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LOW-ENERGY ELASTIC NEUTRINO-NUCLEON AND NUCLEAR SCATTERING AND ITS RELEVANCE FOR SUPERNOVAE

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ABSTRACT

Low-energy ($E_\nu \lesssim 20$ MeV) elastic neutrino nuclear scattering, from the neutron to the Fe - Ni region, is analyzed by using both the conventional extension to hadrons of the Weinberg model (I) and the recent vector model (II). The calculation is relevant for the understanding of supernova theory. In both models, spin (if any) and isospin dependent effects are comfortably below 10% of coherent scattering for $A > 10$. For the neutron, the cross-section for momentum transfer, when compared to the one of the isoscalar current, is: i) more than one order of magnitude larger in model (I); ii) about twice in model (II). It is pointed out that the vector model gives more dramatic effects in the supernova "strategy", both lowering the cross - section in the inner core and increasing it in the outer region.

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1. INTRODUCTION

The weak interactions play a crucial role in astrophysics, and the presence of new components such as neutral currents [1] may have an important effect in this field. It seems that the most spectacular application [2] is to the theory of supernovae. A long standing problem has been the manner in which a supernova can blow off its outer regions while leaving its core to collapse to form a neutron star. These explosions could provide an explanation for the origin of heavy elements and cosmic rays [3].

At the point where the core of the star is neutronized, its collapse to higher densities could be stopped, but the envelope would continue to fall into the core, making impossible that a supernova occurs and can lead to a neutron star. What was suggested [4] to solve this problem is that the envelope could be blown off by neutrinos produced in the core and able to leave it. What is needed is that the neutrinos give a lot of momentum or energy to the outer region, i.e., that the scattering in this region should be very strong, whereas it should be much less important in the core so that the neutrinos might escape.

Weak neutral currents have an interesting feature. In popular models [5]-[7] there is an isoscalar piece that, for non-strange hadrons, is given by the electromagnetic current. Then elastic electron and neutrino cross-sections by $I = 0$ targets are proportional to each other [8]. This means that, like the electromagnetic current, neutral currents allow the coherent scattering of neutrinos by whole nuclei [9]. The coherence is present as long as there are pieces in the weak neutral current which are spin and isospin independent, that is, the vector current of baryon number. This gives the hope that the scattering by the Fe-Ni region is so strong, when compared with the scattering by neutrons, that the explosion occurs. Wilson [10] made exploratory calculations to show the relevance of this argument, and Schramm and Arnett [11] pointed out that degeneracy effects tend to give even greater influence of these neutral currents. Up to now, the approach to the problem for the new ingredient has been through the interaction of neutrinos with the spin and isospin independent piece of the nucleonic current. The differential coherent scattering cross-section is then given by [8],[9]

$$\frac{d\sigma}{d\Omega} = \frac{G^2}{(2\pi)^2}\, a_0^2\, A^2\, E_\nu^2\,(1+\cos\theta) \qquad (1)$$

when the wave length corresponding to the momentum transfer is large compared with the nuclear radius (this is a fairly good approximation for astrophysical neutrino energies, $E_\nu \lesssim 20$ MeV). Here A is the nucleon number, Θ the lab frame scattering angle, and $(G\, a_o)$ the effective coupling constant of this contribution. With G the Fermi value, we have $a_o = \sin^2 \Theta_W$, with the same physical meaning of Θ_W both in the conventional extension to hadrons of the Weinberg model [5),6)] and in the recent vector model of Ref. 7). We shall designate these models by (I) and (II), respectively. Present data [12)] on muonless neutrino interactions give the value $a_o = 0.39 \pm 0.05$ if interpreted in model (I), and $a_o \approx 0.68$ for model (II) [7)].

In this note we want to study the effective neutrino mean free paths for momentum transfer, due to elastic nuclear scattering. The question we ask is whether the values extracted from Eq. (1) are reliable for light elements, in particular for the neutron case where, we shall see, the physics is entirely different. If there is a nucleonic current which is spin and/or isospin dependent, this piece will contribute to the process $\nu A \to \nu A$ as well as the other one. The analysis is made in models (I) and (II), with two different considerations in mind:

i) how the non-isoscalar neutral current piece modifies the simple result (1), in each model separately;

ii) how numerical results for the cross-section of momentum transfer compare between (I) and (II), for a given target.

