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SOME COMMENTS ON THE $n + p \to D + 2\gamma$ ANOMALY

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ABSTRACT

It is shown that the only theoretical contribution to the doubly radiative emission, not included in the two-step transition radiations, is given by charge-bremsstrahlung from the single-photon emission. Due to the pole structure of the amplitude, the experimental energy spectrum is reproduced, but the rate is short of the measured value by more than six orders of magnitude.

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Recently the first observation of the doubly radiative neutron capture by protons has been reported [1] and the theoretical expectations have been discussed by Blomqvist and Ericson [2] (from now on this reference will be denoted by BE). Within the framework of conventional electromagnetic transition theory, BE go through a comparison of theory with the essential experimental features. The outcome of that study is said in the title of their paper: the difference between theoretical expectations and experimental observations is tagged an anomaly. In fact, the gap between theory and experiment is of several orders of magnitude. It is our purpose to make some complementary comments and to discuss the additional contribution, using a different approach, of charge-bremsstrahlung radiation, which eventually will support their final conclusions. We refer to BE for having a more complete view of the problem and its history.

Briefly, the experimental facts are the following: calling Δ the absolute value of the difference between the energies of the two photons, $\Delta = |\omega - \omega'|$, and considering that the measured cross-section covers only a certain range in Δ, i.e.,

$$" \sigma_{\gamma\gamma} " = \int_{\Delta=0}^{\Delta=1\,\mathrm{MeV}} d\Delta \, \frac{d\sigma}{d\Delta} \tag{1}$$

they obtain the values [1]

$$" \sigma_{\gamma\gamma} "^{exp} \Big/ \sigma_{\gamma}^{exp} \simeq 10^{-3} \quad , \quad \left.\frac{d\sigma_{\gamma\gamma}}{d\Delta}\right|_{\Delta=1\,\mathrm{MeV}} \Big/ \left.\frac{d\sigma_{\gamma\gamma}}{d\Delta}\right|_{\Delta=0} \simeq 3 \tag{2}$$

As the neutron is practically at rest, the initial state is an incoherent superposition of the $^1S_0$ and $^3S_1$ np states. The transitions studied by BE are

$$\begin{array}{lcl} ^3S_1 \xrightarrow{E1} {}^3P \xrightarrow{E1} D \\ ^3S_1 \xrightarrow{\quad contact \quad} D \\ ^3S_1 \xrightarrow{M1} {}^1S_0 \xrightarrow{M1} D \\ ^1S_0 \xrightarrow{M1} D \xrightarrow{M1} D \end{array} \tag{3}$$

and all of them proceed with a much smaller rate than that given by the experiment. They distinguish three types of Δ shapes, corresponding to their results for the E1 - E1, ${}^3S_1 \to D$ M1 - M1 and ${}^1S_0 \to D$ M1 - M1 cases. We wish to point out that the experimental spectrum supports the guess that the initial state from which the two photons are emitted is the ${}^1S_0$ state, as there is in this case an argument which, independently of the details of the emission, favours the appearance of a minimum for equipartition $\Delta = 0$. The qualitative reason for this is that, only for $\Delta = 0$, there is a point in the allowed phase space in which the deuteron is at rest, and the amplitude for the process $0^+ \to D\gamma\gamma$ with D at rest vanishes by the same argument by which a spin-1 particle cannot decay into two photons. Of course, in the rest of the $\Lambda = 0$ phase space the deuteron is not at rest, so that the expected minimum can take a non-vanishing value or may even be filled in.

A simple glance at the transitions (3) shows that the radiation from moving charges is missing, which corresponds, when E2 and higher multipoles are neglected, to the radiation schemes

$$\begin{array}{ccccc} {}^1S_0 & \xrightarrow{M1} & D & \xrightarrow{E0} & D \\ {}^1S_0 & \xrightarrow{E0} & {}^1S_0 & \xrightarrow{M1} & D \end{array} \tag{4}$$

[We remember that the usual argument by which an E0 transition is strictly forbidden, i.e., that the transition vector, in the Coulomb gauge, must be proportional to the photon momentum $\vec{k}$, does not apply when there are more momenta to play with, which happens when the process may take place even without this photon.] The processes (4) represent the bremsstrahlung of one photon off the transition process responsible for the emission of the other photon and correspond to the diagrams of Fig. 1 and the crossed ones. In other two photon processes studied up to now (see references in BE), this extra contribution was never considered, but there is in these cases a clear reason for that: the one photon process is highly suppressed and actually the two-photon emission has a different nature and proceeds with a much higher rate than the former one.

