Double Beta Decay And $SU(4)$ Symmetry

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Abstract

The amplitudes for $\beta\beta$ decay with 2ν emission is shown to be related to (p,n) and (n,p) reactions on the initial and final states, respectively. The suppression of both $\beta\beta$ and (n,p) reactions is connected, and its origin is discussed by referring to the $SU(4)$ symmetry. From present data on the first ones, we estimate the forward (n,p) strength of relevance for the $\beta\beta$ problem. The interest of the experimental determination of this strength is emphasized. Assuming a perturbative breaking of the $SU(4)$ symmetry, results are given for $^{76}Ge, ^{82}Se, ^{128}Te$ and ^{130}Te.

PACS numbers 14.60Gh, 23.40 Hc, 25.40 Fq
IFIC/89–20
FTUV 89/34

IPNO/TH 89-55 October 1989

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1 Introduction

Double beta decay is the process by which a nucleus ^AZ in its ground state undergoes a transition to the ground or low lying state of the nucleus $^A(Z+2)$ by a simultaneous change of two neutrons into two protons. A great achievement of experimental science has been the direct observation [1] by Elliott, Hahn and Moe of the decay of ^{82}Se into ^{82}Kr plus two electrons and two antineutrinos with a half-life of $(1.1 \pm {}^{0.8}_{0.3}) \times 10^{20}$ years, in agreement with the geochemical measurements of Kirsten [2], $(1.30 \pm 0.05) \times 10^{20}$ years, and Manuel [3], $(1.0 \pm 0.4) \times 10^{20}$ years, and the cosmochemical (meteorite) measurements of Marti and Murty [4], $(1.2 \pm 0.3) \times 10^{20}$ years. Furthermore, there are results [5] by means of geochemical techniques on ^{128}Te and ^{130}Te, as well as some positive result [6] in ^{76}Ge, $(1.0 \pm {}^{0.3}_{0.2}) \times 10^{20}$ years, in which the total energy of the two electrons is measured.

For this two neutrino mode of $\beta\beta$ decay, the phenomenon is treated as a second order effect of the charged current effective hamiltonian of the standard electroweak interaction. Due to the implications for lepton number non conservation, Majorana character of neutrino and, more generally, for theories beyond the standard electroweak model, the other modes of double beta decay, such as the 0ν mode and the 0ν with emission of a Majoron mode, are particularly interesting. However, in order to correctly interpret the results of experiments on double beta decay, one has to reliably extract the various nuclear matrix elements associated with the different decay processes. In this letter, we restrict ourselves to the discussion of the transition to the ground state of the daughter nucleus for the standard 2ν mode.

Many attempts have been made in the literature to calculate the nuclear matrix element involved. In the shell model calculations [7], the results yield significantly larger values than the ones needed to explain the lifetimes measured either in the laboratory experiment on ^{82}Se decay or by means of geochemical techniques on ^{82}Se , ^{128}Te and ^{130}Te. There is a problem, as yet not fully understood, with this two-neutrino matrix element [and some controversy [8] about the existence of the problem for the corresponding neutrinoless nuclear matrix element]. It has been pointed out [9] that the matrix elements for the 2ν, and to some extent also for the 0ν, $\beta\beta$ decay are suppressed when evaluated in the QRPA with the inclusion of particle-hole and particle-particle components of the neutron-proton interaction. Although these calculations based on QRPA usually give matrix elements smaller than the shell model values, the results depend very sensitively on the value of the particle-particle interaction constant, which is not known with high accuracy. One concludes from this approach that the amplitudes can vary between zero and the shell model value for the allowed transition. Confronted with these problems one feels the necessity of both a phenomenological approach, in which the nuclear matrix element is empirically extracted from other experimental sources, and a more fundamental scheme, with an actual understanding of the suppression of the transition in Nature.

In Section 2 we develop a phenomenological approach in which the amplitudes for $2\nu-\beta\beta$ decay is related to (p,n) and (n,p) reactions on the parent and daughter nucleus, respectively. We shall see that, besides its own virtues, such an approach sheds some light on the theoretical origin of the observed suppression, namely the existence of an (approximate) underlying symmetry responsible for this suppression, which we assume to be the $SU(4)$ symmetry. In Section 3 we calculate the $2\nu-\beta\beta$ amplitudes within a $SU(4)$ scheme for the wave function. We analyze previous results obtained using more realistic wave functions. Finally, in Section 4 we draw the conclusions of this work.

