States of $\rho D^* \bar{D}^*$ with $J = 3$ within the Fixed Center Approximation to the Faddeev equations

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Abstract

We study the interaction of the a ρ and D^*, $\bar{D}^*$ with spins aligned using the Fixed Center Approximation to the Faddeev equations. We select a cluster of $D^* \bar{D}^*$, which is found to be bound in $I = 0$ and can be associated to the X(3915), and let the ρ meson orbit around the D^* and $\bar{D}^*$. In this case we find an $I = 1$ state with mass around 4340 MeV and narrow width of about 50 MeV. We also investigate the case with a cluster of ρD^* and let the $\bar{D}^*$ orbit around the system of the two states. The ρD^* cluster is also found to bind and leads to the $D_2^*(2460)$ state. The addition of the extra $\bar{D}^*$ produces further binding and we find, with admitted uncertainties, a state of $I = 0$ around 4000 MeV, and a less bound narrow state with $I = 1$ around 4200 MeV.

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I. INTRODUCTION

In hadron physics the existence of the three body resonances is drawing much attention for a long time. The proper analysis of the three hadron system can be tackled by combining the Faddeev equations [1] and chiral dynamics [2–4]. However it is quite difficult to solve exactly the Faddeev equations. Recently, the combination of Faddeev equations and chiral dynamics was used to investigate for two meson-one baryon systems [5] and also for three mesons systems [6, 7]. On the other hand, the fixed center approximation (FCA) to the Faddeev equations is technically very simple and a powerful method to explore three hadron systems. This method is especially suitable to study the system in which two of the three particles are bound forming a cluster and this cluster is not much altered by the collision of the third particle [8–10]. This method has proved to be rather reliable for cases like K-deuteron scattering very close to threshold [11, 12]. In recent years the FCA to Faddeev equations has been successfully applied to the study of many three body interactions. For example in the work of the Ref. [13] the $\pi - (\Delta\rho)_{N_{(5/2)^-}(1675)}$ was analyzed by means of the FCA to the Faddeev equations in which the authors give a reasonable explanation for the $\Delta_{(5/2)^+}(2000)$ puzzle. Likewise, the $N\bar{K}K$ system was investigated using the Faddeev equations under the FCA in Ref. [14] and the results are in good agreement with the variational estimation [15] and also the full Faddeev calculation [16, 17].

In the present paper we want to study the $\rho D^*\bar{D}^*$ system. The reason is that the vector-vector interaction is found to be very strong, particularly in the $J = 2$ channel [18, 19]. Due to this, it was possible to see that multi-ρ states with the spins parallel were bound, although the width was increasing with the number of ρ's [20]. The masses and widths obtained were in good agreement with experimental data. The same occurred with K^* multi-ρ states in [21] and with D^* multi-ρ states in [22]. In the case of K^* multi-ρ states one finds good agreement with experiment, but in the case of D^* multi-ρ states only predictions were made and experimental counterparts have not yet been observed.

The $\rho D^*\bar{D}^*$ system is new and has not been studied so far. Yet, studies done for $D^*\bar{D}^*$ interaction in [23] and $D^*\rho$ interaction in [24] have already set the grounds to tackle this interesting system with hidden charm, and we wish to study it here.

II. FORMALISM

In this section we describe the calculation of the three body interaction of the $\rho D^*\bar{D}^*$ system in s-wave and all the spins aligned. In the work of [24], the ρD^* interaction was studied using the hidden gauge formalism [25–27]. In [24] the authors found strong attraction in $I = 1/2$, $J = 2$ which corresponds to the tensor state $D_2^*(2460)$ with $I(J^P) = \frac{1}{2}(2^+)$. Similarly, it was also found in Ref. [23] that the resonance $X(3915)$ ($I^G(J^{PC}) = 0^+(2^{++})$) could be understood as a molecule made of D^* and $\bar{D}^*$ mesons with strong attraction in the $I = 0$, $J = 2$ sector.

We use the FCA to the Faddeev equations to study the $\rho D^*\bar{D}^*$ system. This method is particularly well suited for the system in which a pair of particles clusters together and the cluster is not much modified by the third particle, like in the case of $D_2^*(2460)$ as a ρD^* cluster and $X(3915)$ as a $D^*\bar{D}^*$ cluster.

For the technical details we proceed similarly to Ref. [10] to express the FCA to the Faddeev equations. Hereinafter we are going to interpret the interaction of a particle a_3 with a cluster made of two particles, a_1 and a_2. In this work the particle a_3 will represent the $\rho(\bar{D}^*)$ which scatters from the cluster, $X(3915)$ ($D_2^*(2460)$) and a_1 and a_2 are the $D^*(\rho)$ and $\bar{D}^*(D^*)$ which build up the cluster.