However, for the cases in which the dominant process corresponds to one-photon emission, our contribution can always be related to this single process. This happens in our case as the one-photon transition ${}^1S_0 \xrightarrow{M1} D$ is very strong and thus (4) may become relevant. It is interesting to note that, not only radiative scattering processes, but also radiative hyperon decays [3] ($\Lambda \to p\pi^-\gamma$, $\Sigma^+ \to n\pi^+\gamma$, $\Sigma^- \to n\pi^-\gamma$) are fairly well understood in terms of bremsstrahlung of the charged particles. This is also suggested by the fact that the decay $\Sigma^+ \to p\pi^0\gamma$ has not been observed, as proceeding via charge radiation it would be depressed by a factor $(m_\pi/M_p)^2$. In our case, a very important point, supporting the possibility of this contribution being relevant, is the shape of the experimental spectrum which favours one photon with very low energy, a characteristic fact of the pole approach. At a first glance, the expected order of magnitude could just be the right one. For $\Delta = 0$ and taking $T^{\gamma\gamma}$ constant for the purpose of this estimate, one finds

$$\left.\frac{d\sigma_{\gamma\gamma}}{d\Delta}\right|_{\Delta=0} \Big/ \sigma_\gamma = \frac{\alpha B}{8\pi} \frac{|T^{\gamma\gamma}|^2}{|T^\gamma|^2} \tag{5}$$

where B is the binding energy of the deuteron, α the fine structure constant, and T the invariant matrix elements. Taking the experimental value [1] of 6×10^{-4} MeV^{-1} for (5), this gives

$$|T^{\gamma\gamma}|^2 \simeq |T^\gamma|^2 \ \mathrm{MeV}^{-2} \tag{6}$$

that is, something of the order k^{-1} in the amplitude ratio, where k is the photon energy. A priori, this could come from the pole contribution we are considering. However, what actually happens is that the k^{-1} term contains effectively a velocity factor $|\vec{p}\,'|/M_d$, where $\vec{p}\,'$ is the <u>three-momentum</u> of the recoil deuteron, so that we are going three orders of magnitude down. Let us write the contribution of the diagrams in Fig. 1 in the form

$$\begin{aligned} T^{\gamma\gamma}(p \to p', k, k') = {} & \epsilon^\nu(k)^* \epsilon^\mu(k')^* \eta^\rho(p')^* \\ & \times \left\{ \frac{p'_\nu}{p'.k} T^\gamma_{\mu\rho}(p \to p'+k, k') - \frac{p_\nu}{p.k} T^\gamma_{\mu\rho}(p-k \to p', k') \right. \\ & \left. + \frac{p'_\mu}{p'.k'} T^\gamma_{\nu\rho}(p \to p'+k', k) - \frac{p_\mu}{p.k'} T^\gamma_{\nu\rho}(p-k' \to p', k) \right\} \end{aligned} \tag{7}$$

where the one-photon amplitude from the initial singlet state to the deuteron must take the gauge-invariant form

$$T^{\gamma}_{\mu\rho}(p \to p', k) = A\ (p'.k\ g_{\mu\rho} - p'_{\mu} k_{\rho}) \qquad (8)$$

and the ϵ's and η are the polarization vectors of the photons and the deuteron, respectively. In order to get a gauge-invariant amplitude in (7) the term

$$A\,(g_{\mu\nu}\ k'_{\rho} - g_{\mu\rho}\ k'_{\nu} + g_{\mu\nu}\ k_{\rho} - g_{\nu\rho}\ k_{\mu})$$

