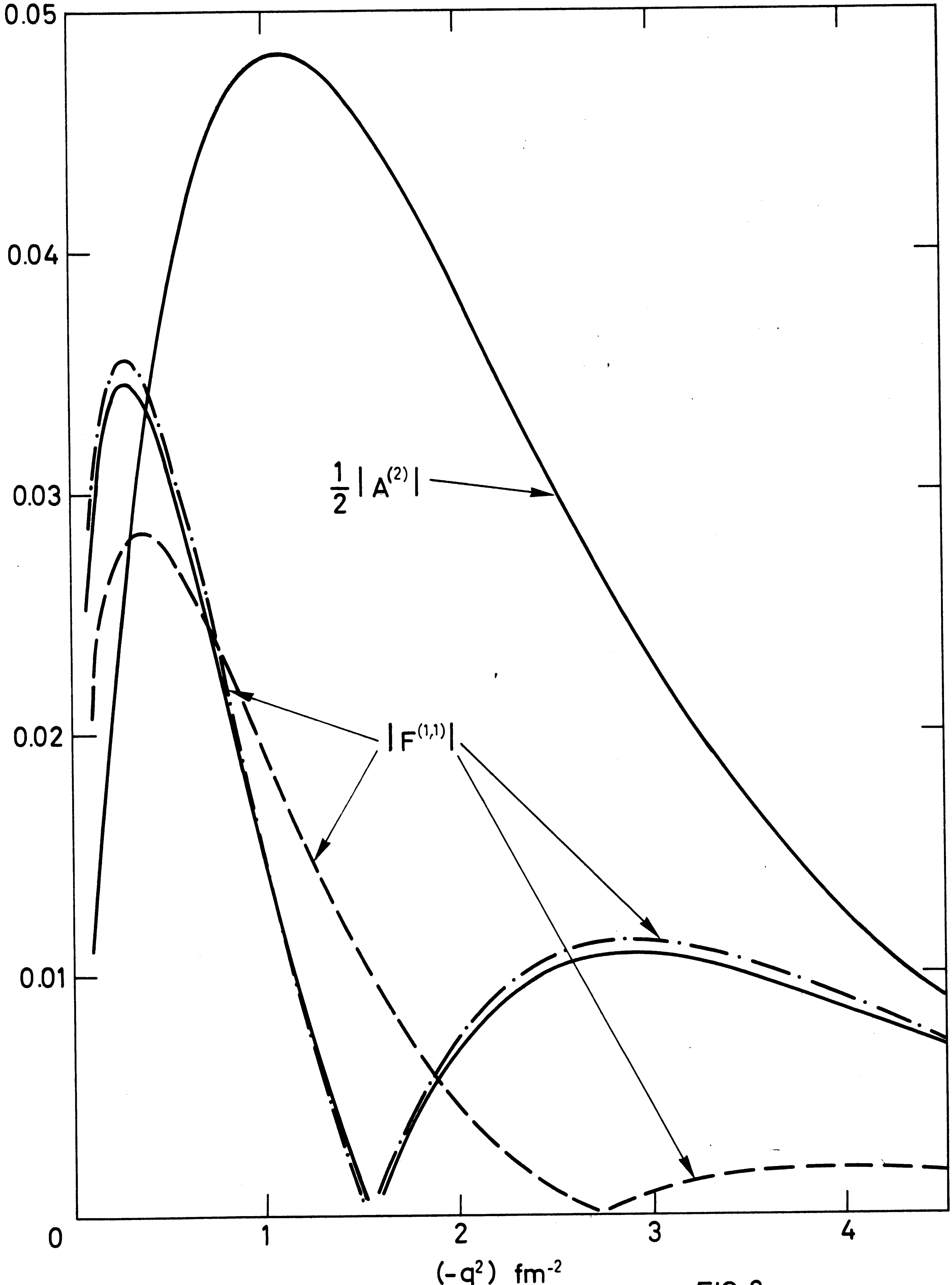
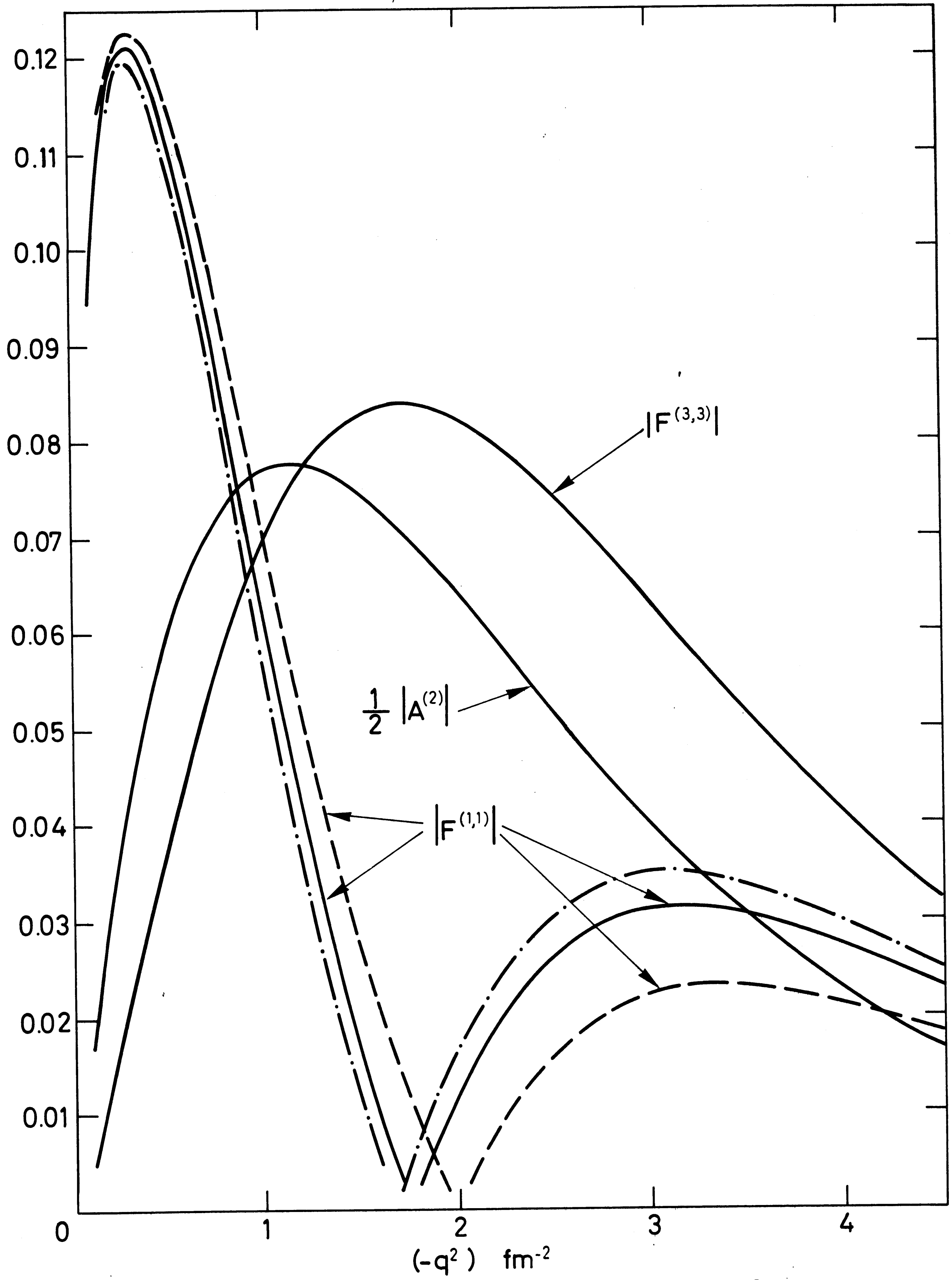