The diagramatic representation of the FCA to the Faddeev equations is shown in Fig. 1. The

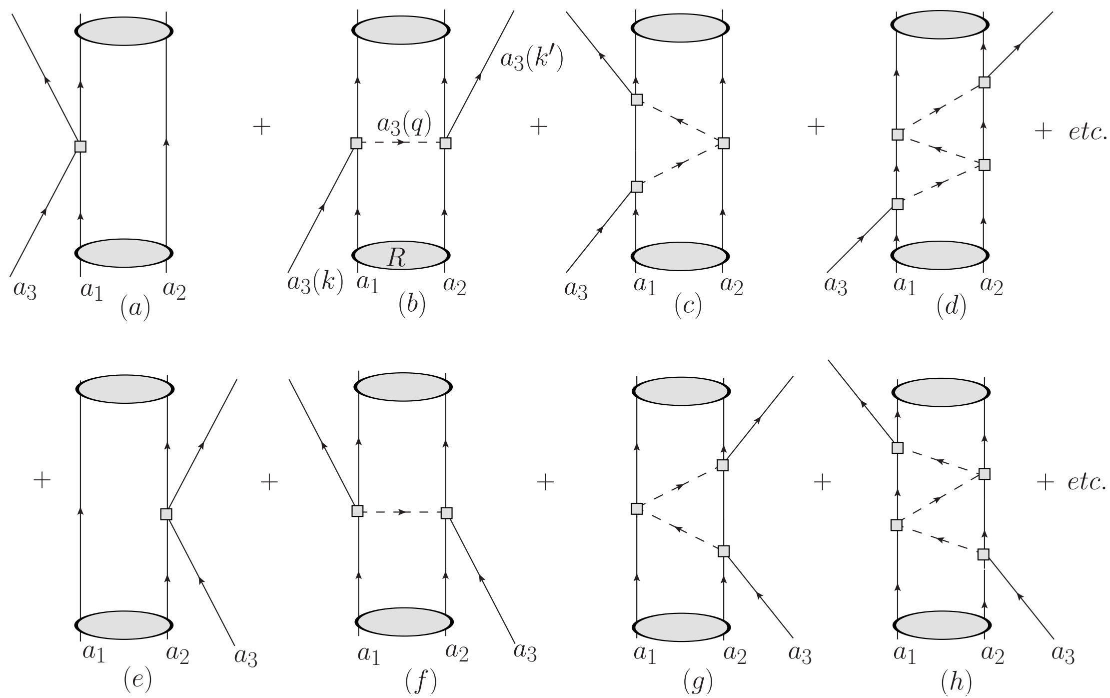

FIG. 1: Diagrammatic representation of the fixed center approximation to Faddeev equations.

particle a_3 rescatters repeatedly with the components of the cluster. In Fig. 1 the thick squared dots represent the unitary scattering amplitudes with coupled channel for the interaction of particle a_3 with particle a_1 (t_1) and a_2 (t_2), respectively, which will be discussed later. In order to write the equations for the total three body scattering amplitude, we define two partition functions T_1, T_2 which sum all diagrams of the series of Fig. 1 which begin with the interaction of particle a_3 with particle a_1 of the cluster (T_1), or with particle a_2 (T_2). Then the FCA equations can be written as a system of coupled equations:

$$\begin{aligned} T_1 &= t_1 + t_1 G_0 T_2, \\ T_2 &= t_2 + t_2 G_0 T_1, \\ T &= T_1 + T_2 \end{aligned} \tag{1}$$

where G_0 is the Green function for the propagator of particle a_3 between the particles a_1 and a_2 which is discussed later on.

The scattering amplitude $\langle \rho D^* \bar{D}^* | t | \rho D^* \bar{D}^* \rangle$ for the single scattering contribution is obtained in terms of the two-body amplitudes t_1, t_2 derived in Refs. [23, 24]. First we explicitly determine the case of $\rho(D^* \bar{D}^*)$ in which $I_{D^* \bar{D}^*} = 0$ and the total isospin of the three body system $I_{\rho(D^* \bar{D}^*)} = 1$. Using the nomenclature $\mid \rho, I, I_z \rangle \otimes \mid D\bar{D}, I, I_z \rangle$ we obtain

$$
\begin{aligned}
\langle\rho(D^*\bar{D}^*)|t|\rho(D^*\bar{D}^*)\rangle &= \langle\rho^+(D^*\bar{D}^*)^{I=0} \mid (\hat{t}_{\rho D^*}+\hat{t}_{\rho\bar{D}^*}) \mid \rho^+(D^*\bar{D}^*)^{I=0}\rangle \\
&= -\langle +1 \mid \otimes \frac{1}{\sqrt{2}}\Big(\langle\frac{1}{2},\frac{1}{2}\mid\langle\frac{1}{2},-\frac{1}{2}\mid - \langle\frac{1}{2},-\frac{1}{2}\mid\langle\frac{1}{2},\frac{1}{2}\mid\Big)(\hat{t}_{\rho D^*}+\hat{t}_{\rho\bar{D}^*}) \\
&\quad (-)\mid +1\rangle \otimes \frac{1}{\sqrt{2}}\Big(\mid\frac{1}{2},\frac{1}{2}\rangle\mid\frac{1}{2},-\frac{1}{2}\rangle - \mid\frac{1}{2},-\frac{1}{2}\rangle\mid\frac{1}{2},\frac{1}{2}\rangle\Big) \\
&= \Big(\frac{2}{3}t^{I=3/2}_{\rho D^*}+\frac{1}{3}t^{I=1/2}_{\rho D^*}\Big)+\Big(\frac{2}{3}t^{I=3/2}_{\rho\bar{D}^*}+\frac{1}{3}t^{I=1/2}_{\rho\bar{D}^*}\Big) \qquad (2)
\end{aligned}
$$

where

$$
\mid D^*\bar{D}^*\rangle^{I=0} = \frac{1}{\sqrt{2}}\mid\frac{1}{2},-\frac{1}{2}\rangle - \frac{1}{\sqrt{2}}\mid -\frac{1}{2},\frac{1}{2}\rangle \qquad (3)
$$

with the nomenclature $\mid I_z\rangle$ for the ρ meson and $\mid I_{z_1}, I_{z_2}\rangle$ for the $D^*\bar{D}^*$ system.

