



Ref.TH.2173-CERN

LONG RANGE POTENTIALS AND THE ELECTROMAGNETIC POLARIZABILITIES

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A B S T R A C T

The long range spin and velocity independent forces of electromagnetic origin which act between any two systems are studied for those cases in which no forces of this type exist to order e^2 . It is shown that they are uniquely determined by the charge, magnetic moment and polarizabilities of both systems not only to the dominant order r^{-n} but also to the next one $r^{-(n+1)}$. These potentials provide the link between Compton scattering polarizabilities (response to real photons) and classically defined polarizabilities (response to static electromagnetic field). The two definitions are shown to be equivalent for neutral spinless systems; the problems arising for a neutral particle with magnetic moment are studied in detail. The $r^{-(n+1)}$ terms have no classical counterpart, since they are due to the relativistic quantum propagation of the system which carries charge or magnetic moment.

The results are of general validity with analyticity, crossing, unitarity and gauge invariance as only inputs. The most general conclusion is that the polarizabilities represent electromagnetic properties of a system at order e^2 , as the charge and magnetic moment do at order e . Thus they give the strength of the response to electric and magnetic fields, independently of the specific characteristics of the electromagnetic agent.

Ref.TH.2173-CERN

9 June 1976

1. - INTRODUCTION

This paper is devoted to the study of the longest range effective potentials which dominate, at large distances, the electromagnetic interaction between two systems, one of which at least is neutral. We were led to this problem when asking such a simple question as the following. The experimental study of low energy neutron-nuclear scattering ¹⁾ is used to obtain the electric polarizability of the neutron, defined as the coefficient of a r^{-4} potential acting between these two systems. Is this coefficient exactly the same parameter as the one which enters the Compton amplitude for the neutron, through low energy theorems? It is well known that the Compton amplitude, up to second order in the photon energy, is determined by two parameters, the so-called electric and magnetic polarizabilities (apart from the mass, charge and magnetic moment). These structure constants may be interpreted as the ones which give the scattering contribution produced by slowly-varying induced electric and magnetic dipoles driven by the electric and magnetic field of the photon. The aim here is to obtain the spin and velocity independent electromagnetic interaction between any two systems, when they are far apart, and to look for the connection with these low energy Compton scattering parameters.

Classical electrodynamics says that an electric field induces an electric dipole in a piece of matter proportional to the applied electric field; the proportionality constant (or tensor for non-isotropic matter) of the linear term is called the electric polarizability. The longest range potential which represents the interaction of neutral isotropic matter with a Coulomb field is then

$$V(r) = -\frac{1}{2} \left(\frac{e^2}{4\pi} \right) \frac{\alpha}{r^4} + O(r^{-6}) \quad (1)$$

where $(e^2/4\pi)$ is the fine structure constant and α is the electric polarizability. The next order term, proportional to r^{-6} , would come from the part of the induced dipole which is quadratic in the applied electric field. If we study the electromagnetic interaction between any two systems, we must reproduce this classical limit (1) when one of the systems is charged and infinitely heavy, and the other is neutral and spinless.

The electric polarizability α also appears in the low energy Compton amplitude for this scalar target. In the laboratory system, and in the Coulomb gauge, the amplitude for a spinless particle of mass m and charge Z is ^{*})

$$T_0 = 2 Z^2 \vec{e} \cdot \vec{e}'^* - \left(\frac{4\pi}{e^2}\right) 2 m \omega \omega' \left[\alpha \vec{e} \cdot \vec{e}'^* + \beta (\hat{k} \times \vec{e})(\hat{k}' \times \vec{e}'^*) \right] + O(\omega^4) \quad (2)$$

where ω , \hat{k} and \vec{e} are the energy, the direction of the momentum and the polarization vector of the incoming photon, and the primed quantities correspond to the outgoing photon. The contribution to the scattering due to the magnetic field of the photon is determined by the magnetic polarizability β . Similar to α , the constant β should be precisely the one of the magnetic dipole induced by a magnetic field in isotropic matter and linear in the field. The potential which represents the interaction of the induced magnetic dipole with a static one $\vec{\mu}$ (which creates the magnetic field) would be

$$V(r) = -\frac{1}{2} \frac{3(\vec{\mu} \cdot \hat{r})^2 + \vec{\mu}^2}{r^6} \beta + O(r^{-8}) \quad (3)$$

A particle with spin has an intrinsic magnetic moment $\vec{\mu}$ proportional to its spin. With μ the quantum mechanical magnetic moment, in natural units, (3) becomes

$$V(r) = -3 \left(\frac{e^2}{4\pi}\right) \left(\frac{\mu}{2M}\right)^2 \frac{\beta}{r^6} + O(r^{-8}) \quad (4)$$

The potential (4) thus corresponds to the longest range velocity and spin independent forces between a neutral infinitely heavy particle with magnetic moment μ (but μ/M finite) and a neutral spinless particle, due to the magnetic moment μ .

At this level, one already realizes that the knowledge of the Compton amplitude (2) for a neutral spinless system determines the response of that system to an external electric or magnetic field. Quantum-mechanically, the counter part of this problem may be studied by considering the

^{*}) The amplitude (2) is the laboratory version of the invariant T matrix. Our normalization conventions correspond to write for the cross-section $d\sigma/d\Omega = (1/4m^2)(e^2/4\pi)^2 (\omega'/\omega)^2 \sum \sum |T|^2$.

interaction of that neutral spinless system with a charged spinless particle or a neutral spin $\frac{1}{2}$ particle, respectively. One question comes to mind immediately. When this last particle (the one which "creates" the field) is not infinitely heavy, what are the quantum modifications to the potential (1) or (4)? In particular, we would like to know which are the terms of the long range potentials which are determined by the two parameters α and β of (2).

We see that, in order to obtain the generalization of the potential (4), we are led to the study of the interaction between two neutral objects. In this situation, there may be a superposition with Van der Waals forces, the ones generated by induced dipoles on both systems. For two spinless particles [in which case, (4) does not exist], London ²⁾ found a r^{-6} potential at long distances, using second order perturbation theory with unretarded electric dipole transitions. However, the problem was taken up again by Casimir and Polder ³⁾, whose aim was to include retardation effects. Working within field theory, but only including electrostatic interactions, they found

$$V(r) = - \frac{23}{4\pi} \frac{\alpha_1 \alpha_2}{r^7} \quad (5)$$

where α_i is the electric polarizability of particle i . Finally, Feinberg and Sucher ⁴⁾ gave the answer to this particular problem by using, as only input, analyticity. This allowed them to obtain, through a dispersion relation technique, the following long-range potential between two neutral spinless systems

$$V(r) = - \frac{1}{4\pi} [23(\alpha_1 \alpha_2 + \beta_1 \beta_2) - 7(\alpha_1 \beta_2 + \alpha_2 \beta_1)] \frac{1}{r^7} + O(r^{-9}) \quad (6)$$

where β_i is the magnetic polarizability of particle i . It is clear from (6) that electric and magnetic fields come in on the same footing and, therefore, the full electromagnetic interaction, and not its electrostatic limit, should be used. The expression (6) turns out to be true also for particles with spin ⁵⁾ as long as only the spin independent potential is considered and as long as no static electric and magnetic multipoles are taken into account, α and β being the average of the diagonal elements of the polarizability tensors. In this context, we shall study the general long range potential

between two neutral systems, one spinless and the other with spin $\frac{1}{2}$, in order to see whether the generalization of (4) might contain a r^{-7} term which would be present in addition to (6).

