CAFPE-105/08
UG-FT-235/08

Obtaining $\sigma \to \gamma\gamma$ Width from Nucleon Polarizabilities [1]

Joaquim Prades[a)] and José Bernabéu[a)]

[a)] Centro Andaluz de Física de las Partículas Elementales (CAFPE) and Departamento de Física Teórica y del Cosmos, Universidad de Granada Campus de Fuente Nueva, E-18002 Granada, Spain.

[b)] Departament de Física Teòrica, IFIC, Universitat de València-CSIC, Apt. de Correus 22085, E-46071 València, Spain.

Abstract

We propose a new method that fixes the coupling to two photons of the recently found lightest QCD resonance, the σ. This coupling provides crucial information for discriminating the yet unknown nature of this special state. Our method uses available data on the nucleon polarizabilities together with analyticity and unitarity. Taking into account all the uncertainties, our result is $\Gamma_{\text{pole}} = 1.2 \pm 0.4$ keV.

September 2008

[1]Work supported in part by MICINN, Spain and FEDER, European Commission (EC) Grant Nos. FPA2005-01678 (J.B.), FPA2006-05294 (J.P.), Consolider-Ingenio 2010 Grant No. CSD2007-00042 – CPAN, by Junta de Andalucía Grants No. P05-FQM 101 (J.P.), P05-FQM 467 (J.P.) and P07-FQM 03048 (J.P.) and by the EC RTN network FLAVIAnet Contract No. MRTN-CT-2006-035482 (J.P.). Invited talk given by J.P. at "14$^{\text{th}}$ International Conference on Quantum Chromodynamics (QCD '08)", 7-12 July 2008, Montpellier, France.

1 Introduction

The lowest isospin $I = 0$ and angular momentum $J = 0$ QCD resonance is usually called the σ and plays a special role in the QCD dynamics and in the QCD non-perturbative vacuum structure. Recently, it has been shown that the σ is also the lowest QCD resonance by fixing the mass and width of this state with a precision of just tens of MeV in Ref. [1]. Making an analytic continuation of the $I = 0$ and $J = 0$ partial wave S-matrix in the region of validity of Roy equations, these authors find a zero at $E = [(441^{+16}_{-8} - i\,(272^{+9}_{-12})]$ MeV on the first Riemann sheet which reflects the σ pole on the second Riemann sheet. Also on the first Riemann sheet, the inverse of the partial wave $\pi\pi$ S-matrix $S = 1 + 2i\,\beta(t)\,T(t)$ has a zero at E^*. Here,

$$T(t) = \frac{1}{\beta(t)\cot[\delta(t)] + \beta(t)}\,, \tag{1}$$

$\delta(t)$ is the scalar-isoscalar $\pi\pi$ phase shift, $\beta(t) = \sqrt{1 - 4m_\pi^2/t}$ and $t = E^2$. The position of the σ resonance pole has been confirmed in [2] at $E = [(484 \pm 17 - i\,(255 \pm 10)]$ MeV.

The nature of the σ remains one of the most intriguing and difficult issues in particle physics. There are have been many proposals about its substructure: $\overline{q} - q$ state, $\pi - \pi$ molecule, $(\overline{qq}) - (qq)$ tetraquark, glueball, and of course, several admixtures of these substructures. The size of $\sigma \to \gamma\gamma$ width can shed light on this question.

$\gamma\gamma \to (\pi\pi)_{I=0,2}$ amplitudes have been calculated using twice-subtracted dispersion relations in [3] and [4] including the recent data on $\pi\pi$ final state interactions which contain the σ pole in the scalar-isoscalar contribution. They get 4.09 ± 0.29 keV [3] and 1.68 ± 0.15 keV [4] for the σ into two photons width. The origin of this discrepancy is discussed in [4]. More recently, the authors of [5] made an amplitude analysis of the world published data on $\gamma\gamma \to \pi^+\pi^-$ and find two regions of solutions. The width $\sigma \to \gamma\gamma$ in these regions are predicted in Ref. [5] to be 3.1 ± 0.5 and 2.4 ± 0.4 keV, respectively.