Second we write the case of $\bar{D}^*(\rho D^*)$ where $I_{\rho D^*}=1/2$ and the total isospin of the three body system $I_{\bar{D}^*(\rho D^*)}=0$ or $I_{\bar{D}^*(\rho D^*)}=1$.

For the total isospin $I=1$ case

$$
\begin{aligned}
\langle\bar{D}^*(\rho D^*)|t|\bar{D}^*(\rho D^*)\rangle &= \langle\bar{D}^*(\rho D^*)^{I=1/2} \mid (\hat{t}_{D^*\rho}+\hat{t}_{D^*\bar{D}^*}) \mid \bar{D}^*(\rho D^*)^{I=1/2}\rangle \\
&= \Big[\frac{1}{\sqrt{2}}\langle\frac{1}{2}\mid\otimes\Big(\frac{1}{\sqrt{3}}\langle 0,-\frac{1}{2}\mid - \sqrt{\frac{2}{3}}\langle -1,\frac{1}{2}\mid\Big) \\
&\quad +\frac{1}{\sqrt{2}}\langle -\frac{1}{2}\mid\otimes\Big(\sqrt{\frac{2}{3}}\langle 1,-\frac{1}{2}\mid - \frac{1}{\sqrt{3}}\langle 0,+\frac{1}{2}\mid\Big)\Big] \\
&\quad \Big(\hat{t}_{D^*\rho}+\hat{t}_{D^*\bar{D}^*}\Big) \\
&\quad \Big[\frac{1}{\sqrt{2}}\mid\frac{1}{2}\rangle\otimes\Big(\frac{1}{\sqrt{3}}\mid 0,-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\mid -1,\frac{1}{2}\rangle\Big) \\
&\quad +\frac{1}{\sqrt{2}}\mid -\frac{1}{2}\rangle\otimes\Big(\sqrt{\frac{2}{3}}\mid 1,-\frac{1}{2}\rangle - \frac{1}{\sqrt{3}}\langle 0,+\frac{1}{2}\rangle\Big)\Big] \\
&= \Big(\frac{8}{9}t^{I=3/2}_{D^*\rho}+\frac{1}{9}t^{I=1/2}_{D^*\rho}\Big)+\Big(\frac{2}{3}t^{I=1}_{D^*\bar{D}^*}+\frac{1}{3}t^{I=0}_{D^*\bar{D}^*}\Big) \qquad (4)
\end{aligned}
$$

with the nomenclature $\mid \bar{D}^*, I_z\rangle\otimes\mid \rho D^*, I_{z_1}, I_{z_2}\rangle$. In the case of the total isospin $I=0$ case we similarly derive

$$
\langle\bar{D}^*(\rho D^*)|t|\bar{D}^*(\rho D^*)\rangle = \Big(t^{I=1/2}_{D^*\rho}\Big)+\Big(t^{I=1}_{D^*\bar{D}^*}\Big). \qquad (5)
$$

Since we use the normalization of Mandl and Shaw [28], which has different weight factors for the particle fields, we need to consider how these factors are adapted to the present problem. It is easy to do this compairing the single scattering, double scattering and full scattering amplitudes. In this case, following the field normalization of Ref. [28], we can obtain the S matrix for the single scattering diagram (Fig. 1 (a) and (e)),

$$
\begin{aligned}
S_1^{(1)} &= -it_1\frac{1}{\mathcal{V}^2}(2\pi)^4\delta^4(k+k_R-k'-k_R') \\
&\quad \times \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega'_{a_3}}}\frac{1}{\sqrt{2\omega_{a_1}}}\frac{1}{\sqrt{2\omega'_{a_1}}}, \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
S_2^{(1)} &= -it_2\frac{1}{\mathcal{V}^2}(2\pi)^4\delta^4(k+k_R-k^{'}-k^{'}_R) \\
&\times \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega^{'}_{a_3}}}\frac{1}{\sqrt{2\omega_{a_2}}}\frac{1}{\sqrt{2\omega^{'}_{a_2}}},
\end{aligned} \tag{7}
$$

where the momentum $k(k')$, the on-shell energy $\omega(\omega^{'})$ refer to the initial (final) particles, respectively, and $\mathcal{V}$ is the volume of the box where the states are normalized to unity. In Eqs. (6) and (7), t_1, t_2 correspond to the first and second terms of the right hand side of Eqs. (4) and (5).