2 A phenomenological approach

The decay rate of the 2ν mode of $\beta\beta$ decay is proportional to the square of the amplitude :

$$M = \sum_K \frac{< f|\sum_m \vec{\sigma}_m \tau_m^+|K >< K|\sum_n \vec{\sigma}_n \tau_n^+|i >}{E_K - (E_i + E_f)/2} , \qquad (1)$$

where $|i>, |f>$ are the given "in" and "out" states of the parent and daughter nuclei respectively, and $|K>$ are all possible intermediates states. Closure is not easily justified in eq.(1) but, nevertheless, it has been used in the shell model calculations. Replacing the energy denominator by an estimated "typical" value, one obtains the closure matrix element :

$$M = \frac{1}{\Delta E} M_{clos.} \ ; \ M_{clos.} =< f|\sum_{m,n} \vec{\sigma}_m \vec{\sigma}_n \tau_m^+ \tau_n^+|i > \ . \qquad (2)$$

Thus, in the closure approximation, the rate of the $2\nu-\beta\beta$ decay is determined by the scalar double Gamow-Teller operator. Vogel, Ericson and Vergados have shown [10] that $M_{clos.}$ determines the extreme low energy tail of the strength distribution corresponding to the double charge exchange Gamow-Teller sum rules. The small fraction of the "real" $\beta\beta$ transition justifies the difficulty of its theoretical understanding. A possible way to study experimentally the double charge exchange strength is through reactions based on strong interactions in which the projectile loses two units of charge and the target gains them or vice versa. However, the experimental difficulties are quite formidable and the interpretation of the reaction mechanism is not straightforward. We do not follow this path in our attempt to connect with experimentally determined quantities.

We realize, instead, that :

$$< K|\sum_n \vec{\sigma}_n \tau_n^+|i > \quad \text{and} \quad < K|\sum_m \vec{\sigma}_m \tau_m^-|f > , \qquad (3)$$

are the probability amplitudes for the forward ${}^AZ(p,n)^A(Z+1)^*$ and ${}^A(Z+2)(n,p)^A(Z+1)^*$ reactions in the parent and daughter nuclei respectively. Empirically, most of the strength observed [11] in the (p,n) reaction lies in a collective state which is manifested as the giant Gamow-Teller resonance induced by the single charge exchange operator $\sum_n \vec{\sigma}_n \tau_n^+$. Although the energy of this relevant intermediate collective state is not always perfectly defined, with some strength distributed in peaks and in the continuum [12] [with a width of a few MeV], the (p,n) transition is fully allowed in the sense that it exhausts typically $\approx 60\%$ of the Gamow-Teller sum rule. One realizes that the suppression of the 2ν-$\beta\beta$ decay matrix element has to have its origin in the second matrix element of eq.(3), that is, the one which can be explored from the (n,p) reaction on the daughter nucleus. Once the relevant intermediate collective state has been identified through the giant Gamow-Teller resonance for the (p,n) reaction on the parent nucleus, we propose the measurement of the strength corresponding to the (n,p) reaction from the daughter nucleus to that intermediate state. One can argue that what is needed in eq.(1), the sum of products of amplitudes divided by the energy differences, cannot be obtained from only the measured strength distribution in these single charge exchange reactions due to sign ambiguities and possible cancellations. Several authors have emphasized the particular importance for $\beta\beta$ decay of the low energy part of the excitation spectrum, due to the energy denominator weight. However, once the transition to the collective state is admitted as the dominant feature, the effect of the energy distribution of the strength can be taken into account by the appropriate integration of a Lorentzian shape, with a Γ−width, weighted with the inverse energy difference factor. In any case, this effect is not expected to be crucial for the $\beta\beta$ problem of the suppression of M. The strength of the ${}^A(Z+2)(n,p)^A(Z+1)^*$ reaction should be at the core of the problem. With this discussion in mind, one gets from eq.(1) that the use of :

$$< K|\sum_n \vec{\sigma}_n\tau_n^+|i> = \delta_{K,GT} < GT|\sum_n \vec{\sigma}_n\tau_n^+|i> , \tag{4}$$

leads to a $2\nu-\beta\beta$ decay matrix element :

$$|M| = | < GT|\sum_n \vec{\sigma}_n\tau_n^+|i> | \frac{E_{GT}-\varepsilon}{(E_{GT}-\varepsilon)^2+\Gamma^2/4}| < GT|\sum_m \vec{\sigma}_m\tau_m^-|f> | , \tag{5}$$

where $\varepsilon = (E_i + E_f)/2$, and E_{GT} is the central value of the giant GT-resonance peak energy. Energy systematics of the GT peak has been discussed in refs. [11,13]. Values of E_{GT} can be obtained from experiment for each nucleus.

