

“SECRET” NEUTRINO INTERACTIONS*

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We review the information about a potentially strong non-standard four-neutrino interaction that can be obtained from available experimental data. By using LEP results and nucleosynthesis data we find that a contact four-fermion neutrino interaction that involve only left-handed neutrinos or both left-handed and right-handed neutrinos cannot be stronger than the standard weak interactions. A much stronger interaction involving only right-handed neutrinos is still allowed.

1 “Secret” neutrino interactions?

In the standard model (SM) the interaction between neutrinos is given by the exchange of the Z -boson and the effective Hamiltonian of the neutrino-neutrino interaction has the form

$$\mathcal{H}_{SM}^{\nu-\nu} = \frac{G_F}{\sqrt{2}} \sum_{\ell, \ell' = e, \mu, \tau} (\bar{\nu}_{\ell L} \gamma_\alpha \nu_{\ell L}) (\bar{\nu}_{\ell' L} \gamma^\alpha \nu_{\ell' L}), \quad (1)$$

where G_F is the Fermi constant. However, it is very difficult to perform direct experimental tests on this interaction.

Many years ago the question was raised^{1,2} whether an additional non-standard four-neutrino interaction exists:

$$\mathcal{H}_I^{\nu-\nu} = F (\bar{\nu} \gamma_\alpha \nu) (\bar{\nu} \gamma^\alpha \nu) \quad (2)$$

Such an effective interaction could arise, for instance, from the exchange of a strongly interacting heavy vector boson, V_μ , coupled to neutrinos only

$$\mathcal{L} = g_V (\bar{\nu}_i \gamma_\mu \nu_i) V^\mu, \quad (3)$$

when it is considered at energy scales much lower than the vector boson mass, m_V , e.i. $q^2 \ll m_V^2$. In this case we have the relation $F = g_V^2/m_V^2$.

*NEUTRINO MIXING (TORINO, MARCH 1999) IN HONOUR OF SAMOIL BILENKY'S 70-TH BIRTHDAY

Interactions mediated by scalars can also be written in the form of $\mathcal{H}_I^{\nu-\nu}$ after a Fierz transformation. The flavour structure could be, however, more general.

Obviously, the possible effect of the exchange of a light particle coupled to neutrinos can not be approximated correctly by the contact four-neutrino interaction of the form (2) at $q^2 \geq m_V^2$. However, in this case the non-standard particles could be produced directly and such models are in general strongly constrained. This is what happens for a class of popular models in which the non-standard interaction among neutrinos is due to the exchange of a massless majoron^{3,4,5,6,7}. In this case very strong bounds on neutrino-majoron coupling constants $g_{\ell\ell}$ follow from searches for massive neutrinos and neutral particles in $K \rightarrow \ell + \dots$. One obtains⁸ $g_{ee}^2 < 1.8 \cdot 10^{-4}$, $g_{\mu\mu}^2 < 2.4 \cdot 10^{-4}$. Consequently majoron bremsstrahlung from neutrinos can give only a small contribution to the invisible Z -width. Majorons with non-vanishing hypercharge^{5,6,7} could potentially give a large contribution to the invisible Z -width because the Z -boson can decay directly to scalars. Therefore, models of this type have already been excluded by LEP data (in the case of the triplet majoron with hypercharge one⁵ this contribution is equivalent to the existence of two additional neutrinos and in the case of the doublet majorons^{6,7} a contribution, equivalent to half the contribution of an additional neutrino, arises). Singlet majorons³ or non-singlet majorons without hypercharge⁴ cannot be excluded by LEP data.

We review in this paper, from a historical point of view, the different bounds set on the exotic four-neutrino contact interaction. In section 2 we review the old low-energy bounds. In section 3 we take a look to the limits obtained from the supernova SN1987A. Sections 4 and 6 are devoted to the limits obtained from the invisible decay width of the Z gauge boson measured at LEP: section 4 by using the four-neutrino decay at tree level and section 6 by using the one loop contribution of the SNI to the two-neutrino decay. In section 5 we briefly discuss the possibility of gaining some information on the SNI by modification of the lepton spectra in the W -boson decay due to the process $W^+ \rightarrow \ell^+ \nu_\ell \nu \bar{\nu}$. In section 7 we present the stringent bounds obtained from nucleosynthesis and finally in section 8 we present our conclusions.

2 Low-energy bounds

In the pioneering paper¹ the contributions of $\mathcal{H}_I^{\nu-\nu}$ to different low-energy processes were analysed and different bounds were set.