From the preceding discussion we conclude that the complete answer is only known for the case of two neutral spinless particles and it corresponds to the potential (6). Our objective, in this work, is to study those cases in which at least one particle is neutral and at least one particle is spinless, because those are the situations for which there is no velocity independent long range (charge-charge or magnetic moment-magnetic moment) interaction of order e^2 . For the sake of clarity, let us specify the spin and charge assignments we shall therefore deal with.

a) Two spinless particles, one charged and one neutral.

The aim is to obtain the generalization of the potential (1) for the case of two systems with finite mass. Is there any quantum effect to the r^{-4} term? Is there a r^{-5} term, also completely determined by the polarizabilities? When studying this case a), we shall prove that the terms of the potential which are determined by the knowledge of the low energy Compton amplitude are not changed if we allow the charged particle to have spin.

b) Two neutral particles, one of spin $\frac{1}{2}$ and one spinless.

Here the aim is to obtain the generalization of (4), associated with the magnetic moment of the spin $\frac{1}{2}$ system. Apart from that contribution, there will be a term identical to (6). We may ask here similar questions to the ones already posed in a).

c) A spinless charged and a spin $\frac{1}{2}$ neutral particle.

This case can be considered as the generalization of a), by including the static multipole moments of the neutral system and not only the induced ones. It is interesting to point out that some modification is expected, when compared with a). The low energy spin averaged Compton scattering amplitude, for a spin $\frac{1}{2}$ particle of mass m , charge Z and anomalous magnetic moment μ , is given by

$$T_S (\text{spin averaged}) = T_0 - \frac{\omega \omega'}{2m^2} \left[(Z + \mu)^2 (1 - \cos \theta) (\hat{k} \times \vec{e}) \cdot (\hat{k}' \times \vec{e}'^*) \right. \\ \left. + \mu (2Z + \mu) (\vec{e} \cdot \vec{e}'^* - (\hat{k} \times \vec{e}) \cdot (\hat{k}' \times \vec{e}'^*)) \right] + O(\omega^4) \quad (7)$$

with T_0 given by the same expression (2) as for the spinless case (we shall put a subscript s on α and β for distinction from the spinless case), and θ is the laboratory frame scattering angle. Furthermore, in defining α_s and β_s for spin $\frac{1}{2}$, some convention is needed. In Eq. (7) we have followed the widespread use of separating out explicitly the so-called Born contribution (in the sense of Feynman diagrams, with the single particle pole and on-shell vertices). This is, in particular, the way in which the analysis of the experimental angular distribution ^{6),7)} for protons is made in order to extract the values of the electric and magnetic polarizabilities. From Eq. (7), we see that the response of a neutral particle with anomalous magnetic moment μ to the electric field of the photon contains a term proportional to μ^2 , in addition to the electric polarizability α_s . The study of case c) will allow us to derive the precise form of the long range neutron-nuclear potential, in terms of the relevant parameters. It will thus provide a tool for the extraction of the value of the electric polarizability of the neutron from low energy neutron-nuclear scattering.

There are several points in the connections we wish to establish which should be emphasized. The main idea underlying the basic approach is, as we shall see, that the knowledge of the low energy real Compton amplitude controls the long range interaction between any two particles, due to the exchange of virtual photons. The fact that the forward longitudinal amplitude, for virtual photons, has to do with the parameters of real photon scattering was already stressed in Ref. 8), where the rôle of the electric polarizability α for this kind of connections was discussed. In our opinion, all this is but a manifestation of the fact that, like the charge and the magnetic moment, the electromagnetic polarizabilities are fundamental structure constants of the system describing the response to any electromagnetic agent. They are not simply a way of parametrizing the low energy real Compton amplitude. But, besides this physical concept, the problem of an unambiguous and precise definition of the parameters remains. To us, for the present approach, it seems more adequate to start from the definitions associated with the real Compton amplitude and then establish the possible connections. The classical definitions of the structure constants as coefficients of the longest range term of the potential are unambiguous for spinless neutral particles, but become blurred when the particle has static electromagnetic multipoles. On the other hand, we shall see that, when we are not in the limit of a static field, the same parameters determine not only the leading order term but also the next order term in the potential.

The paper is organized as follows. In Section 2, we define the effective potential in terms of the centre-of-mass spin averaged scattering amplitude. We show that the long range behaviour is controlled by the absorptive part of that amplitude near $t=0$ ($t \gtrsim 0$) and fixed s , which is determined from two-photon exchange contributions. Section 3 discusses the Compton scattering tensors for scalar and spin $\frac{1}{2}$ objects, with the explicit separation of the single particle poles and the polarizabilities in the corresponding amplitudes. The expressions for the long range potentials are obtained in Section 4, for the three cases discussed above. We shall show that an electric-magnetic symmetry for the results is preserved when the polarizabilities for spin $\frac{1}{2}$ particles are defined in terms of the continuum contributions of the spin averaged amplitudes. Finally, Section 5 gives a recapitulation of the main results and the conclusions obtained from this study.

2. - THE LONG RANGE POTENTIAL

This section is devoted to the kinematics and to the dispersion relation technique which will be used to obtain the long range effective potentials which we are interested in. By definition, long range potentials will be those which behave like an inverse power of r at large distances. In our study, at least one particle is neutral and at least one particle is spinless, so no velocity independent forces of the type we consider can appear in the one-photon exchange approximation. Therefore, the relevant diagram will be the one in which two photons are exchanged between the two objects of masses m and M . This is shown, together with the kinematics, in the Figure. The following auxiliary momenta will be used

$$\begin{aligned}
 P &= \frac{1}{2} (p + p') & ; & & R &= \frac{1}{2} (r + r') \\
 K &= \frac{1}{2} (k + k') & ; & & q &= p' - p = r - r' = k' - k
 \end{aligned}
 \tag{8}$$

and the Mandelstam variables are defined as

$$s = (P + R)^2 & ; & t = q^2 & ; & u = (R - P)^2
 \tag{9}$$

As the particles are on the mass shell, the following relationships for the scalar products hold

$$\begin{aligned} P \cdot q &= R \cdot q = 0 \quad ; \quad P^2 = m^2 - \frac{t}{4} \quad ; \quad R^2 = M^2 - \frac{t}{4} \\ P \cdot R &= \frac{1}{2} (s - M^2 - m^2) + \frac{t}{4} \end{aligned} \quad (10)$$

