

# Transplanckian inflation as gravity echoes

G. Barenboim and O. Vives

*Departament de Física Teòrica and IFIC,*

*Universitat de València-CSIC, E-46100, Burjassot, Spain.*

In this work, we show that, in the presence of non-minimal coupling to gravity, it is possible to generate sizeable tensor modes in single-field models without transplanckian field values. These transplanckian field values apparently needed in Einstein gravity to accommodate the experimental results may only be due to our insistence of imposing a minimal coupling of the inflaton field to gravity in a model with non-minimal couplings. We present three simple single-field models that prove that it is possible accommodate a large tensor-to-scalar ratio without requiring transplanckian field values within the slow-roll regime.

## I. INTRODUCTION

After the recent discovery of tensor modes at BICEP2 experiment [1]<sup>1</sup>, the theory of cosmological inflation [2] can claim to be the current (undisputed) paradigm of early universe cosmology. Inflation cannot only solve most of the problems of the Standard Big Bang Model, but it offers the only available explanation for the origin of the large-scale structure of the universe based on causal physics. Even more, cosmological inflation is a predictive theory. It calls for an almost scale invariant spectrum of curvature perturbations which anticipates the characteristic oscillations in the angular power spectrum of cosmic microwave anisotropy maps, observed with high accuracy by WMAP [3] and Planck [4].

Inflation is simply the assumption that there was a short epoch in the very early universe where the scale factor (space) grew at an accelerated pace, typically in an exponential way. Such an accelerated expansion flattens out and widens up a microscopic size of space, solving the longstanding “size problem” of standard cosmology. Not only that, the accelerated expansion decreases the contribution of any pre-existing curvature to the total energy budget of the universe and therefore turns the spatially flat universe into a local attractor in initial-condition space, solving this way yet another cosmological puzzle, the “flatness problem”. Unfortunately, inflation comes at a cost. Successful models of inflation, *i.e.* successful inflationary potentials require unusual features: the potentials have to be extremely flat so that enough inflation is produced to actually solve the above-

---

<sup>1</sup> Throughout this work, we will assume that although the exact numbers of BICEP2 may change, sizeable tensor modes, *i.e.*  $r \gtrsim .1$  are an actual feature that will stay

mentioned issues, and observations seem to require the inflaton field to travel over transplanckian distances in field space. In fact, following an argument due to Lyth [5]<sup>2</sup> we have,

$$\frac{\Delta\theta}{M_{\text{Pl}}} \gtrsim 5.8 \left(\frac{N_e}{50}\right) \left(\frac{r}{0.2}\right)^{1/2} \quad (1)$$

with  $\Delta\theta$  the variation of the field during inflation,  $r \simeq 13.8 \epsilon$  the tensor-to-scalar ratio with  $\epsilon$  the usual slow-roll parameter,  $N_e$  the number of e-folds of inflation since the relevant scales left the horizon till the end of inflation and  $M_{\text{Pl}} = (16\pi G)^{-1/2}$ . Therefore, the value of  $r = 0.2^{+0.07}_{-0.05}$  measured at BICEP2 implies transplanckian values for the inflaton field,  $\Delta\theta/M_{\text{Pl}} \gtrsim 5.8$ . Fortunately, large (transplanckian) field values do not necessarily involve large (transplanckian) energies, which is the reason why transplanckian field values are not total anathema. In fact, transplanckian field values have been the norm rather than the exception in the inflationary game [7–10]. There is (almost) no single-field inflationary model which can be kept below Planck scale all the way. Yet another problem which has not been devoted enough attention to is the fact that the energy scale of inflation and the Planck scale are not that far from each other and therefore it is easy to imagine that corrections to Planck scale physics are bounded to play a role. Whether this role is significant or not is clearly a debatable issue. Going back to the transplanckian field values, one of the reasons why it is safe to entertain transplanckian field values (once checked that the observables are well behaved) is that a field is, after all, a “dummy” variable, *i.e.* it is “per-se” meaningless. Just a field redefinition will turn its value into the desired domain at no expense, all the observables will remain invariant. Nevertheless, field redefinitions may be gratis observable-wise, but they are not innocent. They will surface somewhere else: in a change of the kinetic terms, the couplings in the potential, etc. In the same way that the mass matrix in the quark sector can be made real, but then the removed (physical) phase will show up in the charged and neutral current interactions, a field redefinition to turn the inflationary field subplanckian may end up shedding light on the shape of gravity close to the Planck scale.

In this work, we conjecture about the possibility that the trasplanckian field values arising in single-field inflationary models may be due to the fact that we are “forcing” our model to have Einstein gravity. We will show that well-behaved and subplanckian modified gravity, as non-minimally coupled scalar fields and/or scalar tensor theories, can become transplanckian once forced to behave as minimally coupled scalar field theories in Einstein gravity. Therefore, the observed tensor-to-scalar ratio can be obtained in single-field inflation models, in the presence of

---

<sup>2</sup> This bound has been generalized in the context of effective field theories of inflation in Ref. [6]. However, for the sake of this work the original bound still holds.

non-minimal couplings to gravity, working always in the subplanckian regime and in the slow-roll approximation.

This work is organised as follows. We begin with a basic review of models with non-minimal coupling to gravity and recall the use of conformal transformations to go from the Jordan frame (with non-minimal coupling to gravity) to the Einstein frame in section II. In section III, we present several realistic examples showing the effect of conformal transformations in the field values, making subplanckian field values in the Jordan frame transplanckian in the Einstein frame. Finally, results and conclusions are summarised in section IV.