Likewise we have the S-matrix for the double scattering diagram as (Fig. 1 (b) or (f))

$$
\begin{aligned}
S^{(2)} &= -i(2\pi)^4\frac{1}{\mathcal{V}^2}\delta^4(k+k_R-k^{'}-k^{'}_R) \\
&\times \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega^{'}_{a_3}}}\frac{1}{\sqrt{2\omega_{a_1}}}\frac{1}{\sqrt{2\omega^{'}_{a_1}}}\frac{1}{\sqrt{2\omega_{a_2}}}\frac{1}{\sqrt{2\omega^{'}_{a_2}}} \\
&\times \int\frac{d^3q}{(2\pi)^3}F_R(q)\frac{1}{q^{0^2}-\vec{q}^{\,2}-m^2_{a_3}+i\epsilon}t_1t_2,
\end{aligned} \tag{8}
$$

where $F_R(q)$ is the form factor of the cluster which represents essentially the Fourier transform of its wave function. The derivation of the form factors can proceed similarly to Refs. [20, 29], where one can also read further discussions and interpretation. The form factor for s-wave functions is given by

$$
\begin{aligned}
F_R(q) &= \frac{1}{\mathcal{N}}\int_{\substack{p<k_{max}\\ |\vec{p}-\vec{q}|<k_{max}}} d^3p\;\frac{1}{2\omega_{a_1}(\vec{p}\,)}\;\frac{1}{2\omega_{a_2}(\vec{p}\,)}\;\frac{1}{M_R-\omega_{a_1}(\vec{p}\,)-\omega_{a_2}(\vec{p}\,)} \\
&\times \left(\frac{1}{2\omega_{a_1}(\vec{p}-\vec{q}\,)}\right)\left(\frac{1}{2\omega_{a_2}(\vec{p}-\vec{q}\,)}\right)\frac{1}{M_R-\omega_{a_1}(\vec{p}-\vec{q}\,)-\omega_{a_2}(\vec{p}-\vec{q}\,)},
\end{aligned} \tag{9}
$$

with the normalization $\mathcal{N}$

$$
\mathcal{N} = \int_{p<k_{max}} d^3p\left[\frac{1}{2\omega_{a_1}(\vec{p}\,)}\;\frac{1}{2\omega_{a_2}(\vec{p}\,)}\;\frac{1}{M_R-\omega_{a_1}(\vec{p}\,)-\omega_{a_2}(\vec{p}\,)}\right]^2 \tag{10}
$$

where ω_{a_1} and ω_{a_2} are the energies of the particles a_1, a_2, and k_{max} is a cutoff that regularizes the integral of Eqs. (9) and (10). This cutoff is the same one needed in the regularization of the loop function of the two particle propagators in the study of the interaction of the two particles of the cluster [29]. In this work we take the cutoff $k_{max} = 1200$ MeV, the same one used to generate the $D_2^*(2460)$ [24].

Similarly, the full three body S-matrix for scattering of particle a_3 with the cluster is given by

$$
\begin{aligned}
S &= -iT\frac{1}{\mathcal{V}^2}(2\pi)^4\delta^4(k+k_R-k^{'}-k^{'}_R) \\
&\quad \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega^{'}_{a_3}}}\frac{1}{\sqrt{2\omega_{R}}}\frac{1}{\sqrt{2\omega^{'}_{R}}}.
\end{aligned} \tag{11}
$$

Compairing this equation with Eqs. (6) and (7), we introduce convenient factors in the elementary amplitudes:

$$\tilde{t}_{1(2)} = \frac{2\ M_R}{2\ m_{a_{1(2)}}} t_{1(2)}. \quad (12)$$

with m_{a_1}, m_{a_2} and M_R the masses of the particle a_1, a_2 and the cluster respectively, where we have taken the approximations, suitable for bound states, $\frac{1}{\sqrt{2\omega_{a_{1(2)}}}} = \frac{1}{\sqrt{2m_{a_{1(2)}}}}$.

Finally solving the set of equations for the FCA to the Faddeev equations, Eqs. (1), we obtain

$$T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2}. \quad (13)$$

Note that the argument of the total amplitude T is regarded as a function of the total invariant mass of the three body system whereas the argument of $t_{1(2)}$ is the invariant masses of the two body systems. In order to obtain the arguments $s_{1(2)}$ of the two body amplitude we share the binding energy among the three particles, proportionally to their masses. Therefore the energy of the particles a_1, a_2 and a_3 become

$$E_{a_3} = m_{a_3} \frac{\sqrt{s}}{(M_R + m_{a_3})} \quad (14)$$

$$E_{a_1} = \frac{\sqrt{s}}{(M_R + m_{a_3})} \frac{m_{a_1}\ M_R}{(m_{a_1} + m_{a_2})} \quad (15)$$

$$E_{a_2} = \frac{\sqrt{s}}{(M_R + m_{a_3})} \frac{m_{a_2}\ M_R}{(m_{a_1} + m_{a_2})} \quad (16)$$

Hence the total energy of the two body system is evaluated as follows

$$s_{1(2)} = (p_{a_3} + p_{a_1(a_2)})^2 = \Big(\frac{\sqrt{s}}{M_R + m_{a_3}}\Big)^2 (m_{a_3} + \frac{m_{a_1(a_2)}\ M_R}{m_{a_1} + m_{a_2}})^2 - \vec{P}^2_{a_2(a_1)} \quad (17)$$

where the approximate value of $\vec{P}_{a_2(a_1)}$ is given by

$$\frac{\vec{P}^2_{a_2(a_1)}}{2\ m_{a_2(a_1)}} \simeq B_{a_2(a_1)} \equiv \frac{m_{a_2(a_1)}\ M_R}{(m_{a_1} + m_{a_2})} \frac{(M_R + m_{a_3} - \sqrt{s})}{(M_R + m_{a_3})} \quad (18)$$

with $B_{a_2(a_1)}$ the binding energy of the particle $a_2(a_1)$.