For nuclei $|f>$ with a large neutron excess, $N - Z >> 1$, the (n,p) transitions are Pauli blocked and thus one expects the strength represented by the last factor of eq.(5) to be small, when compared to the sum rule for Gamow-Teller transitions, $3(N - Z)$. What the 2ν - $\beta\beta$ decay results suggest is that this blocked transition is even smaller in magnitude than present shell model expectations. This is quite contrary to the "first" transition from $|i>$ to $|GT>$, in which only the gross nuclear features are seen ; even the question of where is the missing $\approx$ 40% part of the strength is not important for our purposes, if placed at high energies. The "second" transition from $|GT>$ to $|f>$, however, is going to be sensitive to fine details of the nuclear dynamics and then difficult to control theoretically. Besides all meritory theoretical efforts to understand these minute transition amplitudes, definite experiments to determine the (n,p) transition strength in nuclei of interest such as ^{48}Ti, ^{76}Se, ^{82}Kr, ^{128}Xe and ^{130}Xe , are worth to be undertaken in view of their important and rewarding issues. The collective state $|GT>$ to which measure this transition has been unambiguously identified from the (p,n) reaction in the parent nucleus, such as ^{48}Ca [12], ^{76}Ge, ^{82}Se ^{128}Te and ^{130}Te in ref.[14]. We stress the expected asymmetry in the magnitude of the strength associated with the "first" allowed transition and the "second" blocked transition in 2ν double beta decay.

In presence of residual interactions the Pauli suppression of the (n,p) transition amplitudes does not necessarily work anymore and one should think of an approximate symmetry to be responsible for their forbideness. The relevance of such a symmetry for Gamow-Teller transitions was recently reemphasized (ref.15). The GT operators $\vec{Y}^{\pm} \equiv \sum_m \vec{\sigma}_m \tau_m^{\pm}$ being generators of a spin -isospin supermultiplet $SU(4)$ symmetry, the above forbideness is automatic as long as $|f>$ and the collective $|GT>$-state belong to different supermultiplet representations. This is precisely what is expected in the symmetry limit. For even-even nuclei with $|N-Z| >> 1$, the ground state belongs [16] to the supermultiplet $[y,y,0]$ and has $S=0, T=y=(N-Z)/2$. The ground state is the only state of the nucleus ${}^{A}Z$ belonging to that supermultiplet. In the odd-odd nucleus, ${}^{A}(Z+1)$, there are two states belonging to the supermultiplet $[y,y,0]$, the isobaric analogue state with $T=y, S=0$ and the GT state with $T=y-1, S=1$. The last state corresponds to the intermediate collective excitation for $\beta\beta$ decay. In the next even-even nucleus, ${}^{A}(Z+2)$, the states belonging to $[y,y,0]$ are : the double isobaric analogue $T=y, S=0$ and two double GT states with $T=y-2, S=0$ and $T=y-2, S=2$. None of them corresponds to the ground state of the ${}^{A}(Z+2)$ nucleus, which is the state $T=y-2, S=0$ belonging to the $SU(4)$-supermultiplet $[y-2,y-2,0]$. There are no states belonging to the last supermultiplet $[y-2,y-2,0]$ in the intermediate odd-odd nucleus ${}^{A}(Z+1)$. All these assignments are represented in fig.1, where the states which are linked by the uppering and lowering generators $\vec{Y}^{\pm}$ are explicitly given. This limit would produce a vanishing forward (n,p) transition and, a fortiori, a vanishing 2ν- $\beta\beta$ decay matrix element, so a breaking of the $SU(4)$ supersymmetry able to mix the ground and excited states of the final nucleus in fig.1 is needed.

Lacking experimental results at present on (n,p) reactions in nuclei of interest for $\beta\beta$ decay, we illustrate the phenomenological method proposed in this letter using eq.(5) in the inverse manner. We give in Table 1 the known parameters of eq.(5) from the (p,n) reaction and the 2ν- $\beta\beta$ decay, and we extract the predicted (n,p) transition strength to the Gamow-Teller resonance. We observe the anticipated suppression of these transitions, for which the magnitude of the strength oscillates between $2.5\,10^{-5}$ and $1.2\,10^{-2}$ of the Gamow-Teller sum rule.