## II. INFLATION IN THEORIES WITH NON-MINIMAL COUPLINGS TO GRAVITY.

We start from a general theory with gravity coupled to a single scalar field that will play the role of the inflaton. The action in the Jordan frame, with non-minimal coupling to gravity and assuming canonical kinetic terms<sup>3</sup>, would be,

$$S = - \int d^4x \sqrt{-g} \left[ \frac{k^2}{4} D(\theta) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta + V(\theta) \right] \quad (2)$$

where  $R$  is the scalar curvature and  $\theta$  our scalar field<sup>4</sup>. In the absence of any other sources of matter, and specialising for the case of a Friedmann-Robertson-Walker metric,  $g_{\mu\nu} = \text{diag}\{1, -a(t)^2, -a(t)^2, -a(t)^2\}$  the equations for the Hubble rate and the  $\theta$  field become,

$$\begin{aligned} D(\theta) H^2 &= \frac{\dot{\theta}^2}{3k^2} + \frac{2V(\theta)}{3k^2} - \dot{D}(\theta) H \\ \ddot{\theta} + 3H\dot{\theta} + \frac{k^2}{4} D'(\theta) R + V'(\theta) &= 0 \quad , \end{aligned} \quad (3)$$

with  $k^2 = M_{\text{Pl}}^2/(4\pi)$ , from where it is straightforward to obtain,

$$\dot{H} = -\frac{\dot{\theta}^2}{k^2 D} + \frac{\dot{D}(\theta)}{D(\theta)} \frac{H}{2} - \frac{\ddot{D}(\theta)}{2D(\theta)}. \quad (4)$$

Due to the addition of the extra source for perturbations we have introduced,  $D(\theta)$ , we need to include two more slow-roll parameters as compared to the standard case<sup>5</sup>. The scalar-type

---

<sup>3</sup> The introduction of a non-canonical kinetic term will complicate unnecessarily the theory and is not needed to make our point clear

<sup>4</sup> This action becomes non-renormalizable, once the field is above the cut-off scale in the Einstein frame, a fact that may *per-se* be signaling the need to introduce a non-minimal coupling to gravity as the true driver of inflation as higher corrections are always kept under control in this frame

<sup>5</sup> Throughout this work we assume the slow-roll regime. In principle the field could fast roll and in this case, quantum corrections can become sizeable and the slow-roll solution might stop being an attractor

perturbations will be affected by both of them, although only one ( $\epsilon_3$ ) will be relevant for the tensor perturbations [11, 12],

$$\epsilon_1 = \frac{\dot{H}}{H^2} = \frac{H'\dot{\theta}}{H^2} \quad (5)$$

$$\epsilon_2 = \frac{\ddot{\theta}}{H\dot{\theta}} \quad (6)$$

$$\epsilon_3 = \frac{1}{2} \frac{\dot{D}}{HD} = \frac{1}{2} \frac{D'\dot{\theta}}{HD} \quad (7)$$

$$\epsilon_4 = \frac{1}{2} \frac{\dot{E}}{HE} = \frac{1}{2} \frac{E'\dot{\theta}}{HE}, \quad (8)$$

with  $E = 3k^2(D')^2/2 + D$ . Assuming  $\dot{\epsilon}_i = 0$  and to linear order in the slow-roll parameters

$$n_s = 1 + 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \quad (9)$$

$$n_T = 2(\epsilon_1 - \epsilon_3) \quad (10)$$

$$r = 13.8 |\epsilon_1 - \epsilon_3| \quad (11)$$

As it is well-known, this model, as any non-standard theory of gravity, can be mapped into a standard theory of gravity at the expense of having a more complicated matter sector by a conformal transformation. Such a transformation is not just a coordinate redefinition (being general relativity a covariant theory, a coordinate redefinition would become trivial) rather, it is a transformation that mixes up the matter and gravitational degrees of freedom<sup>6</sup>.

The mapping we are alluding to, takes the original metric  $g_{\mu\nu}$  into a new metric  $\tilde{g}_{\mu\nu}$  according to,

$$\tilde{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}, \quad (12)$$

with  $e^{2\omega} = D(\theta)$ .

The Hubble rate transforms as,

$$\tilde{H} = \frac{H + \dot{D}(\theta)/(2D(\theta))}{\sqrt{D(\theta)}}, \quad (13)$$

with  $\dot{D} = \partial D/\partial t$ , and the canonically normalised field replacing  $\theta$  in the Einstein frame is,

$$\phi(\theta) = \pm \int \sqrt{\frac{3}{2} \left( \frac{D'(\theta)}{D(\theta)} \right)^2 + \frac{2}{k^2 D(\theta)}} d\theta. \quad (14)$$

---

<sup>6</sup> Every single choice of field (and metric) definition among the family of transformations goes under the name of **frame** and obviously the frame where gravity takes the form of Einstein's theory is called Einstein's frame.

In terms of this rescaled field the action takes the form,

$$S = - \int d^4x \sqrt{-\tilde{g}} \left[ \frac{k^2}{4} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \tilde{V}(\phi) \right], \quad (15)$$

with  $\tilde{V}(\phi) = V(\phi(\theta))/D(\theta)^2$ .

It is trivial to show that the slow-roll parameters in both frames are related as,

$$\tilde{\epsilon} = \frac{k^2}{4} \left( \frac{\tilde{H}'}{\tilde{H}} \right)^2 = \epsilon_1 - \epsilon_3 \quad (16)$$

$$\tilde{\eta} = \frac{k^2}{4} \frac{\tilde{H}''}{\tilde{H}} = \epsilon_2 - 3\epsilon_3 + \epsilon_4 \quad (17)$$

$$\tilde{n}_s = 1 + 2(2\tilde{\epsilon} - \tilde{\eta}) = 1 + 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) = n_s \quad (18)$$

$$\tilde{n}_T = 2\tilde{\epsilon} = 2(\epsilon_1 - \epsilon_3) = n_T \quad (19)$$

$$\tilde{r} = 13.8 |\tilde{\epsilon}| = 13.8 |\epsilon_1 - \epsilon_3| = r \quad (20)$$

Leaving all the observables invariant, as it is obvious given the fact that changing from one frame to another one does not correspond to a change in the physics. However as the conformal transformation changes the space-time curvature (and also the scalar/matter field) phenomena that appear to be due to gravity in one frame may appear to be originated in the scalar sector in another. Besides, it is easy to see that as a result of the fact that the inflaton field in Einstein and Jordan frames are related in a highly non-trivial way, it can be expected that subplanckian values in a given frame, may correspond to transplanckian values in the second frame.