2 Method

In Ref. [6], we proposed a new method that fixes the coupling to two photons $g_{\sigma\gamma\gamma}$ of the σ meson found in the $\pi\pi$ scattering amplitude [1, 2] using only available precise experimental data on the nucleon electromagnetic polarizabilities together with analyticity and unitarity. This differs from the analysis of [7], where the properties of the σ meson of a Nambu–Jona-Lasinio model were used. Nucleon electric α and magnetic β polarizabilities are well measured using low energy Compton scattering on protons and neutrons with $\alpha+\beta$ constrained by a forward dispersion relation [8]. The results are [9]: $\alpha^{\mathrm{expt}} = 12.0\ 0.6$ and $\beta^{\mathrm{expt}} = 1.9 \mp 0.5$

for protons and $\alpha^{\rm expt} = 11.6\ 1.5$ and $\beta^{\rm expt} = 3.7 \mp 2.0$ for neutrons. Here and in the rest of the paper, polarizabilities are given in 10^{-4} fm^3 units.

The authors of Ref. [10, 11] wrote a sum rule for $\alpha - \beta$ using a backward dispersion relation for the physical spin-averaged amplitude. The s-channel part of this sum rule is related to the multipole content of the total photo-absorption cross-section. While the t-channel part is related through a dispersion relation to the imaginary part of the amplitude which using unitarity is given by the processes $\gamma\gamma \to \pi\pi$ and $\pi\pi \to \overline{N}N$ [11]. The result is the BEFT sum rule,

$$\begin{aligned}(\alpha-\beta) &= \frac{1}{2\pi^2}\int_{\nu_{\rm th}}^{\infty}\frac{{\rm d}\nu}{\nu^2}\sqrt{1+2\frac{\nu}{M_p}}\,[\sigma(\Delta\pi={\rm yes})-\sigma(\Delta\pi={\rm no})] \\ &+ \int_{4m_\pi^2}^{\infty}\frac{{\rm d}t}{4M_p^2-t}\,\frac{\beta(t)}{t^2}\Big\{|f_+^0(t)||F_0^0(t)| \\ &- \frac{(4M_p^2-t)(t-4m_\pi^2)}{16}\,|f_+^2(t)||F_0^2(t)|\Big\}, \end{aligned} \tag{2}$$

where M_p is the proton mass, the partial wave helicity amplitudes $f_+^0(t)$ and $f_+^2(t)$ for $\overline{N}N \to \pi\pi$ are Frazer and Fulco's [12], and the partial wave helicity amplitudes $F_0^0(t)$ and $F_0^2(t)$ for $\gamma\gamma \to \pi\pi$ were defined in [13]. The s-channel part of the integrand is obtained from that of the forward physical amplitude by changing the sign of the non-parity flip multipoles ($\Delta\pi$ = no) and yields $(\alpha-\beta)^s = -(5.0 \pm 1.0)$ [14]. The "experimental" $(\alpha-\beta)^t$ is therefore 15.1 ± 1.3 for protons and 12.9 ± 2.7 for neutrons. Products of helicity amplitudes in Eq. (2) appear only as moduli products, which can be negative if the phases differ from the $\pi\pi$ phase shift in an odd number of π's. The d-wave contribution to $(\alpha-\beta)^t$ is much smaller than the s-wave one; hence, it is a good approximation to take just the Born result $(\alpha-\beta)^t_2 \simeq -1.7$ [15]. Finally, the "experimental" value for $(\alpha-\beta)^t_0$ is 16.8 ± 1.3 for protons and 14.6 ± 2.7 for neutrons. The input $|F_0^0(t)|$ for $(\alpha-\beta)^t_0$ in Eq. (2) is what we want to fix using this "experimental" value. The other input for this quantity, the Frazer-Fulco's $|f_+^0(t)|$ amplitude, is known with enough accuracy for our purposes from the old determination in [16], though it could be improved using recent data on $\pi\pi$ phase shift. We have assigned a 20 % to the theoretical determination of $(\alpha-\beta)^t_0$ from this source. Notice that the $1/t^2$ factor in the integrand of $(\alpha-\beta)^t_0$ makes the well known low-energy, and to a lesser extent, intermediate-energy contributions to be the dominant ones.