FIG. 2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0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
(-q²) fm⁻²
|F^(3,3)|
1/2 |A^(2)|
|F^(1,1)|

FIG.3

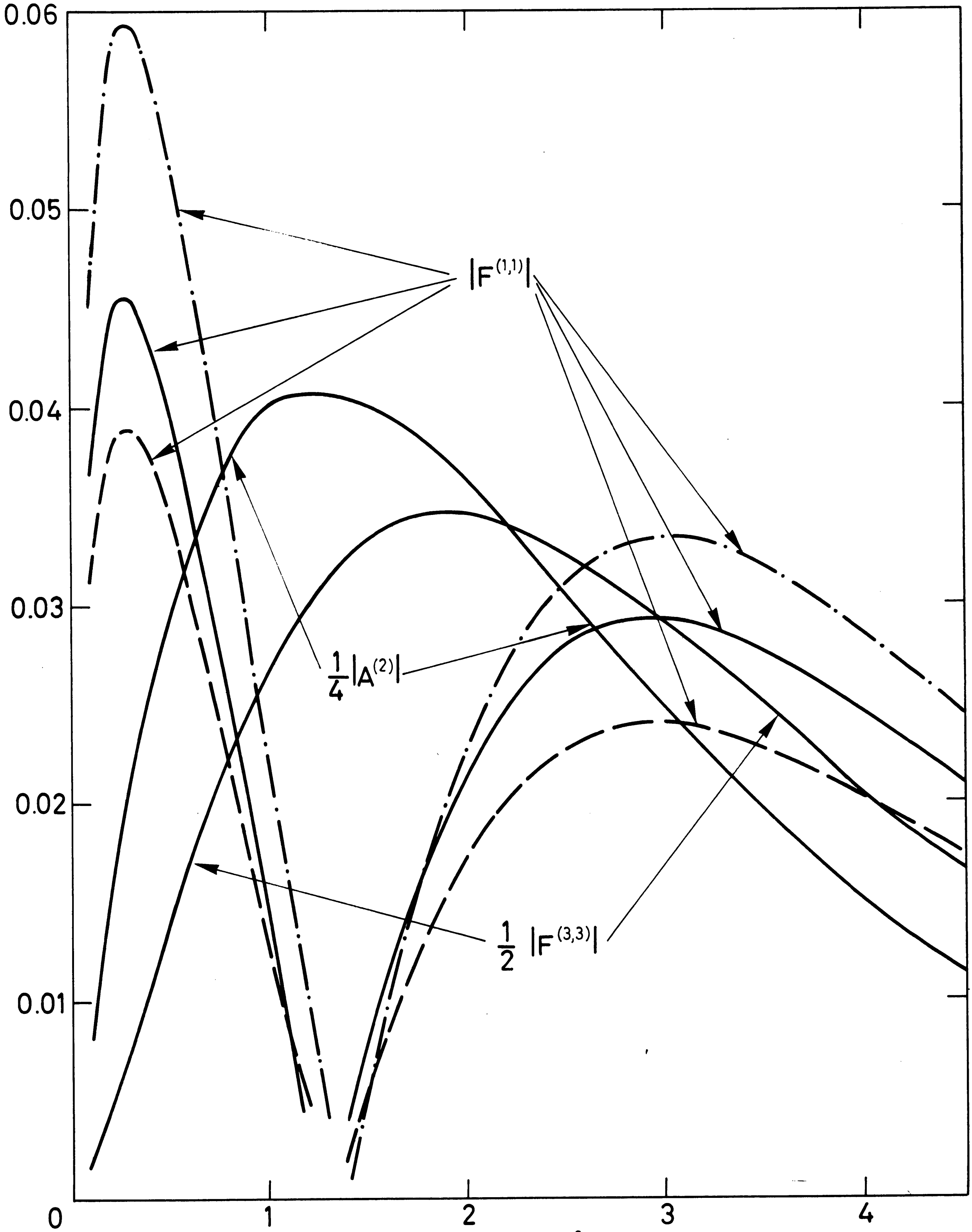

FIG. 4