In Section 2, we review the effective Lagrangian obtained in the two models we discuss, and deduce the differential cross-section of our interest. In Section 3, we express the cross-section for momentum transfer in terms of observed quantities, separating the coherent part from the isospin and spin dependent parts. Finally, in Section 4 we present our results and their relevance to the theory of supernovae.

2. NEUTRAL CURRENT THEORY

In gauge theories with neutral currents for weak interactions, the massive boson Z_μ arises from a combination of the gauge fields W^o_μ and B_μ; the orthogonal combination is the electromagnetic field A_μ. The neutral current, coupled to Z_μ, can be written in terms of the ones associated to W^o_μ and A_μ in the form

$$J_\mu^{Z} = J_\mu^{W^o} - 2 \sin^2 \theta_W \, J_\mu^{e.m.} \tag{2}$$

where Θ_W is the Weinberg angle of the mixing of the gauge bosons to obtain the physical neutral fields. In order to contact with the phenomenology of charged current weak interactions, some identification between the gauge group and a symmetry group for hadrons must be made. The known Cabibbo piece of the hadronic current coupled to the charged fields $W^{\pm}_{\mu}$ is an isovector, with V-A character, for non-strange hadrons. By itself, this piece will originate a neutral current $J^{W^0}_{\mu}$ with the same properties. However, the absence of $\Delta S = 1$ neutral currents indicates that extra pieces must be added to the Cabibbo current. In the conventional extension to hadrons [5] of the Weinberg model, with an extra V-A charmed current, the modification of $J^{W^0}_{\mu}$ only affects the contribution coming from strange (and charmed) constituents [6]. This would not change the properties of $J^{W^0}_{\mu}$ for the nucleon. However, in the recent vector model [7], the charged current also contains an extra V+A charmed piece, from which $J^{W^0}_{\mu}$ receives an isovector V+A contribution for nucleons. In both models, the content of the electromagnetic current is assumed to be well understood, with its isovector and isoscalar vector parts. We summarize this discussion by stating that, for nucleons, model (I) gives a neutral $J^{W^0}_{\mu}$ current which has isovector V-A properties, whereas model (II) contains an extra isovector V+A current, thus giving a pure $\Delta I = 1$ vector contribution to the neutral current from $J^{W^0}_{\mu}$.

As we are interested in a region of momentum transfer much less than the nucleon mass, we will not consider induced terms in the hadronic current. From Eq. (2) and our discussion, we are able to write

$$\langle N(p',\lambda')|\, J_\mu^{Z}\, |N(p,\lambda)\rangle$$

$$= \bar{u}(p',\lambda') \left\{ \begin{matrix} \tau_3 \frac{1}{2}[\gamma_\mu - g_A \gamma_\mu \gamma_5] \\ \tau_3 \gamma_\mu \end{matrix} \; - 2\sin^2\theta_W \frac{1+\tau_3}{2}\gamma_\mu \right\} u(p,\lambda) \tag{3}$$

where $g_A = -1.25$ as determined from charged current processes. For nuclei, using the impulse approximation, we obtain the following charge and current operators

$$J^0 = \sum_{i=1}^{A} \left\{ \begin{array}{l} -a_0 + \frac{1}{2}(1-2a_0)\,\tau_3^{(i)} \\ -a_0 + (1-a_0)\,\tau_3^{(i)} \end{array} \right\} e^{i\vec{q}\cdot\vec{r}_i}$$

$$\vec{J} = \sum_{i=1}^{A} \left\{ \begin{array}{l} g_A/2 \;\; \tau_3^{(i)}\,\vec{\sigma}^{(i)} \\ 0 \end{array} \right\} e^{i\vec{q}\cdot\vec{r}_i} \tag{4}$$

in the non-relativistic limit, with $a_0 = \sin^2 \theta_W$.

For astrophysical neutrino energies $E \lesssim 20$ MeV, the wavelength corresponding to the momentum transfer is larger than the nuclear radius, i.e., $(qR)^2 \ll 1$, so we shall forget retardation effects in the matrix elements of the nuclear currents $\langle A|J^Z_\mu|A\rangle$. These matrix elements have then no $\vec{q}$ dependence. The matrix element of the effective Lagrangian for the process $\nu A \to \nu A$ is given by

$$\langle \mathcal{L}_{eff} \rangle = \frac{G}{\sqrt{2}}\, \bar{\nu}\,\gamma^\mu (1+\gamma_5)\,\nu\, \langle A \,|\, J^Z_\mu \,|\, A \rangle \tag{5}$$