The (n,p) transition strength should be shared between several states. The behaviour of the strength to the GT resonance can be analyzed by counting the number of active nucleons and assuming the $SU(4)$ supermultiplet mixing to be nucleus independent. With this simple minded estimate, one would expect a result for our (n,p) amplitude proportional to $(N-Z)^{-1/2}$, if $N-Z>>1$. This factor, which for some part explains the smaller size of the amplitudes in heavy nuclei, is not sufficient however to account for their overall suppression. The only exception is the case of ^{76}Ge. The corresponding GT strength, $B(GT)=0.28$, when corrected for the factor $(N-Z)^{-1/2}$ mentioned above, largely exceeds the strength for the other nuclei. One has therefore to face a specific nuclear structure effect in ^{76}Ge, or to question the experimental result given in (6). Actual measurements [17] of (n,p) transitions in ^{40}Ca, ^{54}Fe and ^{90}Zr are of no help to clarify the issue, as either $(N-Z)$ is small or they only provide an upper limit to the GT transition.

Our results show that the $SU(4)$ symmetry breaking is not very strong for the Gamow-Teller transition matrix elements : the forbidden transition matrix elements as seen in forward (n,p) reactions, remain very suppressed, even if this breaking is considered to be very strong for the energy levels of medium and heavy nuclei. In that situation, one expects perturbation theory in the $SU(4)$-breaking hamiltonian H' to be valid. This provides a rationale for the theoretical description of 2ν-$\beta\beta$ decay as given in fig.2, where all intermediate states are eigenstates of the supersymmetric H_0 hamiltonian and the connection between the initial and final supermultiplets is provided by the H'-insertion in the final nucleus.

Another phenomenological approach exists in the literature [18], which claims for the relevance of the lowest 1^+ state of the intermediate nucleus. The small value of the GT matrix element for this state can be compensated by a small energy denominator. One criticism to this approach is the large uncertainty in transposing experimental results for some nuclei to actual nuclei of interest for $\beta\beta$-decay. Moreover, one cannot exclude cancellations with contributions of other low-lying states (some cancellation between different contributions of 1^+ low energy states can be observed in fig.11 of the paper of Grotz and Klapdor in Ref.[7]). These features leave the GT state contribution as an essential one.

3 An analysis based on $SU(4)$ symmetry

In this section we present and discuss results obtained according to the scheme described above. Let us consider the simplest case in which the initial and the final states have seniority zero, and let us assume that they have the following structure [16] :

$$
\begin{aligned}
|i> &= |\ell_1^{n_1}[y_1, y_1, 0]\ell_2^{n_2}[y_2, y_2, 0]\ ;[y, y, 0]\ S = 0, T = y, M_T = -y> \\
|f> &= |\ell_1^{n_1+2}[y_1 - 1, y_1 - 1, 0]\ell_2^{n_2-2}[y_2 - 1, y_2 - 1, 0]\ ;[y - 2, y - 2, 0] S = 0, T = y - 2, M_T = 2 - y>
\end{aligned}
\tag{6}
$$

i.e., we consider two active shells ℓ_1 and ℓ_2 , each in a Young tableau of the type $[y_i, y_i, 0]$, where y_i is the half-difference between the number of neutrons and protons in the shell. These configurations correspond to the case in which two neutrons in the shell ℓ_2 of the state $|i>$ have been exchanged into two protons in the shell ℓ_1 to obtain the state $|f>$. The intermediate states are the partners of the state $|i>$ in the supermultiplet $[y, y, 0]$ with $S = 1, T = y - 1$ (Gamow-Teller state) and $S = 0, T = y - 2$ (see Fig.1).