The SNI, contributes to the decays $\pi^+ \rightarrow e^+ \nu_e \bar{\nu} \nu$ and $K^+ \rightarrow l^+ \nu_l \bar{\nu} \nu$ ($l = e, \mu$), and could modify the inclusive lepton energy spectra in K^+ and π^+

decays, which are dominated by standard decays, $K^+(\pi^+) \rightarrow l^+\nu_l$. From an analysis of these spectra the following bounds on the coupling F were obtained ¹

$$|F| \leq 10^7 G_F, \quad |F| \leq 2 \times 10^6 G_F, \quad (4)$$

where G_F denotes the weak Fermi constant.

Similar bounds were found ¹ from the absence of leptons with “wrong” charge in the process $\nu_\mu + N \rightarrow \mu^+ + \nu_\mu + \nu_\mu + X$.

Later on these bounds were improved in a special experiment ⁹ (for more recent experiments see also ¹⁰) searching for the decay $K^+ \rightarrow \mu^+ \nu_\mu \bar{\nu} \nu$. From the negative result of this experiment the following limit was set:

$$F \leq 1.7 \times 10^5 G_F. \quad (5)$$

It seemed at that time that four-neutrino interactions could be much stronger than standard model neutral current interactions.

The reason why bounds on the non-standard neutrino interaction coming from low-energy experiments are so loose is evident. The SNI contributes only to the decays with four particles in the final state, and such processes are strongly suppressed by phase space compared with the standard leptonic π and K decays.

3 Supernova bounds

The detection of (anti)neutrinos from SN1987A stimulated again the interest on SNI. Using that data some new limits were set.

In particular, the detection of neutrinos from SN1987A requires that the value of the mean free path of neutrinos through the cosmic background particles (CBP) is comparable or greater than the distance to the supernova.

Stable neutrinos should be present today as CBP, therefore, a four-neutrino interaction will contribute to the mean free path of supernova neutrinos and a bound can be set¹¹.

If neutrinos have an interaction with neutrinos mediated by heavy vector bosons with mass M and coupling g , a bound on g/M was obtained

$$\frac{g}{M} < \frac{12}{MeV}, \quad (6)$$

which can be translated into the following bound on the constant F :

$$F < 10^{13} G_F. \quad (7)$$

This bound is much worse than the obtained low-energy bounds.

On the other hand, from the estimate of the diffusion time of neutrinos in the supernova and its comparison with the duration of the detected neutrino pulse, an upper bound on neutrino-neutrino cross section $\sigma_{\nu\nu}$ can be obtained¹² (see also ¹³)

$$\sigma_{\nu-\nu} < 10^{-35} \text{ cm}^2 . \quad (8)$$

If this cross section arises from a strong four-neutrino interaction one can obtain the following estimate

$$F < 10^3 G_F , \quad (9)$$

which is two orders of magnitude better than the best of the low-energy bounds but still allows for a rather strong SNI.

4 The decay $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$

The decays $\pi(K) \rightarrow \ell\nu_\ell\nu\bar{\nu}$ with four particles in the final state are strongly suppressed by phase space in comparison with the usual two-body lepton decays. Decays of much heavier particles, such as gauge bosons, will provide a much larger phase space for multi-neutrino production.

If strong four-neutrino interactions exist, four-neutrino decays

$$Z \rightarrow \nu\bar{\nu}\nu\bar{\nu} \quad (10)$$

will also contribute to the invisible width of the Z gauge boson and therefore the strength of such interaction can be constrained¹⁴ from the precise LEP measurement of Γ_{invis} .

To be definite we take the Hamiltonian of $\nu-\nu$ interactions with a general V, A form

$$\mathcal{H}_I^{\nu-\nu} = F \sum_{\ell, \ell' = e, \mu, \tau} (\bar{\nu}_\ell O_{\ell\alpha} \nu_\ell) (\bar{\nu}_{\ell'} O_{\ell'}^\alpha \nu_{\ell'}) . \quad (11)$$

Here

$$O_\ell^\alpha = a_\ell \gamma^\alpha P_L + b_\ell \gamma^\alpha P_R , \quad (12)$$

P_L and P_R are the left and right chirality projectors $P_L = \frac{1}{2}(1 - \gamma_5)$, $P_R = \frac{1}{2}(1 + \gamma_5)$ and F, a_ℓ, b_ℓ are real parameters.