and we are only interested in the differential cross-section. The sum over initial and final polarizations in the square of Eq. (5) implies that there are no interferences among the different components of the matrix elements $\langle A|J^Z_\mu|A\rangle$ and, in view of (4), that there are no vector-axial vector interferences. Thus <u>our results are the same for neutrino and antineutrino scattering</u>. This is a rigorous result in model (II), but it is a consequence of our approximations in model (I). We obtain

$$\frac{1}{2j+1} \sum_{m,m'} |\langle \mathcal{L}_{eff} \rangle|^2 = \frac{4G^2}{2j+1} \sum_{m,m'} \Big\{ [EE' + (\vec{p}\cdot\vec{p}\,')]\, |\langle j,m'|J^0|j,m\rangle|^2 + [EE' - \tfrac{1}{3}(\vec{p}\cdot\vec{p}\,')]\, |\langle j,m'|\vec{J}|j,m\rangle|^2 \Big\} \tag{6}$$

where $(E,\vec{p})$ and $(E',\vec{p}\,')$ are the four momenta of initial and final neutrinos, j is the angular momentum of the nucleus, and $m(m')$ its initial (final) third component.

In terms of the lab frame scattering angle θ, and neglecting nuclear recoil energy, we obtain the following differential cross-section

$$\frac{d\sigma}{d\Omega}=\frac{G^2}{(2\pi)^2}E^2\frac{1}{2j+1}\sum_{m,m'}\left\{(1+\cos\theta)\left|\langle j,m'|\sum_{i=1}^{A}\begin{bmatrix}-a_0+\frac{1-2a_0}{2}\tau_3^{(i)}\\ -a_0+(1-a_0)\tau_3^{(i)}\end{bmatrix}|j,m\rangle\right|^2\right.$$

$$\left.+\left(1-\frac{1}{3}\cos\theta\right)\left|\langle j,m'|\sum_{i=1}^{A}\begin{bmatrix}g_A/2\ \tau_3^{(i)}\vec{\sigma}^{(i)}\\ 0\end{bmatrix}|j,m\rangle\right|^2\right\} \qquad (7)$$

The approximations we made, in order to get Eq. (7), are equivalent to consider, kinematically, the scattering as the one by a fixed centre source of the external weak potential. In elastic scattering, only three momentum is transfered by neutrinos to the nucleus.

3. CROSS-SECTION FOR MOMENTUM TRANSFER

As neutrinos deposit <u>momentum</u> in each elastic interaction, we calculate the cross-section for momentum transfer

$$\sigma_{tr}\equiv\int d\Omega\,(1-\cos\theta)\frac{d\sigma}{d\Omega}=\frac{G^2}{\pi}E^2\,\frac{2}{3}\left\{\begin{bmatrix}-a_0A+\frac{1-2a_0}{2}(Z-N)\\ -a_0A+(1-a_0)(Z-N)\end{bmatrix}^2\right.$$

$$\left.+\frac{5}{3}\,\frac{1}{2j+1}\sum_{m,m'}\left|\langle j,m'|\sum_{i=1}^{A}\begin{bmatrix}g_A/2\ \tau_3^{(i)}\vec{\sigma}^{(i)}\\ 0\end{bmatrix}|j,m\rangle\right|^2\right\} \qquad (8)$$

where $Z(N)$ is the number of protons (neutrons) in the target, and $A=Z+N$. We see that the spin-dependent term, if it exists, is more effective for momentum transfer, as it is already apparent from the angular distribution given in Eq. (7). This is the origin of the factor $5/3$ in front of this term, in Eq. (8).

We shall write this cross-section with the coherent part separated out

$$\sigma_{tr} = \frac{2}{3}\,\frac{G^2}{\pi}\,a_0^2\,A^2\,E^2\left\{[1+T]^2 + S^2\right\} \tag{9}$$

where T indicates the correction coming from pure isospin dependent effects which give interference with the coherent part,

$$T = \frac{N-Z}{a_0 A}\left\{\begin{matrix} \frac{1-2a_0}{2} \\ 1-a_0 \end{matrix}\right\} \tag{10}$$

whereas S refers to the spin and isospin dependent effects, which in model (I) are given by

$$S^2 = \frac{5}{3}\left(\frac{g_A}{2}\right)^2 \frac{1}{a_0^2 A^2}\,\frac{1}{2j+1}\sum_{m,m'}\left|\langle j,m'|\sum_{i=1}^{A}\tau_3^{(i)}\vec{\sigma}^{(i)}|j,m\rangle\right|^2 \tag{11}$$

and they do not exist in model (II).