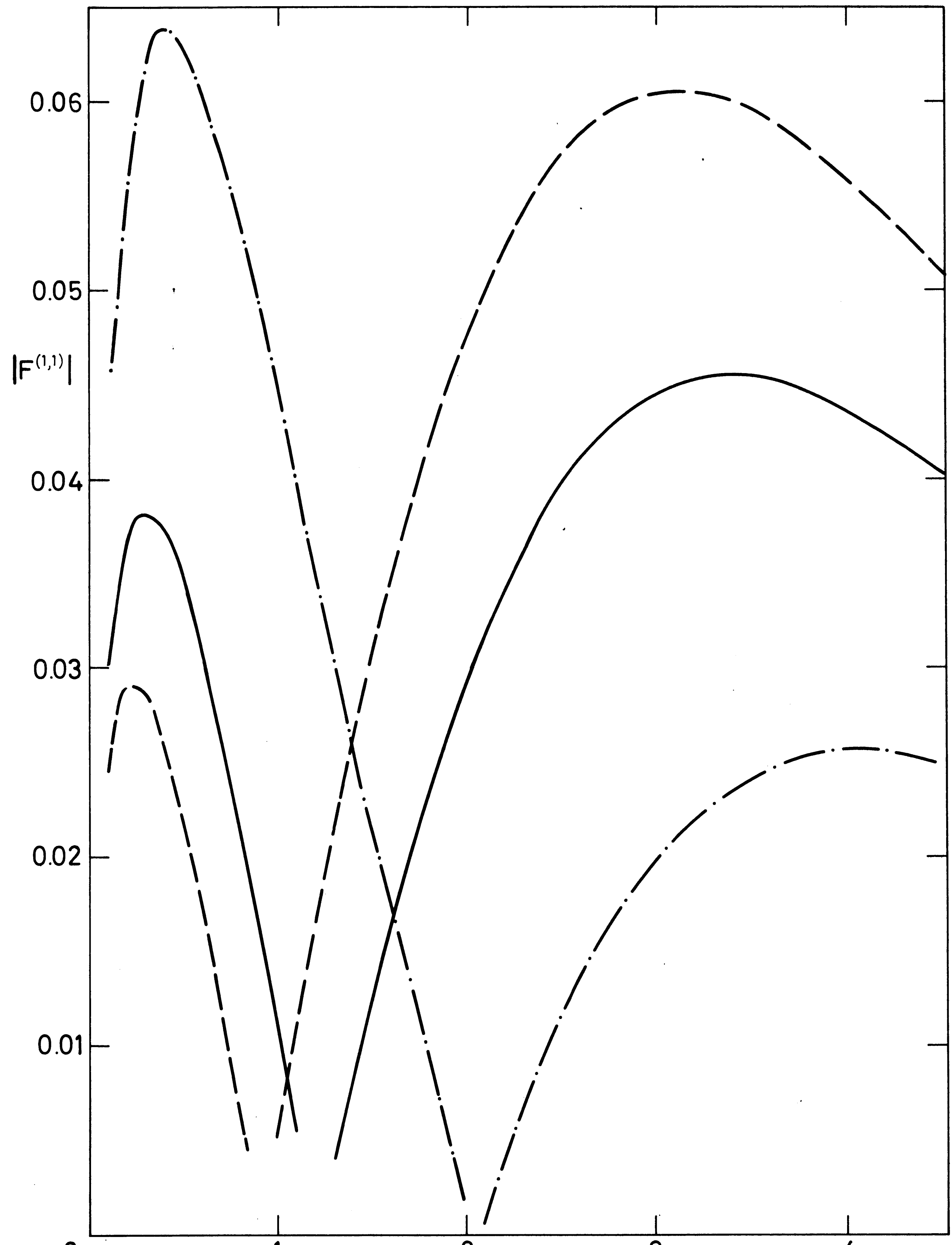

FIG. 5

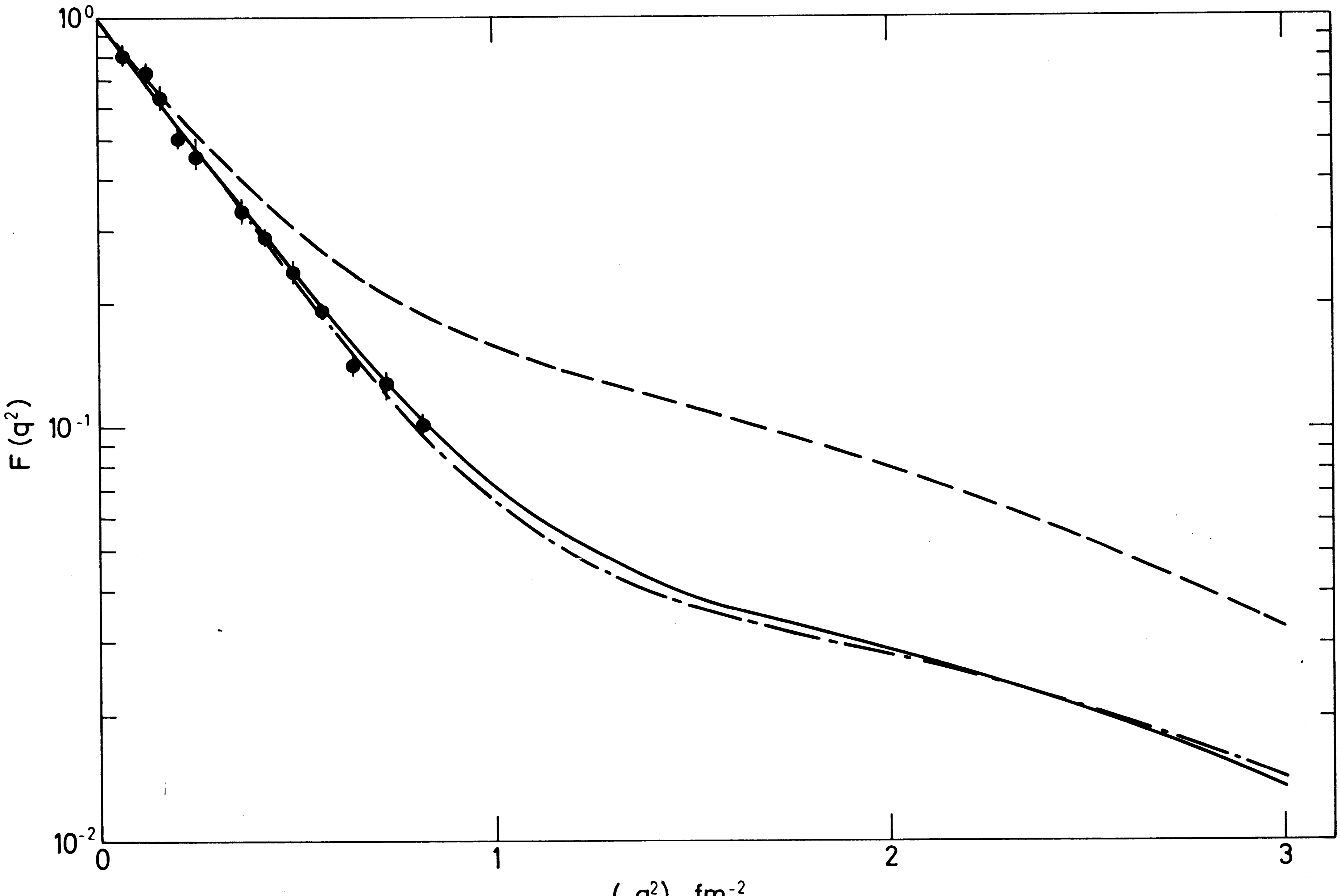