The dynamical breaking of $SU(4)$ symmetry can have several origins : the spin-orbit and the tensor potentials and, concerning the central interaction, the pieces giving different intensities between triplet-singlet channels as well as triplet-triplet and singlet-singlet ones. One can convince one-self that only the differences between triplet-singlet and singlet-triplet parts of the central interaction will contribute to the $\beta\beta$-process with the above choice of states. Such a piece corresponds to the [2,2,0] tableau and we write it as $H' = V(\tau_i\tau_j - \sigma_i\sigma_j)$. After some $SU(4)$ algebra [16], we obtain the following expression for the amplitude

$$M = (-)^{\ell_1+\ell_2} \frac{6}{\pi^2} \frac{C}{(E_f - E_S)(E_{GT} - (E_i + E_f)/2)} \sum_\lambda (\ell_1 \ell_2 \lambda; 00)^2 \, . \\ \cdot \int dq q^2 |R_{12}^\lambda(q)|^2 V(q) \tag{7}$$

where we have defined

$$R_{12}^\lambda(q) = \int dr r^2 j_\lambda(qr) R_{n_1\ell_1}(r) R_{n_2\ell_2}(r) \tag{8}$$

and the coefficient C, which includes the $SU(4)$ parameter dependence, has the following general expression

$$C = \frac{1}{4}\left[\frac{(8\ell_1 + 2y_1 - n_1 + 12)(n_1 - 2y_1 + 4)(8\ell_2 - 2y_2 + 8 - n_2)(n_2 + 2y_2 + 8)y_1y_2}{(y_1+2)(y_2+2)}\right]^{1/2} \tag{9}$$

In the particular case when shell 1 is full in neutrons and shell 2 is empty in protons, this coefficient can more simply be written as

$$C = [(Z_1 + 2)(4\ell_1 + 2 - Z_1)(4\ell_2 + 4 - N_2)N_2]^{1/2} \tag{10}$$

where $Z_1(N_2)$ is the number of protons (neutrons) in the shell ℓ_1 (ℓ_2).

The energy E_S is that of the $S = 0, T = y - 2$ state of the supermultiplet $[y, y, 0]$. Assuming that the same piece of the potential is responsible for the splitting of the $SU(4)$ supermultiplet we can write

$$E_S - E_i = \frac{3}{y+1}(E_{IAS} - E_i) + \frac{2y-1}{y+1}(E_{GT} - E_i) \tag{11}$$

so that the experimental values of E_{IAS} and E_{GT} suffice for the knowledge of E_S. The values of E_{GT} can be deduced from Table 1, whereas the IAS excitation energies for the nucleus we are interested in are 8.24 (^{76}Ge), 9.58(^{82}Ge), 11.88(^{128}Te) and 12.48(^{130}Te) MeV.

In Table 2 we display our results for the amplitude M as given in eq.(7). To get a measure of possible uncertainties we give for ^{82}Se , ^{128}Te and ^{130}Te results corresponding to different assignments which might be equally possible. Woods-Saxon wave functions have been employed. We used as an effective force that one determined by Bertsch and Hamamoto [19], that of Rosenfeld [20] and also the Gogny interaction D_1 [21].

The results given by the interactions of Bertsch-Hamamoto and Rosenfeld are very similar, the interaction of Gogny (which contains a density dependent part), producing slightly higher amplitudes. This stability is obtained in spite of strong cancellations between attractive and repulsive parts of the force. In the following we will particularize the discussion to the results obtained with the Bertsch-Hamamoto force. The results are also sensitive to the number of particle pairs on the active shells. To provide an estimate of the corresponding effect, results for only one active pair have also been reported between parentheses.

A striking feature of our results is their uniform size. Differences are essentially due to firstly, the algebraic factor C related to the number of active nucleons which tends to favor the transition $h^2 \rightarrow f^2$ in Te (a factor $2-3$ with respect to the others), and secondly, to a lesser extent, to radial integrals, which tend to favor the transition $g^2 \rightarrow f^2$ over $g^2 \rightarrow p^2$ by a factor of ~ 1.8.

The right average size of the results with respect to experiment (by factors of 0.2 for ^{76}Ge, 0.2-0.5 for ^{82}Se and 0.4 -2 for ^{130}Te), represents an other striking feature since current predictions generally tend to be too large. We believe that this is specific of our approach, where the $SU(4)$ symmetry breaking is introduced perturbatively. In some other approaches, this symmetry which in any case is only an approximate one, is strongly broken by the choice of the model space (not considering all the $SU(4)$ partners configurations) on the one hand, by retaining only pairing forces on the other hand.

In such a case, enlarging the model space or introducing further parts of the force restores the relevance of the $SU(4)$ symmetry for transitions considered here and one should not be surprised that they tend to reduce previous predictions. The relevance of this symmetry is especially shown by the observed concentration of the Gamow-Teller strength in one state, a feature which is incorporated in our approach.