must be added to the expression within brackets of Eq. (7). As the two-photon amplitude is now gauge invariant we choose the Coulomb gauge $\epsilon^{0} = 0$ for which $p \cdot \epsilon = 0$ and $p' \cdot \epsilon(k) = \vec{k}' \cdot \vec{\epsilon}(k)$. Then we are left with the pole contribution of order $O(|\vec{p}'|/M_d) = O(k'/M_d)$ and a non-pole term of the same order. The square of the two-photon amplitude is then, after summing over polarizations,

$$|T^{\gamma\gamma}|^2/|A|^2 = 2(\omega+\omega')\left[\left(\frac{\omega}{\omega'}\right)^2+\left(\frac{\omega'}{\omega}\right)^2\right]\sin^2\theta + (1+\cos^2\theta)\left[\omega^2+\omega'^2+2\omega\omega'\cos\theta\right] \qquad (9)$$

where ω and ω' are the photon energies and Θ the angle between them. The kinematical configuration corresponding to the deuteron at rest is given by $\omega = \omega'$, $\cos\Theta = -1$, and we see explicitly the vanishing value of (9) in that point, as explained above from general arguments. The energy spectrum is given by

$$\frac{d\sigma_{\gamma\gamma}}{dx}\Big/\sigma_{\gamma} = \frac{\alpha}{3\pi}\left(\frac{B}{M_d}\right)^2\left[\frac{x^3}{1-x} + \frac{(1-x)^3}{x} + x^3(1-x) + x(1-x)^3\right] \qquad (10)$$

where $x = \omega/B \in (0,1)$. We see in Eq. (10) that the rate with which this emission proceeds is roughly $(B/M_d)^2$ times the measured one. The predicted spectrum is plotted in Fig. 2 and corresponds to the quantity between brackets in Eq. (10). This form is precisely the observed one [1], which covers the region between the two arrows in Fig. 2 and with the correct ratio of maximal to minimal values. However, the integration over this region, corresponding to (1), gives a wrong result by more than six orders of magnitude. We have

$$"\sigma_{\gamma\gamma}"/\sigma_\gamma = .5\times 10^{-9} \quad , \quad \left.\frac{d\sigma_{\gamma\gamma}}{d\Delta}\right|_{\Delta=1\,\mathrm{MeV}} \Big/ \left.\frac{d\sigma_{\gamma\gamma}}{d\Delta}\right|_{\Delta=0} = 2.6 \qquad (11)$$

to be compared with (2). It is our conclusion that the experimental rate does not correspond to bremsstrahlung of the one-photon transition.

This calculation allows one to infer the expected order of magnitude of the branching ratio of this contribution to the two-photon emission with respect to the dominant one-photon transition, for any nuclear process. This is given by $Z^2\alpha\, v^2_{recoil} \sim \alpha(Z/A)^2(\Delta E/M_p)^2$, where ΔE is the energy release.

At this stage, one may ask for the contribution of magnetic moments in the vertices associated to the diagrams of Fig. 1. As the three-momentum of the deuteron and the γ energies are of the same order, we expect this contribution to be at the same level as the one coming from charge radiation, apart from the fact that the pole becomes ineffective for $k\to 0$. In fact, this contribution, in the multipolar language of transition radiation, is the last one indicated in (3) and, in the impulse approximation, has been calculated by BE.

BE considered both photons to be transition photons. Here we have considered one transition and one charge-bremsstrahlung photon. What is left is to think about both photons as bremsstrahlung photons. As expected, the only contribution which is left is the well-known gauge contact term which, apart from contributing only when orthogonality of initial and final state is violated, gives in any case a bad shape for the photon spectrum. This corresponds to the conjecture followed by Adler [4] in dealing with the two-photon emission. We discuss now how this result appears.