As we stated before, the G_0 function is the propagator of the particle a_3 inside the cluster as follows

$$G_0 = \frac{1}{2M_R} \int \frac{d^3q}{(2\pi)^3} F_R(q) \frac{1}{q^{0\,2} - \vec{q}^{\,2} - m^2_{a_3} + i\epsilon}. \quad (19)$$

where M_R is the mass of the cluster, and m_{a_3} the mass of the particle a_3. Here the energy of the propagator q^0 is determined at the three body rest frame as

$$q^0 = \frac{s + m^2_{a_3} - M_R^2}{2\sqrt{s}} \quad (20)$$

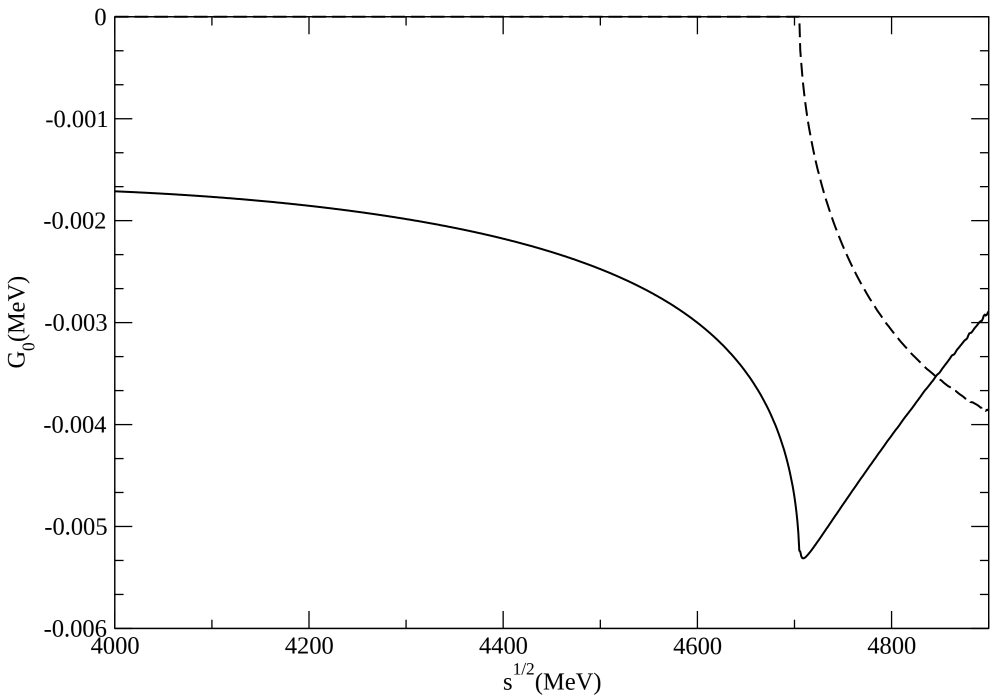

FIG. 2: Real (solid line) and imaginary (dashed line) parts of G_0 function in $\rho(D^*\bar{D}^*)$

with $\sqrt{s}$ the rest energy of the three body system. As an example we depict in Fig. 2 the real and imaginary parts of the G_0 function for the $\rho(D^*\bar{D}^*)$ system. The G_0 function has a similar shape for the $\bar{D}^*(\rho D^*)$ system but with a different threshold.

As we mentioned previously, in the evaluation of the $\rho D^*\bar{D}^*$ three body interaction, the use of the ρD^*, $\rho\bar{D}^*$ and $D^*\bar{D}^*$ unitarized amplitudes has crucial importance. These amplitudes were studied by the coupled channel Bethe-Salpeter equations in Refs. [23, 24] and we use them here. In order to reproduce the $\rho D^*(\rho\bar{D}^*)$ system from the work of [24], the coupled channels used are ρD^* and ωD^* for $I = \frac{1}{2}$ and ρD^* for $I = \frac{3}{2}$ case. In the case of the $D^*\bar{D}^*$ system [19, 23] there are 10 coupled channels, $D^*\bar{D}^*$, $K^*\bar{K}^*$, $\rho\rho$, $\omega\omega$, $\phi\phi$, $J/\Psi J/\Psi$, $\omega J/\Psi$, $\phi J/\Psi$, $\omega\phi$ and $D_s^*\bar{D}_s^*$ for $I = 0$ and six coupled channels $D^*\bar{D}^*$, $K^*\bar{K}^*$, $\rho\rho$, $\rho\omega$, $\rho J/\Psi$ and $\rho\phi$ for $I = 0$.