It is clear that we will face three possible situations.

i) All nuclei with $N=Z \Rightarrow T=0$, $S=0$.

ii) Nuclei $N\neq Z$, but $j=0 \Rightarrow T\neq 0$, $S=0$.

iii) $N\neq Z$ and $j\neq 0 \Rightarrow T\neq 0$, $S\left\{\begin{matrix}\neq 0\\ =0\end{matrix}\right\}$.

For the cases (i) and (ii), we have explicitly the answer because T (if different from zero) is given by (10). However, model (I) in the case (iii) needs the calculation of S, Eq. (11), which involves the use of a particular nuclear model. Instead, we shall rely on the following argument. For $N>Z$ [we have the exceptions of proton and ^3He, but in those targets the calculation of (11) is trivial], the ground state has isospin $I=(N-Z)/2$ with third component $I_3=-I$. The ground state of the analogue nucleus, with the same isospin $(Z\rightleftarrows N)$ and third component $I_3=I$, will be unstable against a superallowed β^+ decay $(I,I)\rightarrow(I,I-1)$. We are interested in the expectation value of $\tau_3\vec{\sigma}$ in the state $|I,-I\rangle$, so it can be related to the matrix element of $\tau_-\vec{\sigma}$ appearing in the β^+ decay considered. We are able to write the connection

$$|\langle I,-I|\sum_{i=1}^{A}\tau_3^{(i)}O^{(i)}|I,-I\rangle|^2 = (N-Z)\,|\langle I,I-1|\sum_{i=1}^{A}\tau_-^{(i)}O^{(i)}|I,I\rangle|^2 \tag{12}$$

With $O^{(i)} \equiv \vec{\sigma}^{(i)}$, the right-hand side of (12) can be extracted from the (ft) value. This means that it is possible to express S^2 in terms of measured quantities; a straightforward calculation gives

$$S^2 = \frac{5}{12a_0^2A^2}(N-Z)\left\{\frac{K}{(ft)} - (N-Z)\right\} \tag{13}$$

where $K = 2\pi^3\ \ell n2/(m_e^5G^2)$, and (ft) is the value for the β^+ transition $(I,I) \to (I,I-1)$ we discussed above.

By using (10) and (13) in Eq. (9) we calculate the modification to the elastic cross-section for momentum transfer, due to spin and isospin dependent effects in the hadronic weak neutral current. It is worth while to discuss here what are the results in the special case of the neutron. We can rewrite $1+T = (2Z/A)+(N-Z)/(2a_0A)$ in model (I) and $1+T = (2Z/A)+(N-Z)/(a_0A)$ in model (II), in which form the separation between the "electromagnetic" contribution and the $\Delta I = 1$ $J_\mu^{W^0}$ piece is apparent; S^2, if it exists, is entirely given by the axial part of $J_\mu^{W^0}$. The isoscalar components came from the electromagnetic current, which is only coupled to protons in the approximation we worked out. This means that the physics for the neutron case is completely different from the one which generates the coherent scattering. All the pieces for the neutron come from the weak neutral current analogue to the charged current. This has the implication that the predicted cross-section, for the neutron, becomes independent of the Weinberg angle, i.e., on the value of a_0. All these comments are the manifestation, on the results, of the assumed structure for the weak neutral current, Eq. (2), independent of the particular model (I) or (II). For the neutron, all contributions arise from $J_\mu^{W^0}$, whereas the isoscalar piece to build up coherent nuclear scattering comes entirely from J_μ^{em}.

4. RESULTS

In order to present the results from the neutron case to the Fe-Ni region, it is convenient to use the separation given in Eq. (9). Deviations of $[1+T]^2$ from 1 indicate pure isospin dependent effects, whereas the deviations of S^2 from zero, in model (I), indicate both spin and isospin dependent effects. The "correction factor" to coherent scattering is obtained by adding these two terms. In the table we give the results for a selected sample of $N \neq Z$ nuclei. The two rows associated to each element correspond to the results obtained in models (I) and (II), respectively, with calculations performed using the values $a_0(I) = 0.4$ and $a_0(II) = 0.7$.

For S^2 the second row is always empty, indicating that this effect does not exist in model (II). In model (I), the result $S^2 = 0$ for scalar nuclei $j = 0$ is explicitly stated. For nuclei in our class (iii), we are able to extract the S^2 value, from Eq. (13), up to ^{39}K, by using the corresponding (ft) value [13]. We see that the value of S^2 is below 1% from ^{23}Na on, so at this level no effect of this term is expected in heavier elements.