On the physical sheet, we can write the twice-subtracted dispersion relation [17],

$$F_0^0(t) = L(t) - \Omega(t)\left[c\,t + \frac{t^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{{\rm d}t'}{t'^2}\,\frac{L(t')\,{\rm Im}\,\Omega^{-1}(t')}{t'-t-i\varepsilon}\right] \tag{3}$$

where c is a subtraction constant fixed by chiral perturbation theory (CHPT) [17, 18], $c = \alpha/48\pi f_\pi^2$ with $\alpha \simeq 1/137$ the fine-structure constant, $f_\pi = 92.4$ MeV the pion decay constant,

$$\Omega(t) = \exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}t'}{t'}\,\frac{\delta(t')}{t'-t-i\varepsilon}\right] \tag{4}$$

is the scalar-isoscalar $\pi\pi$ Omnès function [19] which gives the correct right-hand cut contribution and $L(t)$ is the left-hand cut contribution. In this way we ensure unitarity, the correct analytic structure of $F_0^0(t)$ and that the σ pole properties enter through the scalar-isoscalar phase-shift $\delta(t)$ from $T(t)$ in (1). Here, we shall use a simple analytic expression for $T(t)$, compatible with Roy's equations, which takes a three parameter fit from [2] including both low energy kaon data and high energy data. This fit is valid up to values of t of the order of 1 GeV^2, which is enough for the $(\alpha-\beta)_0^t$ integrand in Eq. (2).

At the σ pole position on the first Riemann sheet [3, 4]

$$F_0^0(t_\sigma) \;=\; e^2\,\sqrt{6}\,\frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}}\,\frac{1}{2i\beta(t_\sigma)}, \tag{5}$$

where e is the electron charge, $g_{\sigma\pi\pi}^2$ is the residue of the $\pi\pi$ scattering amplitude at the σ pole on the second Riemann sheet and $g_{\sigma\gamma\gamma}\,g_{\sigma\pi\pi}$ is proportional to the residue of the $\gamma\gamma\to\pi\pi$ scalar-isoscalar scattering amplitude on the second Riemann sheet. The proportionality factors are such that $g_{\sigma\pi\pi}$ and $g_{\sigma\gamma\gamma}$ agree with those used in [3]. The pole width is given by

$$\Gamma_{\rm pole}(\sigma\to\gamma\gamma) = \frac{\alpha^2|\beta(t_\sigma)\,g_{\sigma\gamma\gamma}^2|}{4M_\sigma} \tag{6}$$

that agrees, modulo normalizations, with that of Ref. [4].

Low's theorem fixes the amplitude $F_0^0(t)$ to be the Born term at very low-energy [20]. As first approximation, we therefore consider the left-hand cut contribution $L(t)$ in (3) to be the Born contribution $L_B(t)$,

$$L_B(t) = e^2\frac{1-\beta(t)^2}{\beta(t)}\log\left(\frac{1+\beta(t)}{1-\beta(t)}\right)\,. \tag{7}$$

This leads to $F_0^0(t)|_B$ which when input in the sum rule (2) gives $(\alpha-\beta)_0^t|_B = 6.7\pm1.2$, where the uncertainty comes mainly from the input data on $|f_+^0(t)|$. This result is 5σ away the "experimental" values quoted above. When analytically continued to complex t, the amplitude $F_0^0(t)|_B$ has a pole at $t_\sigma = \left\{[(474\pm6)-i(254\pm4)]\,\mathrm{MeV}\right\}^2$ with $g_{\sigma\pi\pi} = [(452\pm4)+i(224\pm2)]$ MeV and using (5) we get $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B = (0.49^{+0.03}_{-0.01}) - i\,(0.37\pm0.03)$ which leads to $\Gamma_{\rm pole}(\sigma\to\gamma\gamma)|_B = 2.5\pm0.2$ keV. But this $F_0^0(t)|_B$ does not reproduce the "experimental" $(\alpha-\beta)_0^t$ and hence we need to go beyond the Born approximation $L_B(t)$ for the left-hand cut.

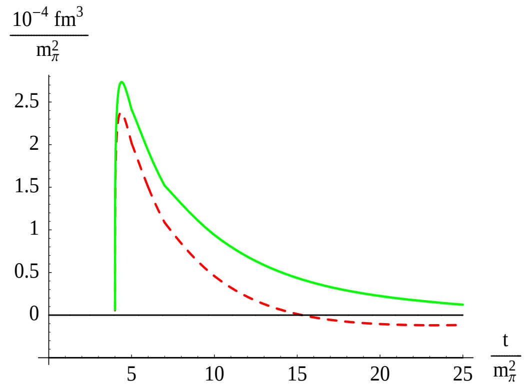

Figure 1: The integrand of $(\alpha-\beta)_0^t$ in (2). The dashed line is when using $L(t) = L_B(t)$ in (3) and the continuous line is when using $L(t) = L_B(t) + L_A(t) + L_V(t)$ in (3) as explained in the text.

