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UNSUBTRACTED DISPERSION RELATION FOR THE LONGITUDINAL COMPTON AMPLITUDE?

J. Bernabéu
CERN - Geneva
and
R. Tarrach *)
Departamento de Fisica Teórica
Universidad de Barcelona

ABSTRACT

It is shown that there is a simple connection between the slope, at $q^2 = 0$, of the longitudinal Compton amplitude and the electric polarizability of the nucleon. The longitudinal subtraction function is thus known to order q^2. The assumption of an unsubtracted dispersion relation for the longitudinal amplitude leads to a sum rule for the electric polarizability. This is a model independent test of the high-energy behaviour of the forward virtual Compton amplitude.

*) Present address: CERN - Geneva

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In the past few years there has been new interest in the determination of the forward virtual Compton amplitudes of the nucleon because of increasing availability of experimental information on the deep inelastic electroproduction structure functions. The situation for the most spectacular [by no means the only one [1]] application of this information, proton-neutron mass difference, was reviewed by Zee [2].

The purpose of the present note is to stress that the low-energy theorem in <u>real</u> Compton scattering -- up to second order in the photon energy -- imposes a restriction on the longitudinal amplitude of <u>virtual</u> photons. In a precise form, we propose a model-independent test of the presence or absence of a subtraction in writing a dispersion relation for the longitudinal Compton amplitude. This test results in a connection between the dispersive integral of the slope (at $q^2 = 0$) of the longitudinal cross-section -- an experimental quantity -- and the electric polarizability, a parameter which is determined by measuring the angular distribution of low-energy real Compton scattering.

Let us remember what the main points are in the analysis of forward virtual amplitudes and where, we believe, is the weakness. Spin averaged forward virtual Compton scattering is determined by the two amplitudes $T_{1,2}(\nu, q^2)$ [usual notation [2]] appearing in the gauge-invariant decomposition of the hadronic tensor. Their absorptive parts $W_i(\nu, q^2)$ are subject to direct experimental measurement through electroproduction data. Experimentally, nothing can be said about the real parts, but for the limit of real photons:

$$T_1(\nu, q^2 \to 0) = \lim_{q^2 \to 0} \frac{\nu^2}{-q^2} T_2(\nu, q^2)$$

In order to get information for them, one usually makes use of dispersion relations. If these need a subtraction or not, is a matter of high-energy behaviour, which is unknown in a model-independent way. However, the low-energy theorem, in particular the Thomson limit, says

$$\lim_{\nu \to 0} \lim_{q^2 \to 0} T_1(\nu, q^2) = -2 Z^2 \tag{1}$$

Z being the charge of the target in natural units. This restriction allows an unsubtracted dispersion relation for $T_2(\nu, q^2)$ whereas the dispersion relation for $T_1(\nu, q^2)$ needs a subtraction (this is compatible with results from a Regge model). This introduces the unknown function $T_1(\nu = 0, q^2)$. The Thomson value (1) only gives information at one point [1]

$$\lim_{q^2 \to 0} T_1(\nu = 0, q^2) = 2 \mu [2Z + \mu] \tag{2}$$

μ being the anomalous magnetic moment. A possible way out of this difficulty has been suggested in the literature by assuming that the so-called longitudinal amplitude

$$T_L(\nu, q^2) = \left(1 - \frac{\nu^2}{q^2}\right) T_2(\nu, q^2) - T_1(\nu, q^2) \tag{3}$$

does not need a subtraction. This is equivalent to say that we know how to calculate the function $T_1(\nu = 0, q^2)$. This hope is based on the possible cancellation of the dominant high-energy terms of $\nu^2 T_2(\nu, q^2)$ and $T_1(\nu, q^2)$, a possibility which is perhaps suggested by a vanishing value of $W_L(\nu \to \infty, q^2)$ if the Regge and Bjorken limits commute and $W_L \xrightarrow[Bj]{} 0$. It is hardly necessary to mention that the Thomson limit does not give any restrictions on the longitudinal amplitude. What we show in this paper is that the wanted model-independent restriction can be obtained from the next order term in the low-energy theorem. In particular, this restriction can be formulated as an experimental test of the presence or absence of a subtraction in the longitudinal amplitude (see below).

Two comments concerning the whole idea of this work. First one refers to the approach. Naively one could say that real photons play no role for the longitudinal amplitude, because only $\lim_{q^2 \to 0} T_2(\nu, q^2)/(-q^2)$ contributes to the physical amplitude. Nevertheless, the slope of $T_L(\nu, q^2)$ is present in the time-time component of the tensor, so it must be related to the general (non-forward) amplitudes describing real Compton scattering. The second remark refers to the result. It is surprising that the high-energy behaviour of the virtual amplitude is, in some way, controlled by the value of a low-energy parameter in real scattering. Let us see how the argument actually works.