The analysis in this paper is done in the framework of an effective field theory neglecting terms suppressed by the cutoff scale, which in the Einstein frame is  $M_{Pl}$ . However, as pointed out recently by Hertzberg in Ref. [13], in the presence of non-minimal couplings to gravity the validity regime of the effective theory may change. As shown in this work, in single field models, the relevant cutoff scale, in the gravitational and kinetic sector, is still the Planck mass. Regarding the potential interactions the situation is model dependent and we will check it in a case by case basis.

### III. SINGLE-FIELD NON-MINIMAL MODELS OF INFLATION

As shown in the previous section, the field values in two different frames are correlated by an non-trivial function in a rather complicated way. Here, we will show that is possible and, in fact, quite natural and easy to find realistic examples of theories with non-minimal coupling to gravity, modified gravity or scalar tensor theories, that have transplanckian field values if we insist

on imposing a minimal coupling to gravity, but are always subplanckian in their “natural” frame. This clearly does not imply that any conformal transformation will turn transplanckian minimally coupled scalar fields into the subplanckian regime once non-minimally coupled to gravity or once allowed to live in a modified-gravity framework but, in our scheme, observations would select a subclass of conformal transformations.

### A. Monomial Potentials

As a first toy-model, we assume that the potential in the Einstein frame is exactly given by the well-known potential  $V(\phi) = \lambda\phi^4$  in the Einstein frame. In the slow-roll regime  $\dot{\phi}^2 \ll V(\phi)$ , using the Eqs. of motion, we have,

$$H = \sqrt{\frac{2\lambda}{3}} \frac{\phi^2}{k} \quad (21)$$

$$\dot{\phi} = -2\sqrt{\frac{2\lambda k^2}{3}} \phi \quad (22)$$

and therefore, we have,

$$\epsilon = \frac{\dot{H}}{H^2} = \frac{H'\dot{\phi}}{H^2} = -\frac{4k^2}{\phi^2} \quad (23)$$

$$\eta = \frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{2k^2}{\phi^2} \quad (24)$$

$$n_s = 1 + 4\epsilon - 2\eta = 1 - \frac{12k^2}{\phi^2} \quad (25)$$

Now, the number of e-foldings fixes the value of the field at which the scales of interest at present left the horizon,

$$N = \int H dt = -\left. \frac{\phi^2}{4k^2} \right|_{\phi_i}^{\phi_f} \simeq \frac{\phi_i^2}{4k^2} \quad (26)$$

Using  $N \simeq 62$  we need  $\phi_i \simeq 11 \times k \simeq 3.1 \times M_{\text{Pl}}$ , and we obtain  $n_s = 1 - 3/N \simeq 0.95$  and  $r \simeq 13 \epsilon = 13/N \simeq 0.21$ . Therefore, we see that this potential would be able to reproduce approximately the observed values for the spectral index and the tensor-to-scalar ratio, but only if the field values during inflation are well above the Planck mass.

However, if our field makes excursions well-beyond the Planck scale, or gets very close to it, we can expect gravitational corrections to come into play and to be very relevant. For example, higher-order curvature invariants could appear, and it is then natural to consider also non-minimal couplings of the inflaton to gravity.

Now, let's assume that our inflaton field,  $\theta$ , has a non-minimal coupling to gravity <sup>7</sup> of the form  $D(\theta) = (1 - \theta^2/(3k^2))$ . The Einstein equations and the  $\theta$  equations of motion in this frame, the Jordan frame, are,

$$\begin{aligned} D(\theta) \tilde{H}^2 &= \frac{\dot{\theta}^2}{3k^2} + \frac{2\tilde{V}(\theta)}{3k^2} - \dot{D}(\theta) \tilde{H} \\ \ddot{\theta} + 3\tilde{H}\dot{\theta} + \frac{k^2}{4} D'(\theta) R + \tilde{V}'(\theta) &= 0 \quad , \end{aligned} \quad (27)$$

with  $\tilde{H}$  the Hubble rate in the Jordan frame related to the Hubble rate in the Einstein frame by Eq. (13). Then, the potential in the Jordan frame is,

$$\tilde{V}(\theta) = \frac{V(\phi(\theta))}{(D(\theta))^2} = \lambda \frac{(\phi(\theta))^4}{(1 - \theta^2/(3k^2))^2} \quad (28)$$

Using this potential we can obtain, following [13], the Jordan-frame cutoff scale which signals the validity regime of the effective theory after taking into account the quantum corrections incorporated a la Coleman-Weimberg. In this case the cutoff is even larger than the Einstein frame one, and it is given by  $\Lambda = 6M_{Pl}$ .

The fields in the Jordan and Einstein frames, are related by Eq. (14), that in this case can be integrated analytically,

$$\phi(\theta) = 2\sqrt{6\pi} k \operatorname{arctanh} \left[ \frac{\theta}{\sqrt{3} k} \right] , \quad (29)$$

or,

$$\theta(\phi) = \sqrt{3} k \tanh \left[ \frac{\phi}{2\sqrt{6\pi} k} \right] . \quad (30)$$

Therefore, we can see clearly that in the Jordan frame, the field  $\theta$  is always subplanckian,  $\theta \leq \sqrt{3/(4\pi)} M_{Pl} \simeq 0.489 M_{Pl}$ , and when  $\phi \simeq 3.1 M_{Pl}$  the Jordan field  $\theta \simeq 0.42 M_{Pl}$ <sup>8</sup>. It is a trivial exercise to show that using Eqs (5–11) and (16–20), and despite the fact the slow-roll parameters are different in both actions, two non-vanishing in one case corresponding to the usual slow-roll parameters, and four in the Jordan frame, all the observables are identical. Moreover, even in the Jordan frame, the potential is approximately quartic in  $\theta$  at low field values,  $\theta/(\sqrt{3}k) \ll 1$ , as can be seen from Eqs. (28) and (29). Therefore, already by the end of inflation, both theories are nearly indistinguishable<sup>9</sup>.