The first corrections to $L_B(t)$ originate in the resonance exchange $\gamma\pi \to R \to \gamma\pi$, with $R = a_1, \rho$ and ω [4, 17]. In the narrow width approximation, the a_1 exchange contribution to $L(t)$ is

$$L_A(t) = e^2 \frac{C}{32\pi f_\pi^2} \left[t + \frac{M_{a_1}^2}{\beta(t)} \log\left(\frac{1+\beta(t)+t_A/t}{1-\beta(t)+t_A/t} \right) \right] \tag{8}$$

while the ρ and ω resonances exchange contribution to $L(t)$ in nonet symmetry ($M_\rho = M_\omega = M_V \simeq 782$ MeV) is

$$L_V(t) = e^2 \frac{4}{3} R_V^2 \left[t - \frac{M_V^2}{\beta(t)} \log\left(\frac{1+\beta(t)+t_V/t}{1-\beta(t)+t_V/t} \right) \right] \tag{9}$$

with $t_R = 2(M_R^2 - m_\pi^2)$. The low energy limit of $L_V(t)$ goes as t^2 and we fix $R_V^2 = 1.49\ \mathrm{GeV}^{-2}$ from the well known $\omega \to \pi\gamma$ decay. Though the low energy limit of $L_A(t)$ goes as t and corresponds to the charged pion electromagnetic polarizability $(\overline{\alpha}-\overline{\beta})_{\pi^\pm}$ or equivalently to $L_9 + L_{10} = (1.4\pm 0.3)\cdot 10^{-3}$ in CHPT [21], we consider $L_A(t)$ as an effective contribution for moderate higher values of t with C a real constant to be determined phenomenologically and not connected to the pion polarizability. This is supported by the fact that the $a_1\pi\gamma$ interaction is not so well known at intermediate energies. We fix the effective C by requiring that the "experimental" value of $(\alpha-\beta)_0^t$ is reproduced within 1.5 standard deviations of the total uncertainty when $L(t)$ in (3) is given by $L(t) = L_B(t) + L_A(t) + L_V(t)$. This procedure leads to $C = 0.59 \pm 0.20$ and the integrand of the sum rule is given in Fig. 1 as a continuous line. Notice that the zero at t_0 in the integrand of $(\alpha-\beta)_0^t$ in (2) when using $F_0^0(t)|_B$ has clearly disappeared now.

The low-energy $\gamma\gamma \to \pi^0\pi^0$ cross-sections obtained when the left-hand cut is either $L_B(t)$ or the full $L(t)$ case studied above are similar [4]. The central values are compatible with data for values of t below $(450\ \text{MeV})^2$ and are above data but compatible within two standard deviations for larger values of t up to $(600\ \text{MeV})^2$ and within one standard deviation for t between $(600\ \text{MeV})^2$ and $(800\ \text{MeV})^2$.

3 Results and Conclusions

The scalar-isoscalar amplitude $F_0^0(t)$, obtained using $L(t) = L_B(t)+L_A(t)+L_V(t)$ as explained above, is analytically continued to the complex plane, and at t_σ on the first Riemann sheet one gets $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = (0.23^{+0.05}_{-0.09}) - i\,(0.30 \pm 0.03)$ which has a smaller absolute value when compared with $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B$ and leads to $\Gamma_{\rm pole}(\sigma \to \gamma\gamma) = (1.0 \pm 0.3)$ keV. The error quoted here is from the uncertainties in the "experimental" value of $(\alpha-\beta)_0^t$ and the inputs of the sum rule (2) only. To obtain the rest of the uncertainty, we vary the σ properties in the $\pi\pi$ scattering as follows. We still use the three parameter fit formula including low energy kaon data and high energy data for $\cot(\delta(t))$ in [2] as input in the amplitude $T(t)$ but with parameter values slightly modified to reproduce the σ pole position $t_\sigma = ([(441\pm6) - i\,(272\pm4)]\ \text{MeV})^2$ found in [1]. In this case, we get $g_{\sigma\pi\pi} = [(480\pm 7) + i\,(191\pm3)]$ MeV. With that $T(t)$ input in the dressed Born amplitude in Eq. (3), one gets $(\alpha-\beta)_0^t|_B = 6.1 \pm 1.1$, $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|_B = (0.57 \pm 0.02) - i(0.41 \pm 0.03)$ and $\Gamma_{\rm pole}(\sigma \to \gamma\gamma)|_B = (3.8 \pm 0.4)$ keV. The integrand of $(\alpha-\beta)_0^t$ in (2) for this case is very similar to the dashed line of Fig. 1. The effective value of C in (8) is $C = 0.62 \pm 0.20$ when fixed to reproduce the "experimental" value of $(\alpha-\beta)_0^t$ within 1.5 standard deviations of the total uncertainty. With this new C, the analytic continuation to t_σ gives $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = (0.31^{+0.05}_{-0.07}) - i\,(0.32 \pm 0.03)$ and $\Gamma_{\rm pole}(\sigma \to \gamma\gamma) = (1.5 \pm 0.4)$ keV. Again, the integrand of $(\alpha-\beta)_0^t$ in (2) for this case is very similar to the continuous curve of Fig. 1.