The first step is to construct a gauge invariant and Lorentz covariant spin averaged virtual Compton scattering amplitude which satisfies crossing and whose invariant functions are free from kinematical zeros, singularities and constraints. We make that à la Bardeen and Tung [3], considering both virtual photons with the same mass. With this tensor, we can describe different limiting processes. In particular, forward real scattering can be obtained through the real photon amplitude (this introduces the polarizability parameter), or through the forward virtual amplitude (which contains the longitudinal component). We obtain

$$\begin{aligned} T^{\mu\nu} \xrightarrow[\substack{q' \to q \\ q^2 \to 0}]{} & -q^\mu q^\nu \left[A_1(0,0,P.Q) + 2(P.Q)\, A_3(0,0,P.Q) \right] \\ & + \left[(P.Q)^2 g^{\mu\nu} - (P.Q)(P^\mu q^\nu + P^\nu q^\mu) \right] A_2(0,0,P.Q) \end{aligned} \tag{4}$$

where q and q' are the incoming and outgoing photon momenta attached to the vertices ν and μ, respectively, $Q=(q+q')/2$, and $P=(p+p')/2$ is the average of the incoming and outgoing nucleon momenta. Crossing implies for the invariant amplitudes

$$\begin{aligned} A_i(q^2, q\cdot q', P\cdot Q) &= A_i(q^2, q\cdot q', -P\cdot Q) \quad ; \quad i=1,2 \\ A_3(q^2, q\cdot q', P\cdot Q) &= -A_3(q^2, q\cdot q', -P\cdot Q) \end{aligned} \tag{5}$$

If the usual gauge invariant decomposition of the forward virtual tensor is used, in terms of $T_2(\nu,q^2)$ and $T_L(\nu,q^2)$, we can identify easily

$$\lim_{q^2\to 0} \frac{T_2(\nu,q^2)}{q^2} = m^2 \, A_2(0,0,m\nu) \tag{6a}$$

$$\lim_{q^2\to 0} \frac{T_L(\nu,q^2)}{q^2} = A_1(0,0,m\nu) + m^2 A_2(0,0,m\nu) + 2m\nu\, A_3(0,0,m\nu) \tag{6b}$$

In a dispersive approach for the right-hand side of Eq. (6b), we get a term $O(\nu^{-2})$ from the pole contribution to the invariant amplitude A_2 and terms of order constant in ν from the pole of A_3 and continuum contribution of $A_1+m^2A_2$. Then we obtain the relation

$$\lim_{q^2\to 0} \frac{\mathrm{Re}\, T_L(\nu,q^2)}{q^2} = \frac{2Z^2}{\nu^2} + \frac{\mu(Z+\mu)}{m^2} + (A_1+P^2A_2)^c + O(\nu^2) \tag{7}$$

where the superscript C means the limit $\nu\to 0$ of the dispersive integral at fixed scattering angle. It has been shown [4] that this parameter $(A_1+P^2A_2)^C$, appearing in the description of real Compton scattering, is related to the electric polarizability of the target α by

$$(A_1+P^2A_2)^c = -\left(\frac{4\pi}{e^2}\right) 2m\alpha - \frac{\mu(2Z+\mu)}{2m^2} \tag{8}$$

in such a way that we get the relation

$$\lim_{q^2\to 0} \frac{\mathrm{Re}\, T_L(\nu,q^2)}{q^2} = \frac{2Z^2}{\nu^2} + \frac{\mu^2}{2m^2} - \left(\frac{4\pi}{e^2}\right) 2m\alpha + O(\nu^2) \tag{9}$$

It is a nice exercise to check that Eqs. (1) to (9), except for Eq. (2) *), are also valid for a spin-zero target, using the natural prescription $\mu = 0$.

The remaining step is to write the dispersion relation for $T_L(\nu, q^2)$. In the general case, with one subtraction and the pole term separated out, we find

$$\lim_{q^2\to 0} \frac{\mathrm{Re}\, T_L(\nu,q^2)}{q^2} = \frac{2Z^2}{\nu^2} + \lim_{q^2\to 0}\left\{\frac{\mathrm{Re}\, T_L(0,q^2)}{q^2} + \frac{8m^2}{q^4} G_E^2(q^2)\right\} + O(\nu^2) \qquad (10)$$

where G_E is the Sachs' electric form factor. In the case of a spin-zero target, $G_E^2(q^2)$ must be replaced by $(1 - q^2/4m^2)\, F^2(q^2)$ [where $F(q^2)$ is the charge form factor].