<sup>7</sup> In this case, although the potential is exactly quartic in the Einstein frame, the effects of this non-minimal coupling would still appear in this frame as higher order terms in the action that we have not considered

<sup>8</sup> Here, we consider only the leading term in the potential. It is clear that higher-order operators could play a role. However, it is always possible to deal with them through a symmetry. For example, in this case, we could impose a  $Z_4$  symmetry.

<sup>9</sup> An interesting possibility would be to have the same non-minimal coupling to gravity to play a role in the current accelerated expansion.

We can repeat the same exercise starting from a quadratic potential  $V(\phi) = \mu^2\phi^2$ . In this case, we obtain  $\epsilon = 1/(2N_e)$ ,  $n_s = 1 - 2/N_e$  and  $r = 6.9/N_e$ , with  $N_e = \phi_i^2/(2k^2)$  the number of e-folds needed for inflation to solve the cosmological problems, which require  $\phi_i \simeq 2.2 \times M_{\text{Pl}}$ . So, again we need transplanckian values for the field in the Einstein frame (less transplanckian than in the previous case due to the flatter potential), but clearly, assuming the same non-minimal coupling to gravity, the relation between the Jordan and Einstein-frame fields is the same as in Eq. (29), and therefore, as in the  $V = \lambda\phi^4$  case, transplanckian field values become subplanckian,  $\theta_i \simeq 0.35 \times M_{\text{Pl}}$ , once allowed to couple to the curvature.

## B. Generic scalar-tensor theories

In the previous model, we have specified the potential in the Einstein frame and the transformation to the Jordan frame, and we have seen that the transplanckian values of the field may be simply due to our attempt to write in Einstein form a theory that has a non-minimal coupling to gravity.

Here we will use a different strategy, we will start from an ansatz that guarantees inflation in the Jordan frame and obtain the potential and the non-minimal coupling to gravity from there.

We start from the requirement that space inflates exponentially with the inflaton field being responsible for it,  $a = \exp(-\theta/b)$  and therefore  $H = \dot{a}/a = -\dot{\theta}/b$  [14].

This ansatz establishes also the number of e-foldings in this scenario, which is given by

$$N_e = \int H dt = - \int \frac{\dot{\theta}}{b} dt = \frac{-1}{b} \int d\theta = \frac{1}{b} (\theta_i - \theta_f) \simeq \frac{\theta_i}{b}, \quad (31)$$

where the fact that the scalar field is rolling down ( $\theta$  is decreasing) becomes transparent and we have chosen  $\theta_f = 0$  at the end of inflation for simplicity.

Using Eqs. (3) and (4), with  $H = -\dot{\theta}/b$  and  $\dot{f} = f'\dot{\theta}$ , we can now obtain the relation between the Hubble rate and the coupling to gravity that will sustain the exponential period of expansion we are longing to have,

$$\frac{H'}{H} = \frac{2b/k^2 + bD'' + D'}{2D - bD'} \quad (32)$$

The following step is clear, we need to choose either a coupling to gravity (as we did in the previous section) or a Hubble rate, and then obtain the other one via this second order differential equation. In this section, we are going to choose the form of the Hubble rate and, from there, obtain the non-minimal coupling to gravity. For simplicity we want to obtain analytic expressions for this coupling, and then not many choices for  $H'/H$  are possible.



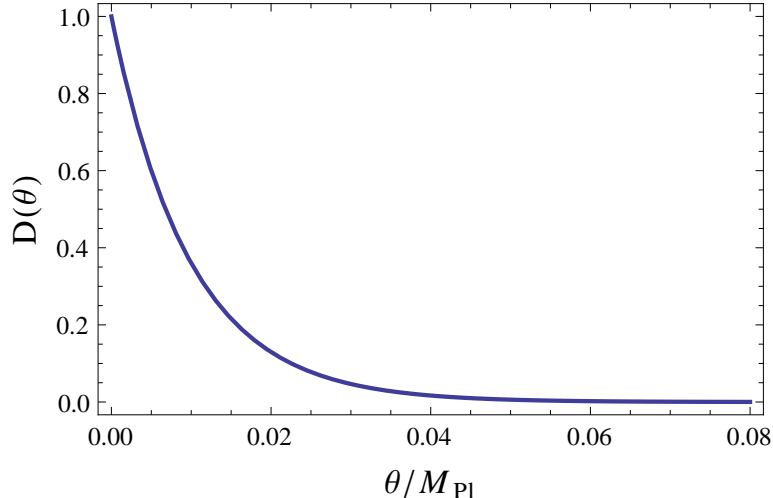


FIG. 1: Coupling of the scalar field to the Ricci scalar curvature  $R$ . The figure is produced for the slow-roll regime,  $\alpha = 0.007$  and taking  $B = 0$ . The value of  $M$  is irrelevant as long as  $M \lesssim 0.1M_{\text{Pl}}$ . The field value is given in units of  $M_{\text{Pl}}$

Unfortunately the most natural and easiest choice,  $H'/H \simeq 0$ , which gives,

$$D(\theta) = -\frac{2b\theta}{k^2} - A b e^{-\theta/b} + B, \quad (33)$$

where  $A$  and  $B$  are the two integration constants, with  $B$  dimensionless and  $[A] = 1/[b] = 1/[\theta]$ , fails phenomenologically. It gives a red spectral index, ( $n_s > 1$ ) and therefore must be abandoned.