Similarly to single β^- transitions, the strength following double β transitions should be concentrated at a rather high excitation energy in the final state and the transition amplitude to this one, according to the scheme represented in fig.1 should be inversely proportional to that excitation energy, about 25-30 MeV in present calculations. In usual calculations ignoring the effect of correlations, this energy excitation may be as low as a few MeV, hence the large predictions they lead to. Probably, truth is in between these two extreme cases, but in any case relying on the $SU(4)$ symmetry provides a natural explanation of the suppression of double β decay amplitudes.

Relying on the $SU(4)$ symmetry may shed some light on other aspects of previous calculations. In most of them, the effect involves the pairing force and therefore the proton-proton or neutron-neutron parts of the nucleon-nucleon interaction. If $SU(4)$ was an exact symmetry, that contribution would exactly be cancelled by the contribution of the proton neutron part of the nucleon-nucleon interaction. In current effective forces, this part has generally a stronger strength than for like particles. As an example, our results have a sign intrinsically opposite to that of most previous calculations and would correspond to a value of the g_{pp} parameter larger than 1. It is important to notice that the smallness of our results with respect to other ones does not imply that g_{pp} would be implicitly very close to 1 in our approach. Forces we used have an $SU(4)$ breaking part which is almost as large as the one giving rise to the pairing force. This point shows that further studies are necessary to reconcile quite different approaches. The first one employs forces between quasi-particles which are fitted to reproduce the energy spectra and some other properties of nuclei. As shown by the recent discovery of the relevance of the particle-particle force for describing β^+ transitions, such an approach may miss essential features of the interaction. The second one employs effective forces which are closer to theoretical expectations. They are too simple to explain with success the detailed spectroscopy of nuclei.

A detailed comparison with present experimental information is difficult due to some ambiguities inherent to our results (different configurations can give amplitudes differing by a factor 2-3). Predicted amplitudes for double β decay will 2ν are not very different for ^{76}Ge and ^{130}Te while the experimental ones differ by a factor 10. With this respect, other calculations are not doing much better. There may be some problem with experiment. The β^+ strength corresponding to the experimental result in ^{76}Ge is particularly large, which is difficult to understand in this scheme. It would probably exceed the strength expected in the low energy range from that $SU(4)$ breaking part of this force relevant here and due to the difference between singlet and triplet spin states. There are other well identified pieces of the force which break the $SU(4)$ symmetry and arise from the spin-orbit and tensor forces. Due to the simple assignment of initial and final states made in our study, which suppose pairs of particles coupled to a total orbital angular momentum $L = 0$, they do not contribute here. They may however contribute with more refined descriptions of initial and final states, incorporating in particular the effect of the single particle spin-orbit splitting. One may then explain experimental results in ^{76}Ge and ^{130}Te respectively by constructive and destructive interferences of the different $SU(4)$ breaking contributions. Obviously, there are several other effects which may explain these features when going beyond the present 1^{rst} order perturbation calculation.

Finally, let us say a few words on the nuclei ^{48}Ca and ^{100}Mo, whose half-lifes have been bounded. We have not considered them because the transitions involve neutrons and protons in the same shell, and the $SU(4)$ algebra needed has not been developed. In particular, the $SU(4)$ fractional parentage coefficients are not tabulated. Nevertheless, we can infer from our scheme that the same $SU(4)$ suppression works also here, since the initial and the final states belong to different supermultiplets.

4 Conclusions

The aim of the present paper was to devise an approach to the determination of $2\nu - \beta\beta$ decay amplitude. Based on the relevance of the GT state within the set of intermediate states, a phenomenological approach has been developed which relies the amplitudes of the reactions (p, n) on the parent nucleus and (n, p) on the daughter one to $\beta\beta$-decay amplitude. The apparent suppression of $\beta\beta$-amplitudes has been related to $SU(4)$ symmetry. This provides a simple scheme in which the various contributions breaking this symmetry can easily be identified and discussed.

Calculations at the level considered here are far from providing accurate results. Nevertheless we stress that in spite of its simplicity this approach gives reasonable results, near the experimental ones. Moreover, it is orthogonal to other approaches used until now in the field and, in this respect, it can help to develop a critical viewpoint about the reliability of present and future estimates. This is of the upmost importance for the interpretation of $\beta\beta$-decay experiments without neutrino emission.