The physical s channel region is defined by

$$\begin{aligned} s &\geq s_0 = (M + m)^2 \\ 2(M^2 + m^2) - s - \frac{(M^2 - m^2)^2}{s} &\leq t \leq 0 \end{aligned} \quad (11)$$

and the physical t channel region by

$$\begin{aligned} t &\geq t_0 = 4 \max(M^2, m^2) \\ M^2 + m^2 - \frac{t}{2} - 2 \sqrt{\frac{t}{4} - M^2} \sqrt{\frac{t}{4} - m^2} &\leq s \\ &\leq M^2 + m^2 - \frac{t}{2} + 2 \sqrt{\frac{t}{4} - M^2} \sqrt{\frac{t}{4} - m^2} \leq 0 \end{aligned} \quad (12)$$

Let us denote by $T(s, t)$ the invariant amplitude which corresponds to the spin averaged part of the diagram of the Figure. The effective potential must reproduce it in the Born approximation. In order to obtain the potential from $T(s, t)$ we need to know the dependence of the amplitude with the momentum transfer squared t . The use of dispersion relations will allow us to obtain the explicit dependence on t . We shall assume that the amplitude $T(s, t)$ satisfies maximum analyticity of the first kind, that means, that all singularities correspond to physical intermediate states and are therefore determined by unitarity. We write the following fixed s dispersion relation

$$T(s, t) = \frac{1}{\pi} \int_0^{\infty} dt' \frac{\text{Abs } T(s, t')}{t' - t} - \frac{1}{\pi} \int_{-\infty}^{(M-m)^2 - s} dt' \frac{\text{Abs } T(s, t')}{t' - t} \quad (13)$$

Possible subtractions in (13) have not been considered for the moment. The lowest mass intermediate state of the u channel [the second term in (13)] has been supposed to be the one given by elastic unitarity. The effective potential is defined as the Fourier transform, in the centre-of-mass system for the s channel, of $T(s, t)$. We are interested in that part which is independent of s , that is

$$V(r) = \frac{1}{(2\pi)^3} \frac{e^4}{4Mm} \int_0^\infty d^3q e^{i\vec{q}\cdot\vec{r}} T(t = -\vec{q}^2) \quad (14)$$

where $T(t)$ is the s independent part of $T(s,t)$. As it will be shown below, for the long range part of the potential we are allowed to forget the left-hand cut (in t) dispersive integral, so that we can write

$$V(r) = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{Mm} \frac{1}{r} \int_0^\infty dt \text{Abs } T(t) e^{-\sqrt{t}r} \quad (15)$$

In obtaining Eq. (15), the Fourier and the dispersive integrations, in (14) and (13), have been exchanged, which is allowed as the denominator is never negative, therefore the integral converges absolutely. Equation (15) is the one which defines the long range, spin and velocity independent effective potential we are interested in. It is worth noting that it gives inverse power behaved potentials only because the dispersive integration starts at $t=0$; if it started at a positive value of t the result would be a superposition of Yukawa type potentials, and therefore, by definition, of short range. This is precisely the kind of argumentation which allows us to forget the left-hand cut in (13). We cannot exchange the Fourier and the dispersive integrations there, as the denominator goes through a zero changing sign. One can, however, define a Majorana exchange potential corresponding to the Fourier transform of the amplitude with respect to $\vec{q}_{\text{exch}} = \vec{p}' + \vec{p}$ instead of $\vec{q} = \vec{p}' - \vec{p}$. The left-hand cut dispersive integral can be exchanged with the \vec{q}_{exch} integration, giving finally an expression similar to Eq. (15), but with the essential difference that the integration region starts at $4Mm$ instead of at zero. Therefore, no long range Majorana exchange potentials are obtained. The essential feature of a long range potential is that the associated amplitude cannot be expanded in natural powers of t , for low values of the momentum transfer t . From Eq. (13) we see that for $|t| < 4mM$, the left-hand cut contribution to $T(s,t)$ can be expanded in a power series in t , so it will not be considered here.

It is also not difficult to convince oneself that even if subtractions are required in the dispersion relation, the result (15) for the long range potential is still valid, as may be seen by dispersing $T(s,t)/(t+c)$, c being any constant. We thus conclude that the long range potentials will be uniquely given by the low t behaviour of the t channel

absorptive part of the scattering amplitude and therefore determined by the two-photon exchange cut. It is to its study that we turn now.

The amplitude $T(s,t)$ associated with the diagram of the Figure is given by

$$T(s,t) = \frac{i}{2} \int \frac{d^4 K}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \frac{1}{k'^2 + i\epsilon} A(s,t, P \cdot K, R \cdot K, q \cdot K, K^2) \quad (16)$$

where $A(s,t, P \cdot K, R \cdot K, q \cdot K, K^2)$ can be written in terms of the virtual Compton scattering tensors of the two particles, of masses m and M , as

$$A(s,t, P \cdot K, R \cdot K, q \cdot K, K^2) = T_{\mu\nu}^{(m)}(P, q, K) T^{(M)\mu\nu}(R, q, K) \quad (17)$$

The absorptive part of the amplitude (16) corresponding to the two-photon intermediate state cut can be calculated, from unitarity, as an integral over real intermediate two-photon states in the following way

$$\text{Abs } T(s,t) = -\frac{\pi^2}{2} \int \frac{d^4 K}{(2\pi)^4} \delta(k^2) \delta(k'^2) \theta(-k^0) \theta(k'^0) A(s,t, P \cdot K, R \cdot K, q \cdot K, K^2) \quad (18)$$

With the conditions specified in Eq. (18), the following identities hold

$$q \cdot K = 0 \quad ; \quad K^2 = -\frac{t}{4} \quad (19)$$

Choosing the t channel centre-of-mass system, for which $q^\mu \equiv (\sqrt{q^2}, 0, 0, 0)$, the absorptive part of the invariant amplitude may be finally written, for $t \geq 0$, as

$$\text{Abs } T(s,t) = -\frac{1}{128\pi^2} \int d\hat{K} A(s,t, \hat{P} \cdot \hat{K}, \hat{R} \cdot \hat{K}) \quad (20)$$

where the integration is performed over the angles of the vector \vec{K} .