The next choice (in order of simplicity) would be  $H'/H = -1/M$ . We can still solve exactly the equation for the coupling to curvature although the solution is not as simple as before,

$$D(\theta) = -\alpha \frac{M^2}{k^2} + e^{\frac{(\alpha-1+\sqrt{\alpha^2-10\alpha+1})\theta}{2\alpha M}} \left( \alpha \frac{M^2}{k^2} + 1 - B \right) + e^{\frac{(\alpha-1-\sqrt{\alpha^2-10\alpha+1})\theta}{2\alpha M}} B, \quad (34)$$

with  $B$  a dimensionless integration constant, which in the following we fix at  $B = 0$  for convenience, and  $\alpha = b/M$ . We have fixed the second integration constant requiring that  $D(\theta_f = 0) = 1$ , so that at the end of inflation we naturally land in an Einstein gravity regime. Despite its rather complicated form, we will see that, for the set of parameters needed to produce the correct inflation phenomenology, the behaviour of  $D(\theta)$  is not as sophisticated as it can appear by looking at its full expression. The number of e-foldings in this scenario can be written in terms of the new mass scale  $M$  and the parameter  $\alpha$  as  $N_e \simeq \theta_i/b = \theta_i/(\alpha M)$ .

For the slow-roll parameters defined in Eqs. (5), we have,

$$\epsilon_1 = \frac{H'\dot{\theta}}{H^2} = -b\frac{H'}{H} = -\alpha \quad (35)$$

$$\epsilon_2 = \frac{\ddot{\theta}}{H\dot{\theta}} = \frac{(\dot{\theta})'}{H} = -b\frac{H'}{H} = -\alpha \quad (36)$$

$$\epsilon_3 = \frac{1}{2} \frac{D'\dot{\theta}}{HD} = -\frac{b}{2} \frac{D'}{D} \simeq -\frac{\left(1 + \frac{M^2}{k^2}\alpha\right) \cdot \omega(\alpha) e^{-\frac{N_e}{2} \cdot \omega(\alpha)}}{4\left(\frac{M^2}{k^2}\alpha - \left(1 + \frac{M^2}{k^2}\alpha\right) e^{-\frac{N_e}{2} \cdot \omega(\alpha)}\right)} \quad (37)$$

$$\epsilon_4 = \frac{1}{2} \frac{E'\dot{\theta}}{HE} = -b\frac{2D' + 3k^2D'D''}{4D + 3k^2(D')^2} \simeq -\left(1 + \frac{M^2}{k^2}\alpha\right) e^{-\frac{N_e}{2} \cdot \omega(\alpha)} \cdot \frac{\frac{M^2}{k^2}\alpha^2 \cdot \omega(\alpha) + 3\left(1 + \frac{M^2}{k^2}\alpha\right) \left((1 - 4\alpha + \alpha^2) \cdot \omega(\alpha) - 4\alpha(1 - \alpha)\right) e^{-\frac{N_e}{2} \cdot \omega(\alpha)}}{4\frac{M^4}{k^4}\alpha^3 - 4\frac{M^2}{k^2}\alpha^2\left(1 + \frac{M^2}{k^2}\alpha\right) e^{-\frac{N_e}{2} \cdot \omega(\alpha)} + 3\left(1 + \frac{M^2}{k^2}\alpha\right)^2 \left((1 - \alpha) \cdot \omega(\alpha) - 4\alpha\right) e^{-N_e \cdot \omega(\alpha)}}, \quad (38)$$

with  $\omega(\alpha) = (1 - \alpha - \sqrt{1 - 10\alpha + \alpha^2})$  and always taking  $B = 0$  in Eq. (34).

These expressions are simplified in the limit  $\alpha \ll 1$ , corresponding to the slow-roll regime,

$$\epsilon_3 \simeq -\alpha e^{-2N_e\alpha} \frac{(1 + 3\alpha)}{\left(\frac{M^2}{k^2}\alpha - e^{-2N_e\alpha}\right)} \quad (39)$$

$$\epsilon_4 \simeq -\alpha e^{-2N_e\alpha} \frac{\frac{M^2}{k^2}(1 + 3\alpha) + 12 e^{-2N_e\alpha}}{\frac{M^4}{k^4}\alpha - \frac{M^2}{k^2} e^{-2N_e\alpha} - 6 e^{-4N_e\alpha}}. \quad (40)$$

The usual slow-roll parameters, in the interesting region  $\frac{M^2}{k^2} < e^{-2N_e\alpha}$  ( *i.e.* basically  $120 \alpha \sim O(1)$ ), become,

$$\tilde{\epsilon} = \epsilon_1 - \epsilon_3 \simeq -2\alpha \quad (41)$$

$$\tilde{\eta} = \epsilon_2 + \epsilon_4 - 3\epsilon_3 \simeq -2\alpha \quad (42)$$

$$n_s = 1 + 4\tilde{\epsilon} - 2\tilde{\eta} \simeq 1 - 4\alpha \quad (43)$$

$$r = 13.8 |\tilde{\epsilon}| \simeq 27.6\alpha. \quad (44)$$

And, for  $\alpha = 0.007$ , we obtain  $r \simeq 0.19$  and  $n_s \simeq 0.97$ . As explained before, the value of  $M$  is irrelevant for these observables as long as  $M \lesssim 0.1M_{\text{Pl}}$ .