Following the ideas of the coupled channels chiral unitary approach, the VV-two body scattering amplitude can be obtained using the Bethe-Salpeter equations in its on-shell factorized form as below

$$t \;=\; (\hat{1} - V\hat{G})^{-1}V \tag{21}$$

where the V is a matrix of the interaction potentials between the channels, which is calculated from the hidden gauge Lagrangian [25–27]. The potential V is a 10×10 matrix in $I = 0$ and 6×6 matrix in $I = 1$ with the amplitudes obtained from the coupled channels for $D^*\bar{D}^*$ case in $J = 2$ [19, 23]. In addition, in the case of the ρD^* the potential V is a 2×2 matrix given by [24].

In Eq. (21) $\widehat{G}$ is a diagonal matrix of the loop function of two mesons in the i channel

$$\widehat{G}_i(P) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_1 + i\epsilon}\frac{1}{(P-q)^2 - m_2 + i\epsilon} \tag{22}$$

where P is the four dimensional momentum of the two vector mesons determined at the rest frame, $P = (\sqrt{s}, 0)$, and m_1 and m_2 are the masses of the vector mesons in the i channel. In order to remove the ultraviolet divergence of the loop function, we use the dimensional regularization scheme and we get

$$\widehat{G}_i(\sqrt{s}) = \frac{1}{16\pi^2}\Big(\alpha_i + Log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} Log\frac{m_2^2}{m_1^2} + \frac{q_i}{\sqrt{s}}\Big(Log\frac{s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}{-s + m_2^2 - m_1^2 + 2q_i\sqrt{s}} + Log\frac{s + m_2^2 - m_1^2 + 2q_i\sqrt{s}}{-s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}\Big)\Big) \quad (23)$$

where q_i is the three momentum of the two vector mesons determined at the center of mass frame evaluated as follows

$$q_i = \frac{\sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}{2\sqrt{s}}. \quad (24)$$

In Eq. (23), μ is a regularization scale and α_i is the subtraction constant. We take $\mu = 1500$ MeV and $\alpha = -1.74$ for the ρD^* to reproduce $D_2^*(2460)$ resonance [24] and $\mu = 1000$ MeV and $\alpha = -2.07$ in the channel including heavy mesons, $\alpha = -1.65$ in the channel including light mesons for $D^*\bar{D}^*$ to obtain the X(3915) state [23]. In Eq. (9), the cutoff regularizes the integral of the form factor. In the present work, we choose the cutoff $k_{max} = 1200$ MeV both for the $D_2^*(2460)$ and $X(3915)$, which produces similar results as the using the chosen subtraction constants in the dimensional regularization.

III. RESULTS

In this section we present the results obtained for the scattering amplitude of the $\rho D^*\bar{D}^*$ system in spin-3. The two-body ρD^*, $\rho\bar{D}^*$ and $D^*\bar{D}^*$ systems were investigated by the coupled channel Bethe-Salpeter equations in Refs. [23, 24]. As we stated before, the resonance $D_2^*(2460)$ was generated as a ρD^* quasibound state or molecule in the isospin 1/2 and spin-2. It was also found that the resonance $X(3915)$ is dynamically generated in $I = 0$ and spin-2 from $D^*\bar{D}^*$ scattering. Therefore, there are two possible cases of three-body scattering of the $\rho D^*\bar{D}^*$ system. One is the $D_2^*(2460) - \bar{D}^*$ and the other one is the $X(3915) - \rho$.

In Fig. 3 we illustrate the modulus squared $|T|^2$ for the $X\rho \to X\rho$ scattering as a function of the total energy of the $\rho D^*\bar{D}^*$ system for the case of $I = 1$ and $J = 3$. The results show a clear peak at $\sqrt{s} = 4338$ MeV about 360 MeV below the threshold of the $X(3915) - \rho$ system. The width of the peak is about 50 MeV.

This looks like a strong binding, but we must keep in mind that the vector-vector interaction in $J = 2$ is indeed very strong [18, 19, 23, 24]. This is why we are studying these superbound states with spins aligned where the spin of any pair is always $J = 2$. The obvious thing is that the $D^*\bar{D}^*$ state is already bound and since the ρD^* also binds to give the $D_2^*(2460)$, the system $\rho D^*\bar{D}^*$ with this configuration will necessarily be bound. This would be the case even if the ρ interacted only with one D^*. In this case we would have a binding of $m_{D^*} + m_\rho - m_{D_2^*} = 320$ MeV. The binding that we get is 360 MeV, which means that we have gained extra 40 MeV binding by the interaction of the ρ with the $\bar{D}^*$. This indicates that there is extra binding from the three body molecular structure. This feature is reminiscent of what happens in Quantum mechanics for the problem of a particle in a well of two attractive δ-functions [30]. For the symmetric solutions, if the two δ's are separated, the binding energy of the two δ potential is the binding of a particle in one δ well. As