FIG.6

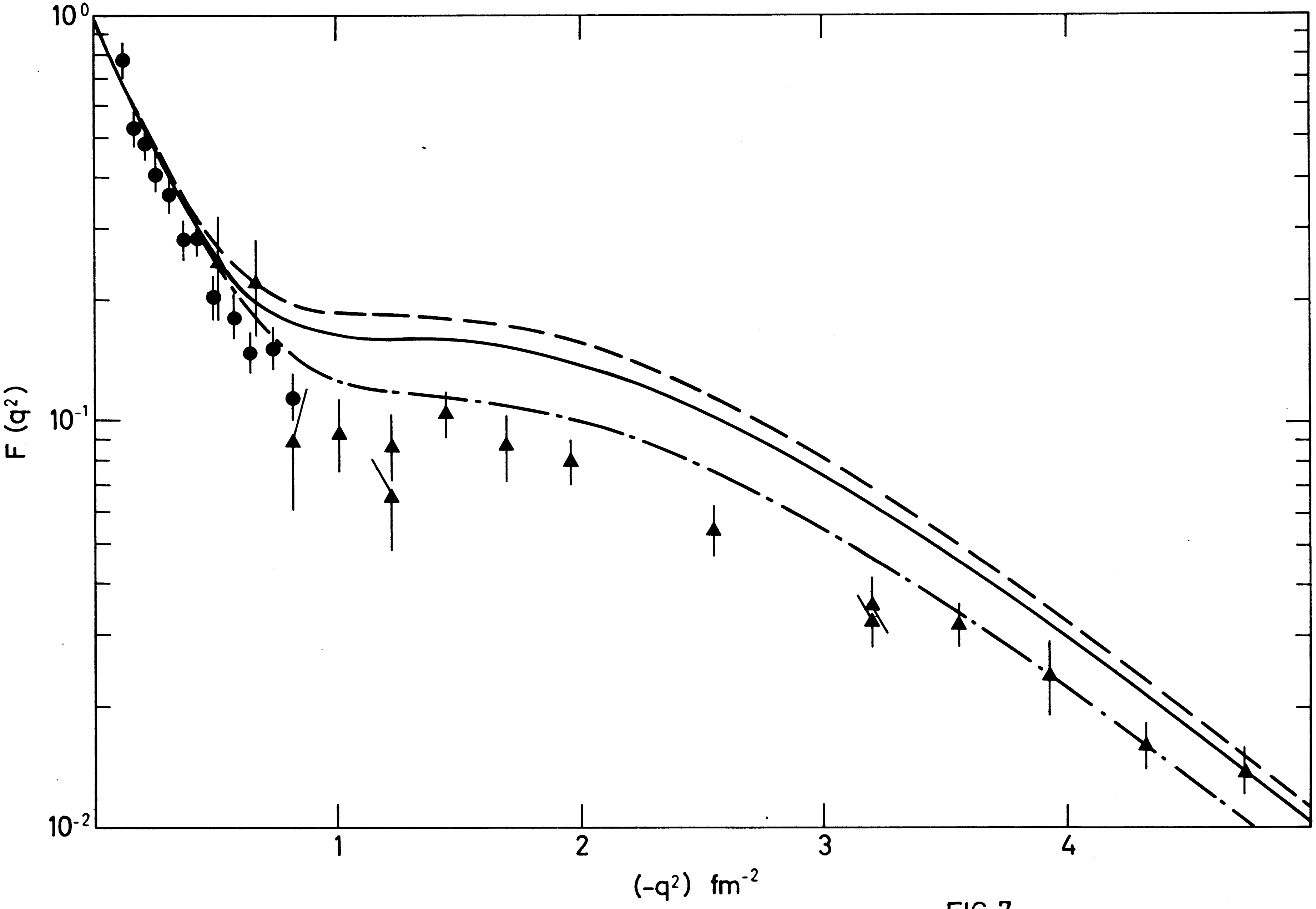

FIG.7