As before, we can get the potential, which has a rather baroque expression in full form, although it is basically an exponential potential  $e^{-\frac{2\theta}{M}}$ ,

$$V(\theta) = \frac{3}{4} k^2 e^{-\frac{2\theta}{M}} \left[ \left(1 + \frac{M^2}{k^2}\alpha\right) (2 + \omega(\alpha)) e^{-\frac{\theta \cdot \omega(\alpha)}{2M\alpha}} - \frac{2}{3} \frac{M^2}{k^2} \alpha (3 + \alpha) \right], \quad (45)$$

as can be seen in the left-side plot in Fig. 2. Here, we see that the potential in the  $\theta$  field is decreasing and seems not able to produce inflation. However, in the Jordan frame, we must take also into account the effects of the non-minimal coupling to gravity, and then the corresponding

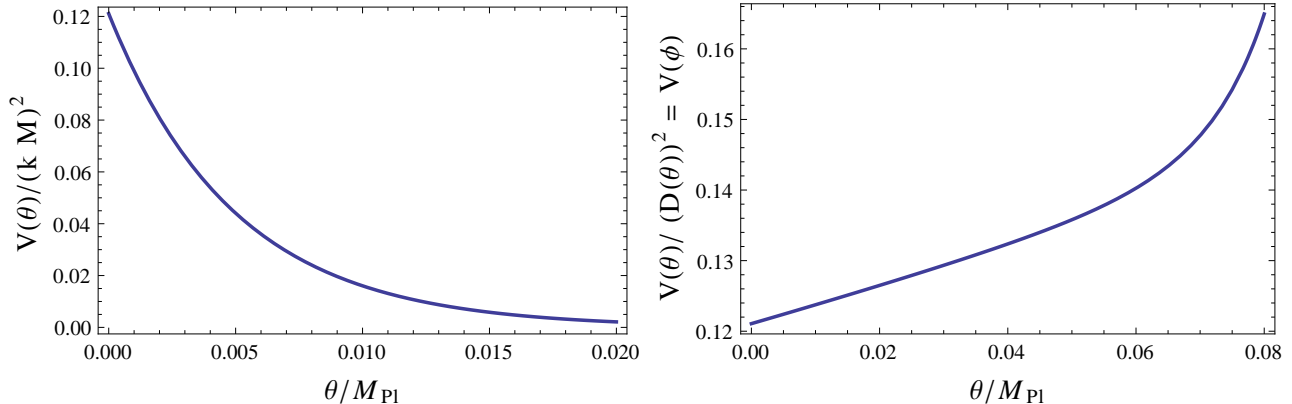


FIG. 2: Potential of the scalar field in the Jordan frame in terms of the non-minimally coupled field (left). The potential in the Einstein frame in terms of the non-minimally coupled variable is shown in the right-side panel. The figures are produced for the slow roll regime,  $\alpha = 0.007$  and taking  $B = 0$ . The value of  $M$  is irrelevant as long as  $M \lesssim 0.1M_{\text{Pl}}$ . The field value is given in units of  $M_{\text{Pl}}$ .

potential in the Einstein frame becomes much more attractive. This is shown in the right-side plot in Fig. 2, where we plot the potential in the Einstein frame as a function of the  $\theta$  field (the Einstein potential in terms of the  $\phi$  field is obtained changing variables using Fig. 3). In this case, the exponential potential is well-behaved and the quantum corrections at one-loop are again small in the region of interest for inflation. The inflaton mass is always smaller than  $M_{\text{Plank}}$ . Thus as shown in Ref. [13] the cutoff is still  $M_{\text{Plank}}$  and there are no UV issues below that scale.

Courtesy of the sophisticated form of  $D(\theta)$ , it is clear that the conformal transformation, which will turn the  $\theta$  field into a more familiar minimally coupled scalar field enjoying Einstein gravity, cannot be carried out analytically. In Fig. 3, we show the relation between both fields, where, as before, our point becomes transparent: the non-minimally coupled field is subplanckian at  $\theta_i = \alpha MN_e \simeq 0.01 M_{\text{Pl}}$ , while the minimally coupled one is not,  $\phi_i \simeq 140 M_{\text{Pl}}$ . At this point, it is important to stress that, in this case, we have not engineered the coupling to gravity in order to support our point. Instead, we have only asked our scale factor to sustain an inflationary period and looked for the simplest possible choice allowing us to solve analytically the second order differential equation which relates the Hubble rate to the curvature coupling. In this context, the emergence of a subplanckian field value in the Jordan frame cannot be considered the result of a fine-tuning.

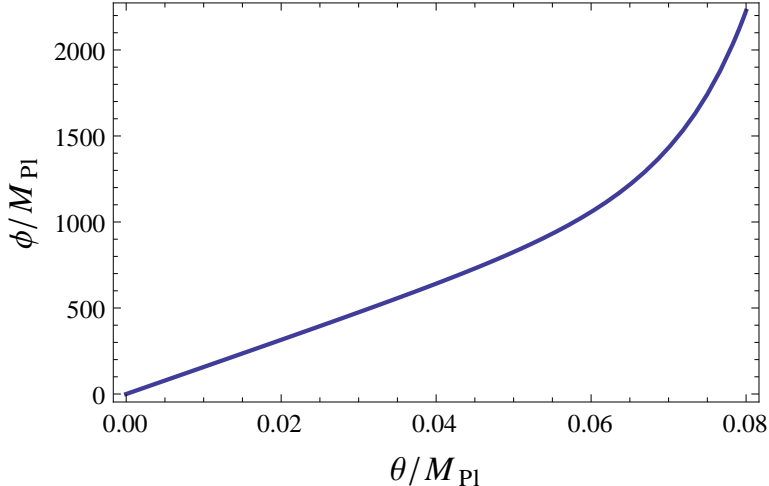


FIG. 3: Minimally coupled scalar field in terms of the non-minimally coupled field. The figure is produced for the slow roll regime,  $\alpha = 0.007$ ,  $M = 0.02$  and taking  $B = 0$ . Both field values are given in units of  $M_{\text{Pl}}$ . From the figure it becomes apparent that while the field in the Einstein frame is transplanckian, the one in the Jordan frame is not.