For the purpose of connecting our approach with the multipolar language of radiation theory, let us apply it first to the one-photon process. The result is very illuminating. Charges do not contribute (as it must be, due to the fact that the process without photon is unphysical) as the three-momentum at the vertex is zero and consequently neither term contributes in the Coulomb gauge. Furthermore, the magnetic moment contributions in external legs give the known result of the traditional M1 transition calculus, when in the last one only the free wave part of the initial scattering wave function is retained. With the same technique of starting with a covariant npd vertex [e.g., the one given by Gourdin et al. [5]], we attach the two photons to external legs and we keep terms up to order charge × magnetic moment (which, a priori, could give the right order of magnitude), making the contribution gauge invariant up to that order. The result is: charges do not contribute, for the same reason as before, the gauge contact term does not connect the scattering state with the deuteron, and the charge × magnetic moment contribution is depressed by a factor (B/M_d) in the amplitude. As it should be clear, after our comment on the one-photon process, this last contribution is just another way of giving the bremsstrahlung (charge) of the one-photon M1 transition (magnetic moment), which has been calculated above in some detail.

Let us finish with the moral resulting from the considerations of this note. The only contribution which is not given by two-step transitions (studied in BE) in this doubly radiative process, corresponds to charge radiation from the single radiative transition. Although the experimental spectral form is well reproduced by this contribution, it fails to explain the branching ratio of two-photon to one-photon emission by more than six orders of magnitude. When this result is combined with the other theoretical contributions, it clearly asks for a confirmation of the experiment. If the present situation were maintained, it would confront us with a major and very serious problem.

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FIGURE CAPTIONS

Fig. 1 : Photon emission from external legs of the single-photon process. The contribution is given by these diagrams plus the crossed ones $[(\nu,k) \rightleftarrows (\mu,k')]$ plus an extra term to restore gauge invariance.

Fig. 2 : Photon energy spectrum as given by our contribution. Experimental data are available between the two arrows. The ratio of values at the arrow and at the minimum is 2.6. Notice the ordinate scale!