For the total probability of the decay of Z -bosons into two neutrino pairs (identical and non-identical) we have found the following expression¹⁴

$$\Gamma(Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}} m_Z^7 F^2 \frac{1}{1024\pi^5} \sum_{\ell, \ell'} \{ (a_\ell^2 a_{\ell'}^2 + a_\ell^4 \delta_{\ell\ell'}) C_1 + a_\ell^2 b_{\ell'}^2 C_2 \} , \quad (13)$$

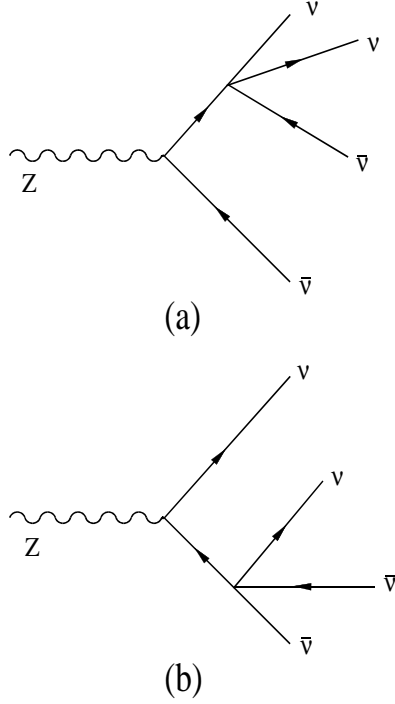


Figure 1. Diagrams contributing to the process $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$.

where

$$C_1 = -\frac{1651}{486} + \frac{28}{81}\pi^2, \quad C_2 = \frac{259}{486} - \frac{4}{81}\pi^2 \quad (14)$$

The summation runs over the three generations $\ell, \ell' = e, \mu, \tau$.

The decays $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$ are not sensitive to pure right-handed $\nu-\nu$ interactions. This is also true for any process involving neutrinos produced through the standard interaction.

Assuming $e-\mu-\tau$ universality in the non-standard $\nu-\nu$ interaction ($a_\ell = a, b_\ell = b, \ell = e, \mu, \tau$), one can rewrite the expression for the decay

width of $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$ in the following form :

$$\Gamma(Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}} m_Z^7 \frac{1}{1024\pi^5} \bar{F}^2 (12C_1 + 9\bar{b}^2 C_2) . \quad (15)$$

Here $\bar{F}^2 \equiv F^2 a^4$ and the parameter $\bar{b}^2 \equiv b^2/a^2$ characterises the relative contribution of right-handed currents into the $\nu - \nu$ interaction.

Assuming that only the standard decays $Z \rightarrow \nu_\ell \bar{\nu}_\ell$ ($\ell = e, \mu, \tau$) and the decays $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$ contribute to the invisible width of the Z -boson Γ_{invis} we can obtain a bound on \bar{F} .

$$\Gamma_{invis} = 3\Gamma(Z \rightarrow \nu_\ell \bar{\nu}_\ell)^{SM} + \Delta\Gamma_{invis} . \quad (16)$$

In our case,

$$\Delta\Gamma_{invis} = \Gamma(Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}) \quad (17)$$

On the other hand this quantity can also be expressed as

$$\Delta\Gamma_{invis} = \Gamma_{invis} - 3 \left(\frac{\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{l}l}} \right)^{SM} \Gamma_{\bar{l}l} . \quad (18)$$

From LEP measurements we have¹⁵

$$\Gamma_{invis} = 500.1 \pm 1.8 \text{ MeV}, \quad \Gamma_{\bar{l}l} = 83.91 \pm 0.1 \text{ MeV}, \quad (19)$$

then, using the ratio of the neutrino and charged leptons partial widths calculated within the SM

$$\left(\frac{\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{l}l}} \right)^{SM} = 1.991 \pm 0.001 . \quad (20)$$

we obtain from eq. (18)

$$\Delta\Gamma_{invis} \simeq -1.1 \pm 1.9 \text{ MeV} . \quad (21)$$

Therefore, from eq. (13), eq. (17) and eq. (21) we obtain for $\bar{b}^2 = 1$ (pure vector or pure axial ν - ν interaction)

$$\bar{F} < 90 G_F , \quad (22)$$

while for pure $V - A$ couplings ($\bar{b}^2 = 0$) we obtain

$$\bar{F} < 160 G_F . \quad (23)$$

The upper bound on this constant is much lower than earlier existing particle physics bounds and one order of magnitude lower than the estimate obtained from the supernova neutrino diffusion time ^a.

^aNote also the important improvement with respect the results obtained in ¹⁴ which is due to the updated values of LEP results we use here.