The problem we are faced with now is the following. Equation (20) for the absorptive part of the amplitude $T(s,t)$ is well defined in the physical t channel region (12), as there the vectors \vec{P} and \vec{R} are

real. On the other hand, we need to know in (15) the behaviour of this absorptive part at $t \gtrsim 0$. If the calculation of (20) were performed in the physical t channel region we would have to continue the result analytically to the region which interests us. Furthermore, this approach would require the knowledge of the explicit dependence of the integrand of (20) in $\hat{P} \cdot \hat{K}$ and $\hat{R} \cdot \hat{K}$, and it would force us to write new dispersion relations for the Compton amplitudes in these variables. Instead, we shall calculate the integral (20) directly in the region of momentum transfer $t \gtrsim 0$, going to zero from positive values, as required in (15). In order to keep the angular variables of integration in Eq. (20) within the limits $-1 \leq \hat{P} \cdot \hat{K}, \hat{R} \cdot \hat{K} \leq 1$, we must also have $-1 \leq \hat{P} \cdot \hat{R} \leq 1$. In the region $0 \leq t \leq 4 \min(M^2, m^2)$, the moduli of \vec{P} and \vec{R} are imaginary, and

$$\hat{P} \cdot \hat{R} = \frac{\frac{s - M^2 - m^2}{2} + \frac{t}{4}}{\sqrt{M^2 - \frac{t}{4}} \sqrt{m^2 - \frac{t}{4}}} \underset{\substack{t \rightarrow 0 \\ s \rightarrow s_0}}{\sim} 1 + \frac{s_0}{8M^2 m^2} t + \frac{s - s_0}{2Mm} \quad (21)$$

With the above conditions, the variables must satisfy

$$\begin{aligned} M^2 + m^2 - \frac{t}{2} - 2\sqrt{M^2 - \frac{t}{4}} \sqrt{m^2 - \frac{t}{4}} &\leq s \\ &\leq M^2 + m^2 - \frac{t}{2} + 2\sqrt{M^2 - \frac{t}{4}} \sqrt{m^2 - \frac{t}{4}} \leq s_0 \end{aligned} \quad (22)$$

This includes the physical threshold value s_0 only when $t=0$. As discussed before, we are interested in the velocity independent part of the long range potential, and thus we shall keep in the amplitude the contribution which is independent of $s - s_0$. As the calculation is strictly valid only for the region (22) in s , one makes an analytic continuation into the $s = s_0$ limiting point. Nowhere, when one goes from the physical t channel region to $0 \leq t \leq 4 \min(M^2, m^2)$ keeping $-1 \leq \hat{P} \cdot \hat{R} \leq 1$, the integration of Eq. (20) encounters any singularities. Therefore absolute convergence is satisfied.

When we calculate Eq. (20) the advantage of the $t \gtrsim 0$ region is that the Compton amplitudes can be expanded around $t=0$. This also means around $P \cdot K = R \cdot K = 0$. The values of the amplitudes at that point are known from physical Compton scattering. Further the explicit dependence in $\hat{P} \cdot \hat{K}$ and $\hat{R} \cdot \hat{K}$ is known from the form of the Compton scattering tensors,

which is imposed by the symmetries of the problem. Because of that expansion in t , we see in Eq. (21) that one cannot go, a priori, to the limit $s = s_0$ before calculating (20), since then $\hat{P} \cdot \hat{R} \geq 1$. The approximation of putting $\hat{P} \cdot \hat{R} = 1$, used in Ref. 4) for the calculation of the Van der Waals forces, is valid only if neither $O(t)$ nor $O(s - s_0)$ terms of $\hat{P} \cdot \hat{R}$ contribute to the leading terms, in t , of $\text{Abs } T(s, t)$. This is not necessarily the case for the problems we are going to study, as the existence of propagators (for the single particle contributions to the Compton amplitudes) could force us to include $O(t)$ terms of $\hat{P} \cdot \hat{R}$. In that situation, it is crucial to keep track of which is a t dependent term, and which is a $(s - s_0)$ dependent term. The first one contributes to the potential in Eq. (15), whereas the second one should be forgotten, once the expression for $\text{Abs } T(s, t)$ has been obtained after the integrations in Eq. (20).

Let us conclude this section by stating the recipe we shall follow for that integration in (20). The external four-vectors P^μ and R^μ are chosen as

$$\begin{aligned} P^\mu &\equiv (0, 0, 0, i\sqrt{P^2}) \\ R^\mu &\equiv \frac{1}{\sqrt{P^2}} (0, 0, i\sqrt{P^2 R^2 - (P \cdot R)^2}, i P \cdot R) \end{aligned} \quad (23)$$

so that, taking $d\hat{K} = d(\cos\theta)d\phi$, the expressions for $P \cdot K$ and $R \cdot K$ become

$$\begin{aligned} P \cdot K &= -i \frac{\sqrt{P^2 t}}{2} \cos \theta \\ R \cdot K &= -i \frac{\sqrt{t}}{2\sqrt{P^2}} P \cdot R \cos \theta - i \frac{\sqrt{t}}{2\sqrt{P^2}} \sqrt{P^2 R^2 - (P \cdot R)^2} \sin \theta \cos \phi \end{aligned} \quad (24)$$

which will be used in the actual calculations. All quantities entering Eqs. (24), P^2 , R^2 and $P \cdot R$, are given in (10) in terms of s and t .

3. - COMPTON SCATTERING TENSORS

In the last section we wrote the effective potential which we are interested in, in terms of the absorptive part of the amplitude $T(s,t)$ on the t channel two-photon cut. This quantity, Eq. (20), is determined by the contraction of the Compton scattering tensors for both systems. As said before, we ask for the spin averaged amplitude, so that we will only need to discuss the form of the scattering matrix for a scalar system. If this is denoted by $T^{\mu\nu}$, satisfying the gauge conditions

$$k'_\mu T^{\mu\nu} = T^{\mu\nu} k_\nu = 0 \quad (25)$$

it is possible to expand the spin averaged tensor, associated with the process $k+r=k'+r'$, in the form

$$T^{\mu\nu} = \sum_{i=1}^4 B_i(q^2, R.K) \mathcal{L}_i^{\mu\nu} \quad (26)$$

where $\mathcal{L}_i^{\mu\nu}$ are the following gauge invariant tensors

$$\begin{aligned} \mathcal{L}_1^{\mu\nu} &= k \cdot k' g^{\mu\nu} - k^\mu k'^\nu \\ \mathcal{L}_2^{\mu\nu} &= k \cdot k' R^\mu R^\nu - R \cdot K (R^\mu K'^\nu + K^\mu R'^\nu) + (R \cdot K)^2 g^{\mu\nu} \\ \mathcal{L}_3^{\mu\nu} &= k \cdot k' (R^\mu K'^\nu + K'^\mu R^\nu) - R \cdot K (K^\mu K'^\nu + K'^\mu K^\nu) \\ \mathcal{L}_4^{\mu\nu} &= K'^\mu K^\nu \end{aligned} \quad (27)$$

and the amplitudes $B_i(q^2, R.K)$ have definite crossing symmetry. Indeed, $B_{1,2,4}$ are symmetric, while B_3 is antisymmetric under the change of sign of $R.K$. In writing (27) we have used that photons are massless, i.e., $k^2 = k'^2 = 0$ [see (18)], but not necessarily transverse. Nevertheless, as we shall have the contracted product of two such tensors, the use of (25) says that only the amplitudes $B_{1,2}$ will contribute to our problem, as it happens in the case of the true physical amplitude. These physical invariant amplitudes $B_{1,2}$ correspond to the ones suggested by Bardeen and Tung⁹⁾ and they are free from kinematical singularities and constraints^{*)}.