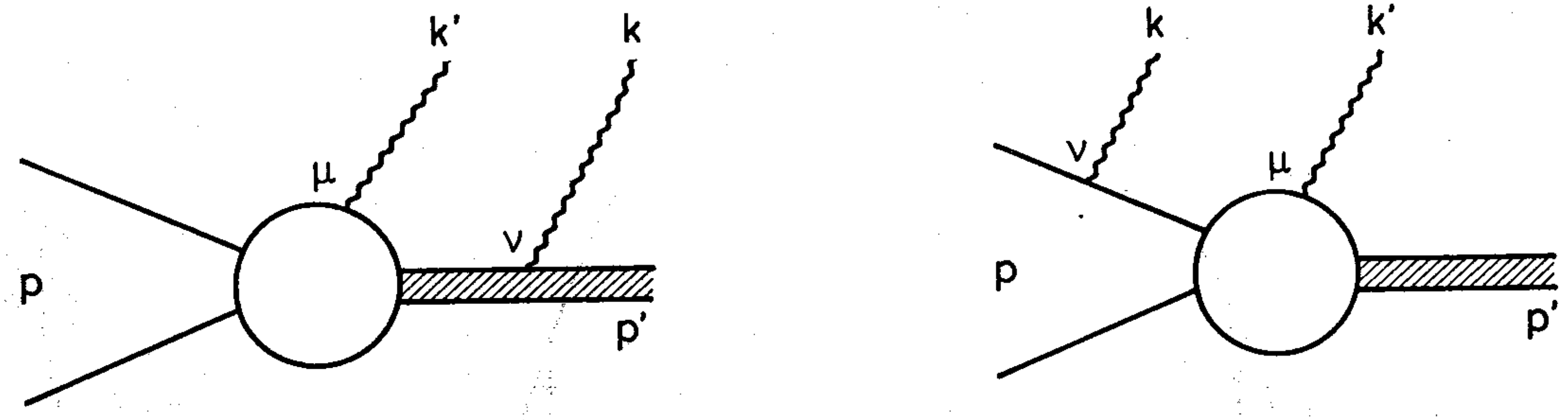

FIG.1

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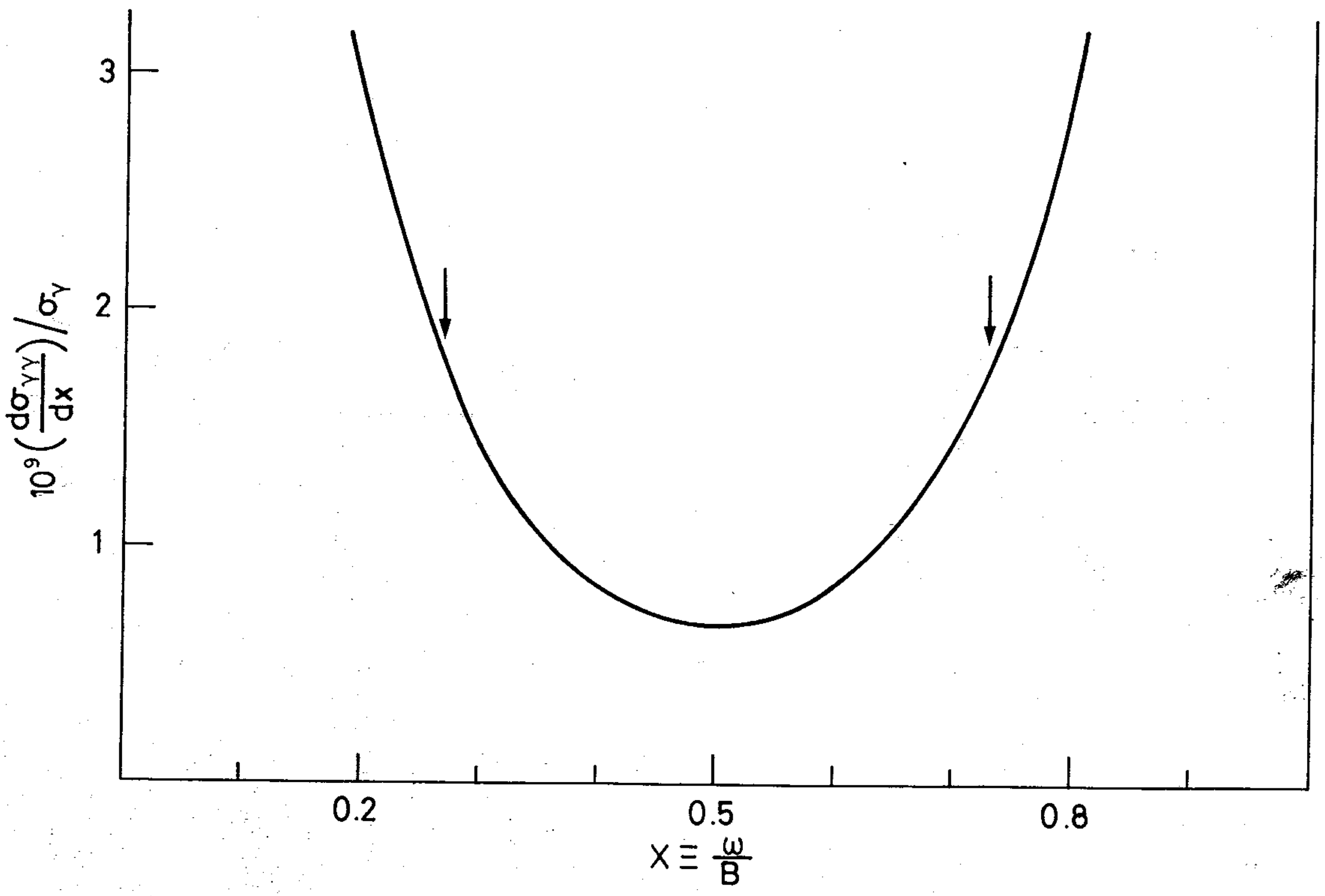
3
2
1
$10^9 \left(\frac{d\sigma_{\gamma\gamma}}{dx}\right) / \sigma_\gamma$
0.2
0.5
0.8
$X \equiv \frac{\omega}{B}$

FIG. 2