From a comparison of Eqs. (9) and (10), it is clear that the subtraction function $\mathrm{Re}\, T_L(0,q^2)$ is known to order q^2

$$\mathrm{Re}\, T_L(0,q^2) = \frac{8m^2}{-q^2} G_E^2(q^2) - q^2\left[\left(\frac{4\pi}{e^2}\right) 2m\alpha - \frac{\mu^2}{2m^2}\right] + O(q^4) \qquad (11)$$

Equation (11) must be understood as a knowledge of the longitudinal subtraction function, when $q^2 \to 0$, due to the low-energy theorem, in quite the same way as the Thomson limit gives the value of $T_1(0,q^2)$ at the point $q^2 = 0$, Eq. (2). The new ingredient now is the presence of the electric polarizability parameter α.

The other way round: if the dispersion relation for $T_L(\nu,q^2)$ is written without a subtraction, we obtain a sum rule for α, relating experimental quantities on both sides of the equation. We have

$$\lim_{q^2\to 0} \frac{\mathrm{Re}\, T_L(\nu,q^2)}{q^2} = \frac{2Z^2}{\nu^2} + 4\,P\int_{\nu_{th}}^{\infty} \frac{d\nu'}{\nu'}\left[\lim_{q^2\to 0} \frac{W_L(\nu',q^2)}{q^2}\right] + O(\nu^2) \qquad (12)$$

[no subtraction]

where $W_L = (2\pi)^{-1}\, \mathrm{Im}\, T_L$ and ν_{th} is the threshold inelastic excitation at the limit of real photons $q^2 = 0$. Identifying the right-hand side of

*) For a spin-zero target, the value of T_1 at the origin $\nu = 0$, $q^2 = 0$, becomes independent of the order in which we let $-q^2$ and ν go to zero, so that Eq. (2) becomes the Thomson limit (1).

Eqs. (9) and (12) we obtain finally

$$\alpha - \left(\frac{e^2}{4\pi}\right)\frac{\mu^2}{4m^3} = \frac{1}{2\pi^2}\int_{\nu_{th}}^{\infty} d\nu \lim_{q^2 \to 0} \frac{\sigma_L(\nu, q^2)}{-q^2} \qquad (13)$$

[no subtraction]

where σ_L is the so-called longitudinal cross-section. Equation (13) is a direct consequence of the low-energy theorem and an unsubtracted dispersion relation for $T_L(\nu,q^2)$. The validity or non-validity of this sum rule for α is therefore a clean test of the latter assumption. This was precisely our original motivation in looking for model-independent restrictions. Equation (13) provides the relation.

Let us now discuss the present experimental situation, in the case of the proton. The value of α is not well known and a better determination is needed [5]. Experimentally we can say [6) *)] that its value is at the level of 10^{-3} fm^3. The magnetic moment term gives a contribution of 5.4×10^{-5} fm^3. The right-hand side of Eq. (13) must be determined from inelastic electron scattering at low-momentum transfers and all energy transfers. Backward angle measurements should be welcome for giving the value of σ_L. From Bloom's report [7], it seems that the ratio $R(\nu,q^2) = \sigma_L(\nu,q^2)/ /\sigma_T(\nu,q^2)$ increases smoothly from $q^2 = 0$ up to a relative maximum value in the q^2 distribution when averaged for values of W between 2 and 4 GeV [W is the invariant mass, i.e., $W^2 = m^2 + 2m\nu$ at $q^2 = 0$]. If we extract, from these results, the value of the slope of R [at $q^2 = 0$], we get a contribution from that region of W, to the right-hand side of Eq. (13), which is about 1.5×10^{-4} fm^3. In the resonance region, there is a high value of the slope of σ_L at the peak of the first resonance [8]. Since this fact may imply a sizeable contribution to the integral [in fact, the most important one if this point in W is not an isolated case], it is of extreme interest to make a close experimental study of this energy transfer region. We ask for these measurements in order to settle the question posed by Eq. (13). We think that the main importance of this relation is not the fact that we have here a sum rule for the electric polarizability, but that it is a <u>model-independent restriction</u> obtained from the assumption of an unsubtracted dispersion relation for $T_L(\nu,q^2)$. The character of this dispersion relation is crucial, if we hope to understand processes with virtual photons.

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In the meantime and from a pragmatic point of view, if the unsubtracted dispersion relation is accepted, it follows from Eq. (13) and present experiments that a significant positive lower limit for the electric polarizability of the proton is

$$\alpha > 2 \times 10^{-4} \ \mathrm{fm}^3 \tag{14}$$

since the integrand is positive-definite: any new determination of the slope of σ_L, for different values of W, adds a positive contribution to the lower limit (14).

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