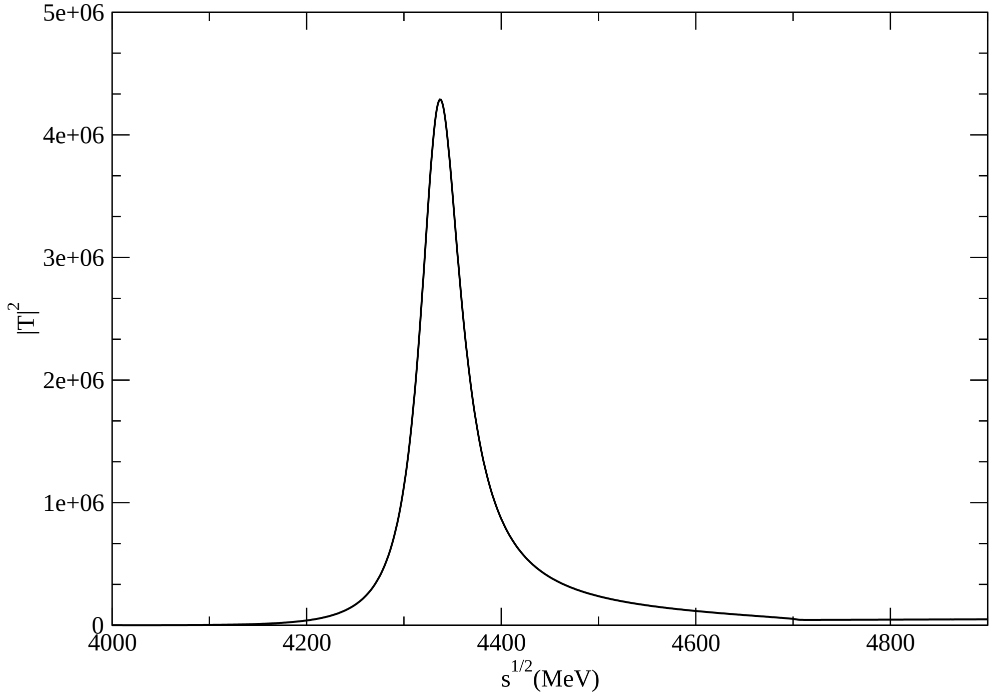

FIG. 3: Modulus squared of the $\rho(D^*\bar{D}^*)$ scattering amplitude with total isospin $I = 1$.

the two δ potentials get closer, the particle starts orbiting the two potential wells and the binding energy grows. This is what we observe here, indicating an extra binding from the orbiting of the ρ around the D^* and $\bar{D}^*$.

In Figs. 4 and 5 we show the results of $|T|^2$ for the case of $D_2^*\bar{D}^* \to D_2^*\bar{D}^*$ in total isospin $I = 0$ and $I = 1$, respectively. We find a peak around 4000 MeV which is about 470 MeV below the $D_2^*(2460)$ and $\bar{D}^*$ threshold for the isospin $I = 0$ case. The width of this state is quite large about 250 MeV. If we conduct the same exercise as before, the D^* with a ρ would be bound by 320 MeV, and the D^* with the $\bar{D}^*$ by about 63 MeV. We would think that the D^* is orbiting the ρ where it is more bound and get some extra binding from orbiting the D^*. It is not clear why one passes to 470 MeV binding. It is also unclear why the width is much larger. Probably we have to accept that in this case, since the D^* is heavier than the ρ, there are limitations to the applications of the FCA, and we should accept this result as an indication that we could now have a state more bound than in the former case, which fulfills all the conditions for a reliable application of the FCA and hence is more reliable, but we cannot be certain about the mass and the width.

For the isospin $I = 1$ case, we see a clear peak around 4195 MeV, and the width is around 60 MeV. The position of the peak is about 270 MeV below the $D_2^*(2460)$ and $\bar{D}^*$ threshold. These results are more intuitive than before. We can apply the same argumentation as before, but now according to Eqs. (4) and (5), we can see that the weight of the $t_{\rho D^*}^{I=1/2}$ amplitude in $\langle\bar{D}^*(\rho D^*)|t|\bar{D}^*(\rho D^*)\rangle$ for $I = 0$ is unity while for $I = 1$ it is 1/9, and this is the amplitude that contains the attractions that binds the $D_2^*(2460)$. With the caveat about the arguments used before for the case of $I = 0$, it looks clear that the binding should be smaller than for the case of $I = 0$ and the width is also similar to that of the $\rho(D^*\bar{D}^*)$ molecule.