*) This is, apart from the zero in $R.K = 0$ due to crossing, also true for B_3 but not for B_4 , which has the kinematical zero $B_4(0,0) = 0$ [see Ref. 10].

Following the notation of the last section, the relevant quantity A in (17), contraction of the two tensors $T^{(m)}$ and $T^{(M)}$, can be written as

$$\begin{aligned}
 A = & \frac{q^4}{4} [2 A_1 (q^2, P \cdot K) B_1 (q^2, R \cdot K) \\
 & + R^2 A_1 (q^2, P \cdot K) B_2 (q^2, R \cdot K) + P^2 A_2 (q^2, P \cdot K) B_1 (q^2, R \cdot K)] \\
 & + [2 (P \cdot K)^2 (R \cdot K)^2 - \frac{q^2}{4} (P^2 (R \cdot K)^2 + R^2 (P \cdot K)^2) + \frac{q^4}{4} (P \cdot R)^2 \\
 & + 2 q^2 (P \cdot R) (P \cdot K) (R \cdot K)] A_2 (q^2, P \cdot K) B_2 (q^2, R \cdot K) \quad (28)
 \end{aligned}$$

where A_i and B_j are the invariant amplitudes associated with the particles of mass m and M , respectively, and all the invariants appearing in (28) can be written in terms of s , q^2 and the two angles of the \hat{K} integration of Eq. (20).

We are interested in the knowledge of these invariant amplitudes for scalar and spin $\frac{1}{2}$ systems. The single particle pole contribution may be calculated explicitly. In a dispersive approach, their residues can be deduced by comparison with the Compton scattering Feynman diagrams. Once these pole terms have been separated out, the remaining contribution to the amplitude, the continuum contribution, is finite in $q^2 = R \cdot K = 0$ and may therefore be Mc Laurin expanded. In the following we will discuss this program explicitly.

3.1. - Spin 0 system

For scalar particles, and after due consideration of a contact term, the pole contribution reads

$$\begin{aligned}
 T^{\mu\nu} (\text{pole}) = & Z^c [(2R+k)^\mu (2R+k')^\nu D_s \\
 & + (2R-k)^\mu (2R-k')^\nu D_u - 2g^{\mu\nu}] \quad (29)
 \end{aligned}$$

where z is the charge of the particle and

$$D_s = \left(2R.K - \frac{q^2}{z} \right)^{-1} ; \quad D_u = \left(-2R.K - \frac{q^2}{z} \right)^{-1} \quad (30)$$

correspond to the single particle exchanges in the s and u channels (of the photon-particle system), respectively. By comparison of the tensor (26) with (29) we find the following pole contributions to the amplitudes

$$\begin{aligned} B_1^L(q^2, R.K) &= -z^2 (D_s + D_u) \\ B_2^L(q^2, R.K) &= 8z^2 D_s D_u \end{aligned} \quad (31)$$

The remaining contribution to the amplitudes is finite in the limit $q^2, R.K \rightarrow 0$, and the associated structure constants are the so-called electric, α , and magnetic, β , polarizabilities. In precise terms, we define these constants by ¹¹⁾

$$\begin{aligned} \alpha &\equiv \frac{1}{2M} \left(\frac{e^2}{4\pi} \right) (B_1 + R^2 B_2)^c \\ \beta &\equiv -\frac{1}{2M} \left(\frac{e^2}{4\pi} \right) B_1^c \end{aligned} \quad (32)$$

where the superscript means the limiting value (at $q^2 = R.K = 0$) of the continuum contributions. The knowledge of the analytic properties of the invariant amplitudes implies that, beyond (31) and (32), next order terms of these amplitudes go at least like q^2 or like $(R.K)^2$.

For real Compton scattering, the contraction of (26) with the polarization vectors of the photons leads to the result given in (2), thus providing a tool to measure the structure constants α and β from the Compton scattering differential cross-section at low enough energies. In our problem, the knowledge of (31) and (32) allows us to determine the relevant quantity A in (28) up to a certain order in $q^2, P.K$ and $R.K$ (which is that order depends on the system being considered, charged or not).

3.2. - Spin $\frac{1}{2}$ system

At first sight, the introduction of spin seems to complicate the rather simple considerations of the previous subsection. In fact, the Compton scattering matrix on a spin $\frac{1}{2}$ particle consists of six different gauge invariant tensors for transverse photons. The six amplitudes given by Bardeen and Tung⁹⁾ are free from kinematical problems, and their single particle poles are unambiguously identified in terms of the charge Z and the anomalous magnetic moment μ of the particle. To second order in the photon energy, it is enough to consider the continuum contributions of only two amplitudes, and the corresponding structure constants define the electromagnetic polarizabilities of the particle, α_s and β_s .

However, when only the spin independent part of the Compton scattering matrix is considered, the tensor may be taken to be the one of a spinless particle and the expansions (26) and (27) may be used. The associated spin averaged amplitudes $B_{1,2}(q^2, R.K)$ are related to the ones defined by Bardeen and Tung through

$$T^{\mu\nu} = \frac{M}{4R^2} \text{Tr} \left[(\not{x} + M) (\not{x}' + M) T_S^{\mu\nu} \right] \quad (33)$$

where $x' = \gamma_\mu \gamma^\mu$ and γ^μ are the Dirac matrices. With the electromagnetic polarizabilities defined as said before, we obtain¹¹⁾

$$\begin{aligned} \alpha_s + \beta_s &\equiv \frac{1}{2M} \left(\frac{e^2}{4\pi} \right) (R^2 B_2)^c \\ \alpha_s - \beta_s &\equiv -\frac{\mu(2Z + \mu)}{2M^3} \left(\frac{e^2}{4\pi} \right) + \frac{1}{2M} \left(\frac{e^2}{4\pi} \right) (2B_1 + R^2 B_2)^c \quad (34) \end{aligned}$$

with the same meaning for the superscript as in the previous subsection. Apart from the term containing the anomalous magnetic moment, which has its origin in the spin of the particle, the result (34) reproduces the one obtained for scalar particles (32). An alternative way of defining the polarizabilities of a spin $\frac{1}{2}$ particle is to pursue further the analogy with spin 0 and to use its definition (32) also for spin $\frac{1}{2}$ particles. These polarizabilities, which we will denote by $\bar{\alpha}$ and $\bar{\beta}$, are therefore given only in terms of the continuum contributions of the spin averaged amplitudes. They are related to the previous ones as follows

$$\begin{aligned}\bar{\alpha} &= \alpha_S + \frac{\mu(2Z + \mu)}{4M^3} \left(\frac{e^2}{4\pi} \right) \\ \bar{\beta} &= \beta_S - \frac{\mu(2Z + \mu)}{4M^3} \left(\frac{e^2}{4\pi} \right)\end{aligned}\tag{35}$$

This freedom in the definition of the polarizabilities of a spin $\frac{1}{2}$ particle is due to the fact that the pole contributions to the amplitudes of the complete spin $\frac{1}{2}$ Compton scattering matrix generate both pole and continuum contributions to the spin averaged amplitudes.