The authors thank Dr. Flanders for providing them with his experimental results prior to publication and also Dr. Hecht for valuable information about $SU(4)$ algebra. Three of us (J.B., J.N., S.N.) acknowledge the warm hospitality of the Division de Physique Théorique de l'IPN, where this work was started. This work is partly supported by CICYT (Spain) under grants AE-0021 and AE87-0027-3.

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Table Caption

Table 1 Inputs used to derive the (n,p) transition amplitudes given in the last column as discussed in the text. Results for GT strength and energy parameters are from ref.[12] for ^{48}Ca and from ref.[14] for ^{76}Ge, ^{78}Ge,^{82}Se,^{128}Te and ^{130}Te.

Table 2 Double β decay amplitudes calculated with different effective forces. Relevant transitions considered for the nuclei of interest are indicated in the first column, while experimental results are reminded in the last one.

Figure Captions

Fig.1 Energy levels diagram corresponding to the $SU(4)$ supermultiplet involved in a $\beta\beta$ transition. The mixing between them is provided by H'.

Fig.2 Feynman diagram representation of the double β transition, with the insertion in the final state of the $SU(4)$ symmetry breaking interaction.

Table 1

	$\mid<GT\|\vec{Y}^{+}\|i>\mid$	$(E_{GT}-\epsilon)$ (MeV)	Γ (MeV)	$\|M\|$ (MeV^{-1})	$\mid<GT\|\vec{Y}^{-}\|f>\mid$	
^{48}Ca	4.1[12]	11.3	5	<0.050	<0.14	^{48}Ti
^{76}Ge	4.5[14]	13.1	5.1	0.17[6]	0.53	^{76}Se
^{82}Se	4.7[14]	13.6	4.7	0.083[10]	0.25	^{82}Kr
^{128}Te	6.6[14]	14.8	5.1	$<0.026-0.033$[5]	$<$ 0.059-0.075	^{128}Xe
^{130}Te	7.0[14]	15.4	4.9	0.019-0.024[5]	0.043-0.054	^{130}Xe

Table 2

	Transition	Gogny	Rosenfeld	Bertsch-Hamamoto		Experimental result
^{76}Ge	$g^2 \rightarrow f^2$	0.053	0.042	0.036	(0.027)	0.17
^{82}Se	$g^2 \rightarrow p^2$	0.030	0.016	0.014	(0.009)	0.083
	$g^2 \rightarrow f^2$	0.059	0.046	0.040	(0.024)	
^{128}Te	$h^2 \rightarrow g^2$	0.055	0.049	0.041	(0.017)	$<$0.026-0.033
	$d^2 \rightarrow g^2$	-0.014	-0.010	-0.008	(-0.005)	
^{130}Te	$h^2 \rightarrow g^2$	0.054	0.048	0.041	(0.017)	0.019 -0.024
	$d^2 \rightarrow g^2$	-0.013	-0.009	-0.008	(-0.005)	