### C. $f(R)$ gravity models

Unlike the previous cases, where inflation was given by the interaction between the matter sector and the modified gravity sector, to finish we will consider the case where inflation is entirely nourished by gravity [15],

$$L = \frac{k^2}{4} f(R). \quad (46)$$

In this case, equations for the background become,

$$\begin{aligned} H^2 &= \frac{1}{3F(R)} \left( \frac{RF(R) - f(R)}{2} - 3H\dot{F}(R) \right), \\ \dot{H} &= -\frac{1}{2F(R)} \left( \ddot{F}(R) - H\dot{F}(R) \right), \end{aligned} \quad (47)$$

where  $F(R) = \partial f(R)/\partial R$  and  $R = -6(2H^2 + \dot{H})$ . Face value, this case is quite distant from the previous ones, as now there is no conformal transformation capable of driving us to the Einstein frame. However, once we depart in a non-trivial way from the standard gravity, the field equations for  $R$  become higher-order effectively, signalling the presence of additional degrees of freedom. This feature can be taken care by the introduction of an auxiliary scalar field and going, as an intermediate step, through a Brans-Dicke form of our model [15]. Then, in a similar way as in the previous cases, a conformal transformation will take us away from our  $f(R)$  gravity to the kingdom of Einstein gravity plus a minimally coupled scalar field with a specific potential. In the case we

are studying, under a conformal transformation, the metric is redefined as,

$$\hat{g}_{ab} = \Omega^2 g_{ab}, \quad (48)$$

where  $\Omega$  is a spacetime position-dependent factor and is defined to be,

$$\Omega^2 = F(R) = \exp\left(\sqrt{\frac{2}{3k^2}}\phi\right), \quad (49)$$

and  $\phi$  is the new dynamical variable we have obtained after conformal transformation of the Brans-Dicke auxiliary field,

$$\phi = \sqrt{\frac{3k^2}{2}} \ln F(R). \quad (50)$$

The Lagrangian in terms of the  $\phi$  field is given by,

$$L = -\left(\frac{k^2}{4}\hat{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right), \quad (51)$$

where  $\hat{R}$  is the conformally transformed Ricci scalar and the potential has the form,

$$V(\phi) = \frac{k^2}{4} \frac{f(R) - RF(R)}{F^2(R)}. \quad (52)$$

For the sake of concreteness and to make our point even more transparent, we will consider the following ad-hoc gravity during inflation <sup>10</sup>,

$$f(R) = R\left(1 + (R/M^2)^{5/4}\right), \quad (53)$$

where  $M$  is an arbitrary mass scale and we assume that during inflation the second term dominates over the first one, implying that during inflation  $H^2 \gg M^2$ . Clearly, once the inflationary phase is over, we smoothly approach Einstein gravity. In this case,

$$F(R) = \exp\left(\sqrt{\frac{2}{3}}\phi\right), \quad (54)$$

and

$$V(\phi) = \frac{-5k^2}{54} \left(\frac{2}{3}\right)^{3/5} M^2 e^{-2\sqrt{\frac{2}{3}}\phi} \left(e^{\sqrt{\frac{2}{3}}\phi} - 1\right)^{9/5}. \quad (55)$$

As before, the form of the potential looks rather complicated but its shape is pretty simple, as can be seen from figure 4. In fact, already by eye, we can guess that this kind of potential should be

---

<sup>10</sup> Although every single inflationary model is a toy model, we would like to stress that there is not a physics motivation for the proposed modification of gravity. At the same time, we should also bear in mind that physics without assumptions or caveats is unimaginable. We thus, left the reader judge by himself the degree of skepticism that is appropriate when considering modified gravity models of inflation like the one presented here.

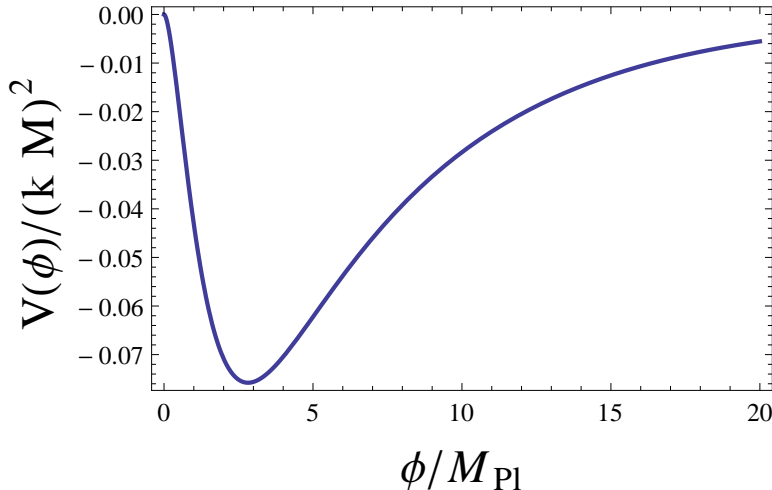


FIG. 4: Potential for the scalar field introduced in the conformal transformation. From the figure it is obvious that the potential does have the right shape to inflate. However transplanckian masses are involved.

able to accommodate a decent period of inflation. As seen in Model B, the cutoff scale in this kind of potentials is again  $M_{Pl}$  and the effective theory is UV safe.