The residues at the pole of the single particle exchange contributions are obtained by spin averaging, Eq. (33), the corresponding Feynman graphs. The result is

$$\begin{aligned}B_1^{\mathcal{L}} &= \mu(2Z + \mu)(D_S + D_U) \\ R^2 B_2^{\mathcal{L}} &= -\mu(2Z + \mu)(D_S + D_U) + gZ^2 M^2 D_S D_U\end{aligned}\tag{36}$$

where D_S and D_U have been given in (30). It is straightforward to reproduce the spin independent Compton amplitude given in the Introduction, Eq. (7), from the preceding expressions. The relation is ^{*}

$$T_S(\text{spin averaged}) = \frac{\sqrt{R^2}}{M} \epsilon^{\mu*}(k', \lambda') T_{\mu\nu} \epsilon^\nu(k, \lambda)\tag{37}$$

where $\epsilon^\nu(k, \lambda)$ is the polarization vector of a photon of four-momentum k and helicity λ and similarly for $\epsilon^{\mu*}(k', \lambda')$. $T_{\mu\nu}$ is given by (26) and the relevant contributions to the amplitudes by (36) and (34).

It should be noticed that it is precisely $\bar{\alpha}$ the structure constant which gives the strength of the response of a spin $\frac{1}{2}$ particle to an electric field, as may be immediately seen from (7). In this sense, i.e., using Compton scattering amplitudes in order to define the polarizabilities,

^{*}) The reason for the factor $\sqrt{R^2}/M$ is that it allows us to obtain cross-sections from T_S with the same conventions as for a spinless particle. In fact, it could have been absorbed by (33), but spoiling then the similarity of the definitions of the spin 0 and $\frac{1}{2}$ polarizabilities, (32) and (34).

$\bar{\alpha}$ appears to be the generalization to include spin (and therefore magnetic moment) of the spinless electric polarizability α . However, the combination of invariant amplitudes appearing in our problem, Eq. (28), is a priori different. It is therefore not clear if the generalization of the potential (1) will also give $\bar{\alpha}$ as extension of the spinless electric polarizability α to include magnetic moment. In fact, as we will see, the interpretation of the result in terms of the polarizabilities, in the potential approach, is more delicate.

4. - CALCULATION AND RESULTS

With the integrand of Eq. (20), given in Eq. (28), in terms of Compton amplitudes, we are now able to carry out the integrations. In that way, we obtain the absorptive part of the amplitude, $\text{Abs } T(s,t)$, in the region $t \gtrsim 0$. Since we are interested in velocity independent potentials, the analytic continuation to $s = s_0$ of the relevant part of $\text{Abs } T(s,t)$ is trivially performed by maintaining only the $s - s_0$ independent terms. They are then integrated according to Eq. (15) to give the sought long range potentials. This will be the content of the present section. We shall present the results separately for the three cases mentioned in the Introduction.

We would like to mention here that two alternative approaches have been followed in the cases in which a spin $\frac{1}{2}$ particle is considered. The difference between the two approaches lies in the way in which the spin independent part of the Compton amplitude is obtained. This part may be defined by spin-averaging in a covariant way, as done in Eq. (33), and which means that the axes of quantization of the spins of the in and outgoing particles are related by the boost which transforms their four-momenta into each other, leaving invariant any four vector orthogonal to these. The alternative definition is obtained by spin-averaging over the non-relativistic spin, that is, by keeping the spin (Pauli matrices) independent part of the complete scattering matrix⁹⁾, once the non-relativistic reduction of the amplitude has been performed. Since both approaches reproduce the low energy Compton amplitude given in Eq. (7), we expect them to give the same results for the potential up to the order in which it is determined by the polarizabilities. We will present the absorptive part of the amplitude in both fashions, thus shedding light on different aspects of the problem.

4.1. - Two spinless particles, one of them neutral, the other charged

The integrand of Eq. (20) is obtained from Eq. (28) substituting the amplitudes of the charged particle (whose mass will be denoted by M) by its pole contribution (31), and the amplitudes of the neutral particle (of mass m and polarizabilities α and β) by its polarizability contribution (32). The angular integration is easily performed and the result at $s = s_0$ and expanded around $q^2 = 0$ is

$$\text{Abs } T(q^2) = -\frac{Z^2}{8} \left(\frac{4\pi}{e^2} \right) m M \sqrt{q^2} \left(\alpha - \frac{\sqrt{q^2}}{6M\pi} (11\alpha + 5\beta) \right) + O(q^2 \sqrt{q^2}) \quad (38)$$

Z being the charge of the charged particle. The potential is trivially obtained from (38) by integration of q^2 , Eq. (15), and reads

$$V(r) = Z^2 \left(\frac{e^2}{4\pi} \right) \left[-\frac{1}{2} \frac{\alpha}{r^4} + \frac{1}{4\pi} \frac{11\alpha + 5\beta}{M r^5} \right] + O\left(\frac{1}{r^6}\right) \quad (39)$$

The following conclusions may be drawn from (39). The first term does not depend on the masses of the two particles and therefore coincides with the classical result (1) even without taking the static field limit $M \rightarrow \infty$. The second term is a "quantum effect", its origin being found in the propagation of the charged particle. It vanishes in the static field limit, not only because it does not have a classical counterpart but also because it is sensitive to the magnetic field of the photons (as it depends on β), which is not present in a static electric field. It is completely determined by the polarizabilities.

We will postpone the discussion of the higher order terms, which cannot be obtained in terms of the polarizabilities, to the end of this section, since it applies also to the other two cases we still want to study.

Before passing to the next case, we would like to remark that the potential (39) is unchanged if we allow the charged particle to have spin, in our case spin $\frac{1}{2}$, since the terms proportional to Z_μ or to μ^2 are at least of order $q^2 \sqrt{q^2}$ in the absorptive part of the amplitude. This may easily be seen from Eq. (36) and repeating the steps which led to Eq. (38) or, for the part proportional to μ^2 [and therefore for Z_μ , as they go together, Eq. (37)], going over to the next subsection.