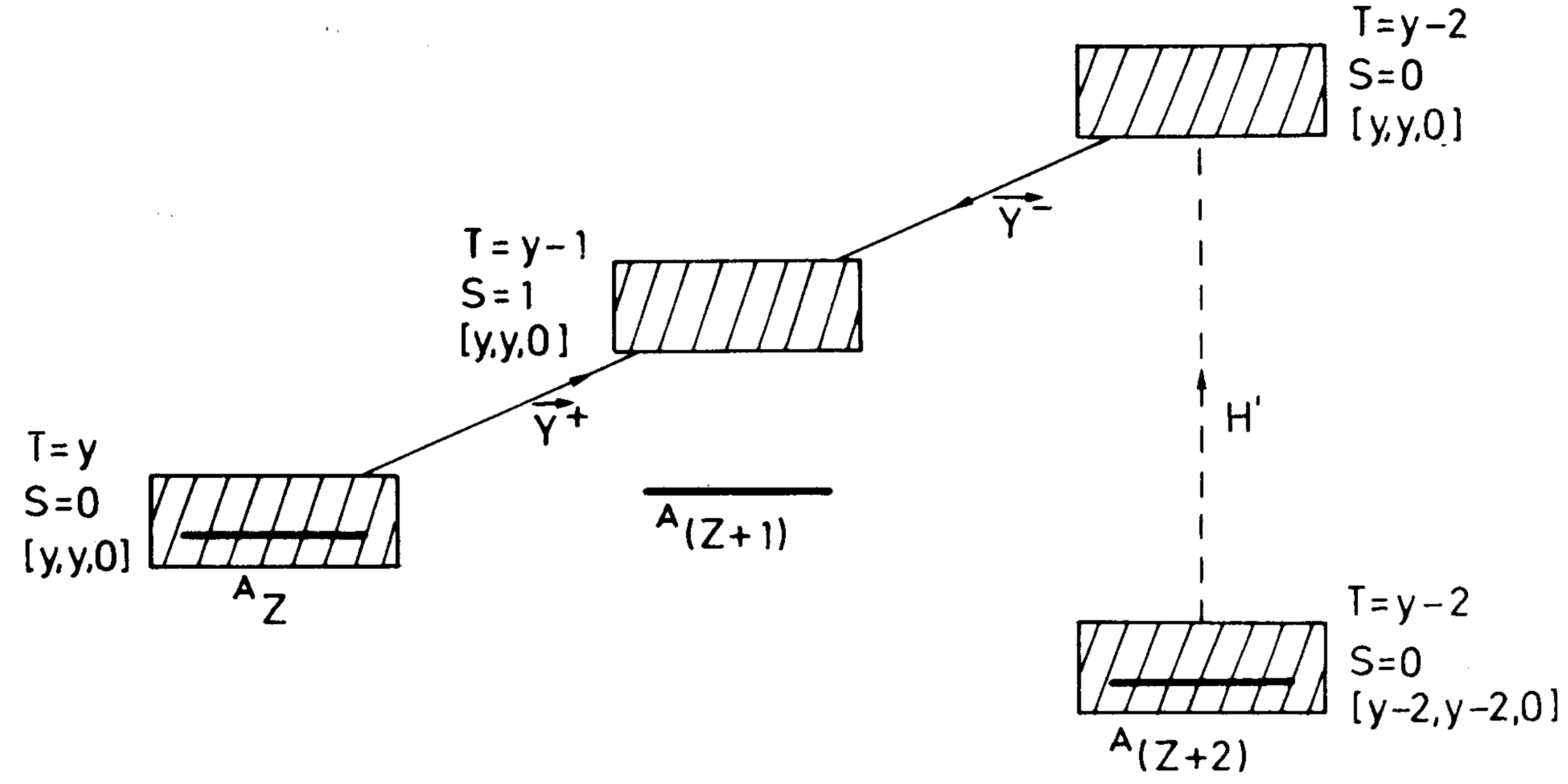

Fig.1

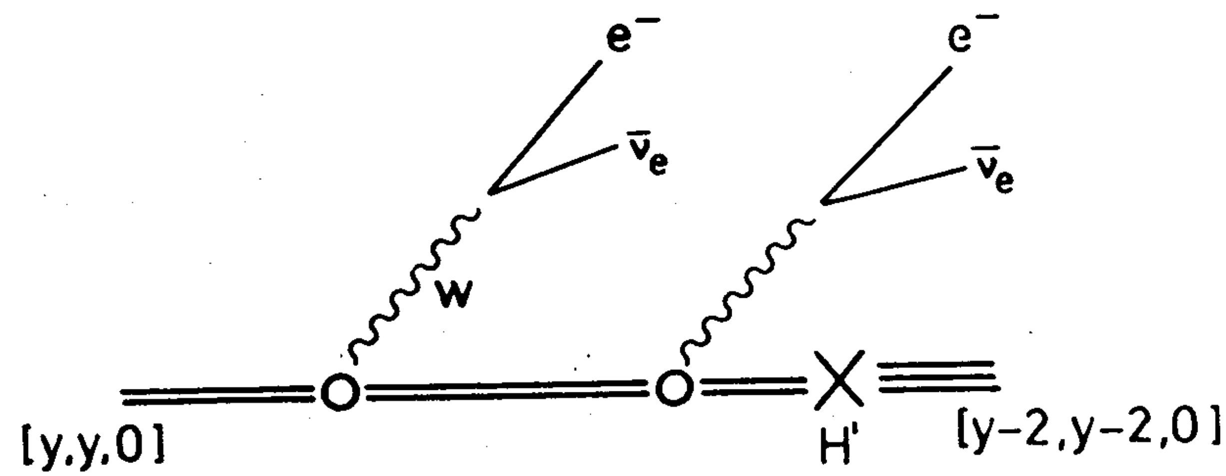

Fig.2