From the table we see that isospin dependent effects are somewhat larger in model (II). This is understood because the vector current of $J_\mu^{W^0}$ is twice that of model (I) and, although $a_0(I) < a_0(II)$, we have $2a_0(I) > a_0(II)$. However, in very light elements $(A < 10)$, the effects of S^2 are largely dominating in model (I). For the neutron, the cross-section for momentum transfer becomes 14 times larger than the one extracted from the "coherent scattering" formula, Eq. (1). This tremendous factor comes essentially from the spin-isospin dependent term present in the axial current, which contains several enhancement factors: larger matrix element, more effective angular distribution and it is not suppressed by a_0^2, when compared with the isoscalar part. In model (II), things are not so drastic. The result for the neutron is about twice the one from Eq. (1), the change coming from isospin dependence of the neutral current. For $A > 10$, the modifications to coherent scattering are always below 10%, and that independent of the particular model. The table and the discussion above answers the first question we posed in the Introduction.

In connection with the second point, we see that in heavy nuclei, in which the non-isoscalar piece is only marginal, that the cross-section for momentum transfer is essentially determined by the value of a_0 in each model, then

$$\left.\frac{\sigma_{tr}(II)}{\sigma_{tr}(I)}\right|_{A>10} \approx 3.1 \tag{14}$$

For the neutron, however, the results for this cross-section are, as pointed out before, independent of a_0. The result gives

$$\left.\frac{\sigma_{tr}\,(\mathrm{II})}{\sigma_{tr}\,(\mathrm{I})}\right|_{\mathrm{neutron}} = 0.45 \tag{15}$$

Thus, in the neutronized inner core of the supernova, the effective neutrino mean free path becomes larger in model (II) when compared with model (I). In the past, an estimate for this mean free path was extracted from Eq. (1) with $a_0 = a_0$ (I). We have seen that it becomes about 14 times shorter if one uses model (I) and about six times shorter with model (II).

The comparison (14) and (15) shows that the veotor model is more effective than the conventional model, in order to do the job needed in the supernova mechanism, in both aspects: neutrinos may escape more easily from the inner core, <u>and</u> the scattering is stronger in the outer region.

The difference between the neutrino opacities in the neutronized core and the high A region is <u>not</u> as great as was thought from the "coherent scattering" formula, Eq. (1). In model (II) the difference remains quite dramatic, but it goes down if one believes model (I). For example, the ratio of mean free paths, due to elastic scattering, for neutron matter and for $A \approx 60$ elements is about 4 or 5 (instead of 60), and it becomes 30 in (II) for the same neutrino energy. In any case, the difference is there and the big cross-section in the outer layers remains the same (I), or is increased (II).

We hope that the remarks and results contained in this paper may be useful for understanding the mechanism, because it appears that the elastic neutral current scattering discussed here is a very important process, <u>both</u> for neutrons and for heavy elements. A word of caution, however. Detailed answers may strongly depend on the model one chooses, and thus we will need to wait until the quantum number content of the hadronic neutral current is established.

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Table

Pure isospin dependent effects $[(1+T)^2-1]$ and spin-isospin dependent effects $[S^2]$ in the neutrino - nuclear cross-section for momentum transfer (see the text)

^{A}Z	$(1+T)^2$	S^2	^{A}Z	$(1+T)^2$	S^2
n	1.56 2.04	12.18	^{35}Cl	1.01 1.02	0.00
p	0.56 0.32	12.18	^{37}Cl	1.04 1.07	0.00
^{3}H	1.17 1.31	1.28	^{39}K	1.01 1.02	0.00
^{3}He	0.85 0.73	1.28	^{40}Ar	1.05 1.09	0
^{7}Li	1.07 1.13	0.13	^{45}Sc	1.03 1.06	
^{9}Be	1.06 1.10		^{48}Ti	1.04 1.07	0
^{11}B	1.05 1.08	0.01	^{51}V	1.05 1.09	
^{19}F	1.03 1.05	0.02	^{52}Cr	1.04 1.07	0
^{23}Na	1.02 1.04	0.00	^{55}Mn	1.05 1.08	0
^{26}Mg	1.04 1.07	0	^{56}Fe	1.04 1.06	
^{27}Al	1.02 1.03	0.00	^{58}Ni	1.02 1.03	0
^{31}P	1.02 1.03	0.00	^{59}Co	1.04 1.07	
			^{60}Ni	1.03 1.06	0

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