4.2. - Two neutral particles, one of them spinless, the other with spin $\frac{1}{2}$

The essential difference with respect to the previous case is that here both the pole and the polarizability contributions have to be taken into account for the spin $\frac{1}{2}$ particle. This is so because the dominant contribution from the polarizabilities is of the same order as the non-dominant, but still uniquely determined, contribution from the magnetic moment μ . Let us denote by M , $\bar{\alpha}$ and $\bar{\beta}$ the mass and polarizabilities of the spin $\frac{1}{2}$ particle and by m , α and β the corresponding magnitudes for the spinless particle. Using Eqs. (28), (32) and (36) to obtain the integrand of Eq. (20), the result of the angular integration at $s = s_0$ and expanded around $q^2 = 0$ is

$$\begin{aligned} \text{Abs } T(q^2) = & -\frac{1}{16} \left(\frac{\mu}{2M} \right)^2 \left(\frac{4\pi}{e^2} \right) M m q^2 \sqrt{q^2} \left(\beta - \frac{\sqrt{q^2}}{6M\pi} (11\beta + 5\alpha) \right) \\ & - \frac{1}{960\pi} \left(\frac{4\pi}{e^2} \right)^2 M m q^4 \left(23(\alpha\bar{\alpha} + \beta\bar{\beta}) - 7(\alpha\bar{\beta} + \bar{\alpha}\beta) \right) + O(q^4 \sqrt{q^2}) \end{aligned} \quad (40)$$

It is at this point instructive to write down the same absorptive amplitude as obtained in the non-relativistic approach to spin independence. Written in terms of the polarizabilities α_s and β_s , it reads

$$\begin{aligned} \text{Abs } T(q^2) = & -\frac{1}{16} \left(\frac{\mu}{2M} \right)^2 \left(\frac{4\pi}{e^2} \right) M m q^2 \sqrt{q^2} \left(\beta - \frac{\sqrt{q^2}}{3M\pi} (7\beta + \alpha) \right) \\ & - \frac{1}{960\pi} \left(\frac{4\pi}{e^2} \right)^2 M m q^4 \left(23(\alpha\alpha_s + \beta\beta_s) - 7(\alpha\beta_s + \alpha_s\beta) \right) + O(q^4 \sqrt{q^2}) \end{aligned} \quad (41)$$

Using the connection (35) it is seen that both results (40) and (41) are identical. However, comparing Eqs. (40) and (41) to Eq. (38), the advantage of the covariant procedure, or, more precisely, of the definition of the polarizabilities suggested by the covariant approach to spin independence (i.e., of using $\bar{\alpha}$ and $\bar{\beta}$ instead of α_s and β_s) becomes conspicuous.

It is enough to remark the electric-magnetic symmetry of the first part of Eq. (40) with respect to Eq. (38), since we go from one expression to the other simply by making the substitutions $\frac{1}{2}(\mu/2M)^2 q^2 \rightleftharpoons Z^2$ and $\alpha \rightleftharpoons \beta$. We therefore choose to obtain the potential from Eq. (40), getting, after a trivial integration,

$$V(r) = \left(\frac{\mu}{2M} \right)^2 \left(\frac{e^2}{4\pi} \right) \left[-3 \frac{\beta}{r^6} + \frac{5}{2\pi} \frac{5\alpha + 11\beta}{M r^7} \right] - \frac{1}{4\pi} \frac{1}{r^7} \left[23(\alpha \bar{\alpha} + \beta \bar{\beta}) - 7(\alpha \bar{\beta} + \bar{\alpha} \beta) \right] + O\left(\frac{1}{r^8}\right) \quad (42)$$

Similarly to the previous subsection, the following comments apply to the result.

The first term does not depend on the masses and therefore coincides with the classical result (4) even without taking the static field limit $M \rightarrow \infty$ (but μ/M fixed).

The second term is, as in the previous case, a "quantum effect" and all the comments made there apply also here simply substituting "electric" by "magnetic" and vice versa. The particular combination $(5\alpha + 11\beta)$ which is the coefficient of this term, depends on the definition chosen for the polarizabilities of the spin $\frac{1}{2}$ particle, which has been $\bar{\alpha}$ and $\bar{\beta}$, as shown in the last term, which is nothing else than the Van der Waals potential of Eq. (6).

4.3. - A spinless charged and a spin $\frac{1}{2}$ neutral particle

As in the previous case we can distinguish two contributions, one due to the magnetic moment (pole) and the other due to the polarizabilities (continuum) of the spin $\frac{1}{2}$ particle. The difference with respect to the previous subsection is that now the other, spinless particle is charged and not neutral as there, and therefore also contributes with a pole to the integrand of (20). A consequence of the presence of at least one pole in the integrand (28) is that both afore-mentioned contributions overlap already in the dominant long range term and not only in the second one as in (42).

We take the spinless particle to have mass M and charge Z and the spin $\frac{1}{2}$ one to have mass m and magnetic moment μ . Using again the results of Section 3 one can perform the integrations, which, due to the existence of double poles are more involved in this case. Keeping only the $s - s_0$ independent part and expanding around $q^2 = 0$ we obtain

$$\begin{aligned} \text{Abs } T(q^2) = & -\frac{Z^2}{8} \left(\frac{\mu}{2m}\right)^2 \sqrt{q^2} \left(M + m + 3 \frac{\sqrt{q^2}}{\hbar}\right) \\ & - \frac{Z^2}{8} \left(\frac{4\pi}{e^2}\right) m M \sqrt{q^2} \left(\bar{\alpha} - \frac{\sqrt{q^2}}{6M\hbar} (11\bar{\alpha} + 5\bar{\beta})\right) + O(q^2 \sqrt{q^2}) \end{aligned} \quad (43)$$

Again, and following the alternative approach, one finds

$$\begin{aligned} \text{Abs } T(q^2) = & -\frac{Z^2}{8} \left(\frac{\mu}{2m}\right)^2 \sqrt{q^2} \left(2M + m + 2 \frac{\sqrt{q^2}}{\hbar}\right) \\ & - \frac{Z^2}{8} \left(\frac{4\pi}{e^2}\right) m M \sqrt{q^2} \left(\alpha_s - \frac{\sqrt{q^2}}{6M\hbar} (11\alpha_s + 5\beta_s)\right) + O(q^2 \sqrt{q^2}) \end{aligned} \quad (44)$$

which again turns out to be equivalent to Eq. (43) through the relations (35). We choose to calculate the potential from Eq. (43) and not from Eq. (44), since this emphasizes the symmetry in the masses of both systems. The result is

$$\begin{aligned} V(r) = & -\frac{Z^2}{2mM} \left(\frac{\mu}{2m}\right)^2 \left(\frac{e^2}{4\hbar}\right)^2 \left[\frac{M+m}{r^4} + \frac{9}{\hbar} \frac{1}{r^5} \right] \\ & + Z^2 \left(\frac{e^2}{4\hbar}\right) \left[-\frac{1}{2} \frac{\bar{\alpha}}{r^4} + \frac{1}{4\hbar} \frac{11\bar{\alpha} + 5\bar{\beta}}{M r^5} \right] + O\left(\frac{1}{r^6}\right) \end{aligned} \quad (45)$$

This result will answer some of the questions we asked in the Introduction. First, by comparison with Eq. (39), and defining the electric polarizability in the classical way as coefficient of the potential (1), we conclude that the inclusion of spin, and therefore magnetic moment, leads to the substitution

$$\alpha \rightarrow \bar{\alpha} + \left(\frac{\mu}{2m}\right)^2 \left(\frac{e^2}{4\pi}\right) \frac{M+m}{mM} \quad (46)$$

This, of course, cannot be considered a constant, characterizing a physical property of the particle of mass m as also the mass of the charged particle, M , appears in (46). However, the nice feature of the decomposition shown in (46) is that the second term depends only on the reduced mass of the system, whereas this would not have been the case if we had chosen (44) and therefore the polarizability α_s for writing the potential (45).