Making a weighted average of the two cases discussed above, we get

$$\Gamma_{\rm pole}(\sigma \to \gamma\gamma) = (1.2 \pm 0.4)\ \text{keV}\,. \tag{10}$$

Acknowledgments

It is a pleasure to thank Stephan Narison for the invitation to this very enjoyable conference. We also thank Heiri Leutwyler, José A. Oller, José Ramón Peláez and Mike Pennington for useful discussions and sharing unpublished results.

References

[1] I. Caprini, G. Colangelo, and H. Leutwyler Phys. Rev. Lett. **96** (2006) 132001.

[2] R. García-Martín, J.R. Peláez, and F.J. Ynduráin, Phys. Rev. D **76** (2007) 074034.

[3] M.R. Pennington, Phys. Rev. Lett. **97** (2006) 011601; Mod. Phys. Lett. A **22** (2007) 1439.

[4] J.A. Oller, L. Roca, and C. Schat, Phys. Lett. B **659** (2008) 201; AIP Conf. Proc. **1030** (2008) 147; J.A. Oller and L. Roca, arXiv:0804.0309 [hep-ph]; private communication.

[5] M.R. Pennington, T. Mori, S. Uehara and Y. Watanabe, Eur. Phys. J. C **56** (2008) 1.

[6] J. Bernabéu and J. Prades, Phys. Rev. Lett. **100** (2008) 241804.

[7] M. Schumacher, Eur. Phys. J. A **34** (2007) 293; ibid. **31** (2007) 327; ibid. **30** (2006) 413.

[8] M. Damashek and F.J. Gilman, Phys. Rev. D **1** (1970) 1319.

[9] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.

[10] J. Bernabéu, T.E.O. Ericson, and C. Ferro Fontan, Phys. Lett. **49 B** (1974) 381.

[11] J. Bernabéu and R. Tarrach, Phys. Lett. **69 B** (1977) 484.

[12] W.R. Frazer and J.R. Fulco, Phys. Rev. **117** (1960) 1603.

[13] O. Babelon *et al.* Nucl. Phys. B **114** (1976) 252.

[14] M. Schumacher, Prog. Part. Nucl. Phys. **55** (2005) 567; M.I. Levchuk *et al.*, arXiv:hep-ph/0511193.

[15] B.R. Holstein and A.M. Nathan, Phys. Rev. D **49** (1994) 6101.

[16] G.E. Bohannon, Phys. Rev. D **14** (1976) 126; G.E. Bohannon and P. Signell, Phys. Rev. D **10** (1974) 815.

[17] D. Morgan and M.R. Pennington, Phys. Lett. B **272** (1991) 134; J.F. Donoghue and B.R. Holstein, Phys. Rev. D **48** (1993) 137.

[18] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.

[19] R. Omnès, Nuovo Cimento **8** (1958) 316.

[20] F. Low, Phys. Rev. **96** (1954) 1428; M. Gell-Mann, M.L. Goldberger, Phys. Rev.**96** (1954) 1433; H.D.I. Abarbanel, M.L. Goldberger, Phys. Rev. **165** (1968) 1594.

[21] J. Bijnens and F. Cornet, Nucl. Phys. B **296** (1988) 557.