But we can do way better than guessing; the analysis of our potential is straightforward<sup>11</sup>,

$$\epsilon_V = \frac{k^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{1}{300} \left( 11 - 9 \coth \left( \frac{\phi}{\sqrt{6}} \right) \right)^2, \quad (56)$$

$$\eta_V = k^2 \frac{V''}{V} = \frac{2}{75} \left( \frac{36}{\left( e^{\sqrt{\frac{2}{3}}\phi} - 1 \right)^2} - \frac{63}{e^{\sqrt{\frac{2}{3}}\phi} - 1} + 1 \right), \quad (57)$$

$$n_s = 1 - 6\epsilon_V + 2\eta_V = \frac{1}{150} \left( 198 \coth \left( \frac{\phi}{\sqrt{6}} \right) - 171 \operatorname{csch}^2 \left( \frac{\phi}{\sqrt{6}} \right) - 52 \right). \quad (58)$$

The modes we are interested in studying are those that left the horizon 60 efoldings before the end of inflation, where  $\phi_{\text{end}}$ , the field value at the end of inflation, is calculated by asking  $\epsilon_V = 1$ . Then, the field value at horizon exit is obtained from,

$$N = 62 = \int_{\phi_{\text{end}}}^{\phi_{\text{hor}}} 5\sqrt{\frac{3}{2}} \left( -\frac{9}{e^{\sqrt{\frac{2}{3}}\phi} - 10} - 1 \right) d\phi, \quad (59)$$

which is independent of the specific value of  $M$  and, for our choice of parameters, is well beyond

<sup>11</sup> Notice that unlike the previous cases, now we are not using the Hamilton-Jacobi formalism any longer and therefore we need to resort to another set of slow-roll parameters, the ones calculated directly from the potential. Their relation with the spectral index takes into account this difference (see for instance [16]).

$M_{\text{Pl}}, \phi_{\text{hor}} \approx 15 M_{\text{Pl}}$ , giving,

$$n_s = 0.97, \quad r = 0.16. \quad (60)$$

Once again, we see that an innocent modification of gravity, when casted as a minimally coupled scalar field rolling down a potential, ends up giving transplanckian field values. Once more, we would like to stress that, as in the previous example, we have not designed a modification of gravity able to accommodate transplanckian field values. We have just chosen an  $f(R)$  capable of producing sizeable tensor modes and found that this corresponds to transplanckian field values once analysed as a minimally coupled scalar field.

#### IV. CONCLUSIONS

In this work, we have explored the possibility that the transplanckian field values needed to accommodate the experimental results in minimally coupled single-field inflation models are only due to our insistence of imposing a minimal coupling of the inflaton field to gravity. If the theory responsible for inflation includes a non-minimal coupling to gravity, the energies and field values can be subplanckian during the full inflation era in the Jordan frame, while they may appear transplanckian in the Einstein frame.

We are perfectly aware that the field value by itself carries no information, it is after all a “dummy” variable, but the fact that its vacuum expectation value turns out to be well above the Planck mass may be telling us that it is gravity (or its couplings to gravity), and not only the inflaton potential couplings, the true drivers of inflation.

We have shown (Section III A: Monomial potentials ) that not only it is possible to turn the most popular inflationary potentials ( $\phi^4, \phi^2$ ) into the desired regime by choosing an appropriate coupling to gravity, but also that scalar tensor theories, designed exclusively to sustain inflation by asking the scale factor to grow exponentially (Section III B: Generic scalar-tensor theories), also turn subplanckian even in the simplest cases (let us remind the reader that the case  $H'/H = 0$  was discarded, not because it does not satisfy our conjecture, but because it leads to a spectral index larger than 1). We have also presented a case (Section III C:  $f(R)$  gravity models) where gravity itself is solely responsible for inflation, and again results in transplanckian field values once interpreted as single-field inflation.

In summary, we have seen that is possible, and in fact quite natural and easy, to find realistic examples of theories with non-minimal coupling to gravity that have transplanckian field values if

we insist on imposing a minimal coupling to gravity, but are always subplanckian in their “natural” frame. Thus, we have proven that single-field inflation models can still accommodate a large tensor-to-scalar ratio with subplanckian field values in the presence of non-minimal coupling to gravity.

### Acknowledgements

The authors are grateful to Scott Dodelson and Joe Lykken for useful discussions. We acknowledge support from the MEC and FEDER (EC) Grants FPA2011-23596 and the Generalitat Valenciana under grant PROMETEOII/2013/017. G.B. acknowledges partial support from the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442).

- 
- [1] P. A. R. Ade *et al.* [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].
  - [2] A. H. Guth, Phys. Rev. D **23**, 347 (1981);  
A. D. Linde, Phys. Lett. B **108**, 389 (1982);  
For a recent review, see: A. D. Linde, Phys. Rept. **333** (2000) 575.
  - [3] C. L. Bennett *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **208**, 20 (2013) [arXiv:1212.5225 [astro-ph.CO]].
  - [4] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
  - [5] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997) [hep-ph/9606387].
  - [6] D. Baumann and D. Green, JCAP **1205**, 017 (2012) [arXiv:1111.3040 [hep-th]].
  - [7] A. Mazumdar and J. Rocher, Phys. Rept. **497**, 85 (2011) [arXiv:1001.0993 [hep-ph]].
  - [8] A. Kehagias and A. Riotto, arXiv:1403.4811 [astro-ph.CO].
  - [9] S. Antusch and D. Nolde, arXiv:1404.1821 [hep-ph].
  - [10] D. Chialva and A. Mazumdar, arXiv:1405.0513 [hep-th].
  - [11] J. -c. Hwang and H. Noh, Phys. Rev. D **54**, 1460 (1996).
  - [12] H. Noh and J. -c. Hwang, Phys. Lett. B **515**, 231 (2001) [astro-ph/0107069].
  - [13] M. P. Hertzberg, JHEP **1011**, 023 (2010) [arXiv:1002.2995 [hep-ph]].
  - [14] G. Barenboim and J. Lykken, JCAP **0803**, 017 (2008) [arXiv:0711.3653 [astro-ph]].
  - [15] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010) [arXiv:0805.1726 [gr-qc]].
  - [16] A. R. Liddle, P. Parsons and J. D. Barrow, Phys. Rev. D **50**, 7222 (1994) [astro-ph/9408015].