In the classical limit of a static electric field acting on a particle of mass m and magnetic moment μ , Eq. (46) gives

$$\alpha \rightarrow \bar{\alpha} + \frac{1}{m} \left(\frac{\mu}{2m}\right)^2 \left(\frac{e^2}{4\pi}\right) \quad (47)$$

which is neither $\bar{\alpha}$ nor α_s ! As in all previous cases it is seen in (45) that only the dominant r^{-4} term is left in this limit.

We would like to end this section with some general comments which apply to all obtained potentials and to their calculation. The approximation mentioned in Section 2, $\hat{P} \cdot \hat{R} = 1$, could have been used in all cases, except in the only one in which it would really simplify the integrations : when a double pole is present (subsection 4.3). Its use there transforms, through the relation $s_0 - s \approx (s_0/4mM)t$ implied by Eq. (21), a velocity dependent term into a momentum transfer dependent one. This would contribute to the longest range term giving therefore a wrong result.

The other comment refers to those higher order terms in all obtained potentials which cannot be determined in terms of the polarizabilities alone. This is due to the fact that also higher order terms in the McLaurin expansion of the continuum contributions of the Compton amplitudes enter into them, whereas only the first term of these expansions corresponds to the polarizabilities. The scale of these expansions may be

taken to be a typical excitation energy E^* of the corresponding particle. Thus all obtained potentials dominate over the unknown higher order contributions only for $E^*r \gg 1$. On the other hand, a different and avoidable expansion in q^2 has been done in writing the absorptive part of the amplitude $T(s,t)$, since we wanted to present the potentials as powers of r . In this expansion the scale is given by the mass M of the particle which has charge or magnetic moment and therefore pole contributions to its Compton amplitudes. This limits further the region of validity of the potentials to the region where $Mr \gg 1$ is satisfied. It should not be forgotten that in the cases in which this limitation is stronger than the first one ($M < E^*$) it may be useful to avoid it by presenting the results without having performed the expansion in q^2 which is its origin.

5. - CONCLUSIONS

In this work the spin and velocity independent long range potentials between two systems, of which one at least is neutral and one spinless, have been obtained. The main outcome is that their two longest range terms are completely determined by the mass, charge, magnetic moment and electromagnetic polarizabilities of both objects. Thus, the physical parameters entering the electromagnetic interaction between two systems, when they are far apart, are the same as the ones appearing in the description of low energy Compton scattering on each system. It is the knowledge of the analytic behaviour of the Compton scattering amplitudes that allows us to determine the two leading order terms of the potentials in terms of only the limiting value, at threshold, of these amplitudes, i.e., in terms of the polarizabilities. The first term is classical and it survives without modification in the limit of a static electric or magnetic field acting on the neutral system, reproducing Eqs. (1) or (4), respectively. However, the next to the leading order term is of the form \hbar/Mcr times a term of the type of the leading one, M being the mass of the particle creating the field. Therefore, this second term is a relativistic quantum effect generated by a charged particle [r^{-5} in Eq. (39)] or by a neutral one with spin [r^{-7} in Eq. (42)] due to its magnetic moment. It disappears, however, in the static field limit $M \rightarrow \infty$. We thus may conclude that the polarizabilities represent general electromagnetic properties of the system, since they give its complete quantum electrodynamic behaviour (together with

the static electromagnetic multipoles) to order e^2 at low energies or long distances, similarly to what the charge and magnetic moment do to order e .

For the case of particles with spin, we have seen that the definition of the polarizabilities $\bar{\alpha}$ and $\bar{\beta}$ in terms of the continuum contribution of the spin averaged amplitudes for Compton scattering produces more symmetric results, in the long range potentials, than their usual definition α_s and β_s in terms of what is missing once the complete Born contribution (in the sense of Feynman graphs) has been separated out. In particular, the interaction of a neutral particle with spin with a charged one is given, to leading order in r , by $\bar{\alpha}$ plus a term which only depends on the reduced mass of the system (and the magnetic moment of the particle with spin). However, in the limit of a static electric field acting on this neutral particle with magnetic moment, the coefficient of the r^{-4} potential which survives is given by (47), and it is neither α_s nor $\bar{\alpha}$. This coefficient is therefore the quantity which is measured in the scattering of neutrons by heavy nuclei at low enough energies, when one looks for a linear term in the momentum transfer $|\vec{q}|$ in the amplitude ¹⁾.

We have discussed, at the end of Section 4, the region of validity of the inverse power expansion of the potentials. Apart from the condition $E^*r \gg 1$, which comes from the expansion of the continuum contribution to the Compton amplitudes in order to single out the polarizabilities, an additional expansion in q^2 of the absorptive part of the total amplitude has been made in order to present the potentials as inverse powers of r , implying the condition $Mr \gg 1$, with M the mass of the charged or spinning particle. This provides a simple explanation of why the two-photon exchange contribution to the neutron electron amplitude ¹²⁾, although controlled by the neutron polarizability, is not given ^{13),14)} by the classical potential r^{-4} . In fact, this potential would be meaningful only for distances larger than the Compton wave length of the electron. With this example in mind, it could be interesting to obtain the form of the potential with the only limitation $r \gg (E^*)^{-1}$ ($\sim m_\pi^{-1}$ for the neutron), irrespective of whether the separation r is larger or smaller than M_e^{-1} . This would, however, spoil the inverse power expansion in r of the potential, which allowed us to compare with the classical limits.

ACKNOWLEDGEMENTS

The authors would like to acknowledge several fruitful discussions with J.S. Bell, G. Domokos and T.E.O. Ericson. One of us (R.T.) is indebted to the CERN Theoretical Physics Division for hospitality and to the GIFT for financial support.

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FIGURE CAPTION

Two-photon exchange contribution to the elastic scattering amplitude for two particles of masses m and M .

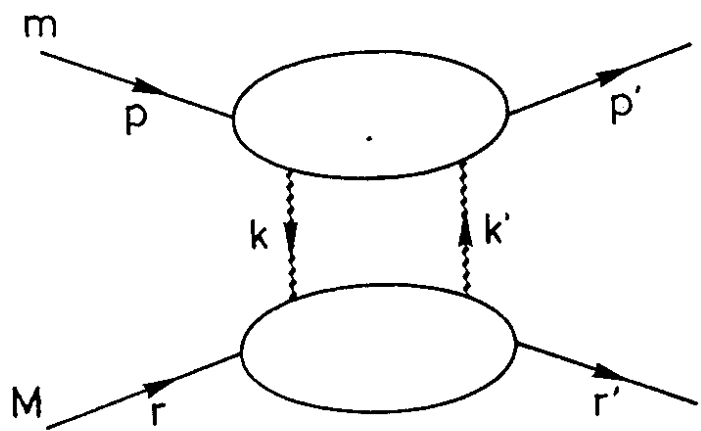


FIG. 1