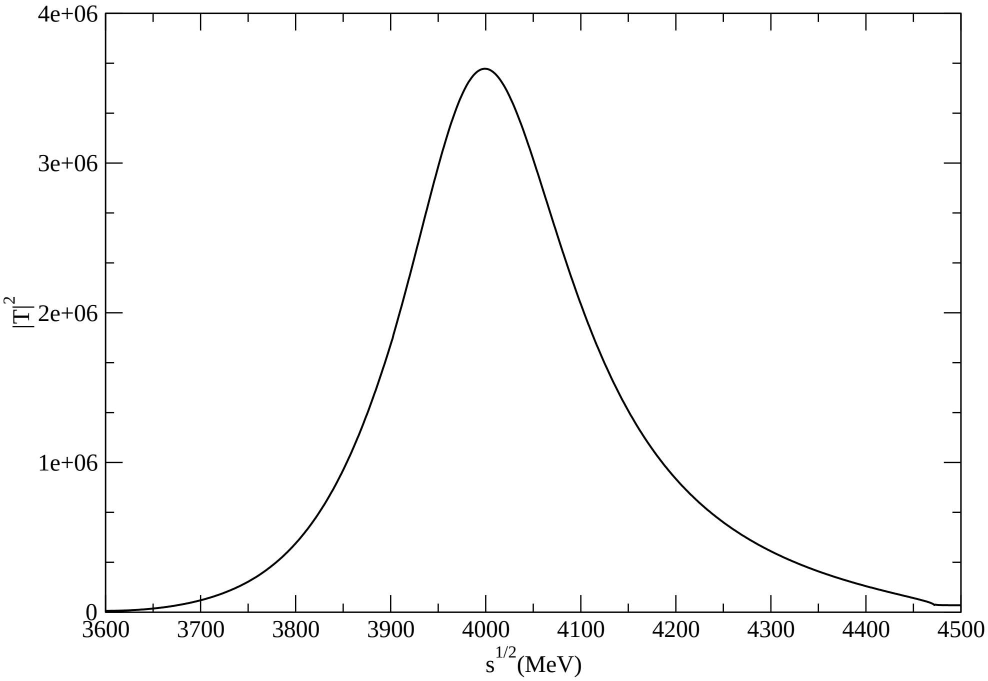

FIG. 4: Modulus squared of the $\bar{D}^*(\rho D^*)$ scattering amplitude with total isospin $I = 0$.

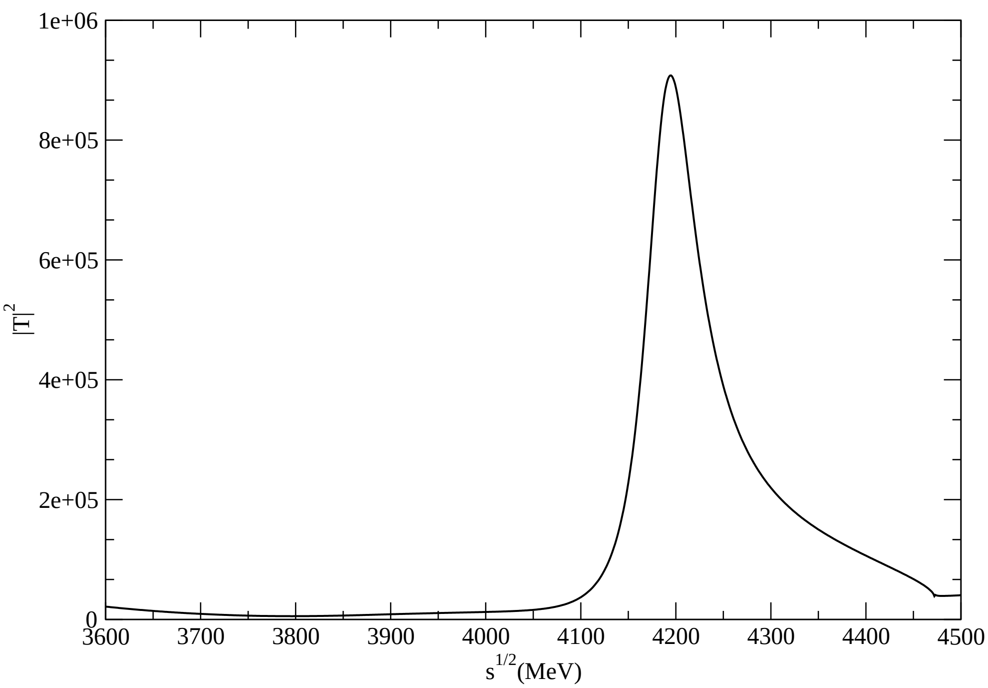

FIG. 5: Modulus squared of the $\bar{D}^*(\rho D^*)$ scattering amplitude with total isospin $I = 1$.

IV. CONCLUSIONS

We studied the interaction of the ρ and D^*, $\bar{D}^*$ with spins aligned using the Fixed Center Approximation to the Faddeev equations. This guarantees that we have $J = 2$ for any of the pairs, where the vector-vector interaction is strongest, and leads to three body states with $J = 3$. We first select a cluster of $D^*\bar{D}^*$, which is found to be bound in $I = 0$ and can be associated to the X(3915), and then let the ρ meson orbit around the D^* and $\bar{D}^*$. In this case the FCA produces an amplitude for ρ-$D^*\bar{D}^*$ scattering which has a clear and narrow peak around 4340 MeV. The case of a $\bar{D}^*$ orbiting around a cluster of ρD^* is more uncertain because the mass of the external particle is heavier than the one of the ρ in the cluster, and the FCA is less reliable. In this case the cluster makes the $D_2^*(2460)$ state, and we point at some qualitative results, with an $I = 0$ state around 4000 MeV and an $I = 1$ state around 4200 MeV. The results obtained for the $I = 1$ state with the ρ orbiting around the X(3915) should be realistic since the ρ is lighter than the constituents of the cluster. In the other case our results should be taken as indicative, but strong arguments are given that these states should be strongly bound.

The results obtained here should serve to stimulate calculations with more accurate three body tools, as those of [5–7, 16, 17] which could make predictions on these interesting $J = 3$ states. One should recall at this point that states with increasing spin number already exist in the light sector [20] and in the strange sector [21]. What we have done here it to extend this to the hidden charm sector. Parallelly, it would also be interesting to investigate states of large spin in the region of mass investigated here. The results obtained in this work provide sufficient support for a devoted search of such states.

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