5 $W^+ \rightarrow \ell^+ \nu \nu \bar{\nu}$

Four-neutrino interactions will also give rise to the decay

$$W^+ \rightarrow \ell^+ \nu \ell \nu \bar{\nu} \quad (24)$$

Using our effective Hamiltonian for the SNI we get the following lepton spectrum in the rest frame of the W .

$$\begin{aligned} \frac{d\Gamma}{dE} &= \frac{1}{9} \frac{1}{(2\pi)^5} \frac{G_F}{\sqrt{2}} F^2 m_W^2 a_\ell^2 \left[a_\ell^2 + \sum_{\ell'} (a_{\ell'}^2 + b_{\ell'}^2) \right] \\ &\times \sqrt{E^2 - m_\ell^2} (E_0 - E) (3m_W E - 2E^2 - m_\ell^2), \quad (25) \end{aligned}$$

where E is the total energy of the charged lepton, m_W and m_ℓ are the masses of the W -boson and the lepton and $E_0 = (m_W^2 + m_\ell^2)/2m_W$ is the maximum energy of the lepton.

The search for decays of the W -boson with a single lepton with energy less than E_0 could give additional information about $\nu - \nu$ interactions. This analysis could be done using present LEP II data.

6 “Secret” neutrino interactions at one loop

All previous bounds are extracted from processes in which the new interaction is the only relevant one and, therefore, observables depend quadratically on the coupling F . Obviously, if the new interaction enters in loop corrections to a SM process, modifications come through its interference with the SM amplitude and, then, the deviations from the SM predictions will depend linearly on the coupling F .

For example, ν - ν interactions will contribute to the decay $Z \rightarrow \bar{\nu}\nu$ at the one-loop level and consequently to the invisible width of the Z -boson¹⁶.

It is very simple to estimate the order of magnitude of the corresponding contribution of the non-standard interactions at one-loop:

$$\frac{\Delta\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{\nu}\nu}} \approx \frac{FM_Z^2}{(4\pi)^2}. \quad (26)$$

As the invisible width of the Z -boson is now measured with an accuracy better than 1%, one finds the following bound on the non-standard coupling F :

$$F \leq (1-10) \cdot G_F. \quad (27)$$

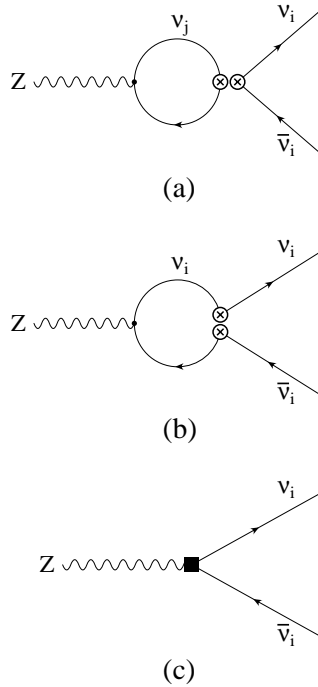


Figure 2. Diagrams that give contributions to the $Z\bar{\nu}\nu$ vertex in the presence of the non-standard four-neutrino interaction. In diagram (a), neutrinos of different flavours are running in the loop.

Therefore, one expects stronger bounds of F coming from the one-loop analysis than those which follow from its contribution to the invisible width of the Z -boson at tree-level.

Although four-fermion interactions are not renormalizable in the “text-book-sense”, still one can obtain some information on their couplings by using them at the one-loop level^{16,17}. This is done by considering additional dimension-six operator(s) which mix with the four-fermion operator under the renormalization group and serve as counterterms to cancel the divergences arising from the use of the four-fermion operator in the loop. The price to be paid is the introduction of more unknown parameters in the analysis which depend on the details of the full theory giving rise to the effective theory. However, if the scale of new physics and the EW scale are well separated, the

dominant contributions are the logarithmic terms coming from the running between the two scales. These contributions are quite model independent and can be unambiguously computed in the effective theory.

The partial decay width of the Z -boson into two neutrinos can be written in the following form:

$$\Gamma(Z \rightarrow \bar{\nu}\nu) = \Gamma^{SM}(Z \rightarrow \bar{\nu}\nu) + \Delta\Gamma_{\bar{\nu}\nu} , \quad (28)$$

where $\Gamma^{SM}(Z \rightarrow \bar{\nu}\nu)$ is the SM contribution and $\Delta\Gamma_{\bar{\nu}\nu}$ contains the effects of the non-standard operators.

At lowest order these effects come from the interference of the non-standard amplitude with the SM amplitude and we have

$$\Delta\Gamma_{\bar{\nu}\nu} = \Gamma^{SM}(Z \rightarrow \bar{\nu}\nu) 2\text{Re} \{g_L(M_Z^2)\} , \quad (29)$$

where

$$\text{Re} \{g_L(q^2)\} = G_F q^2 (c_2 + c_1 \kappa + c_1 \gamma \log(M_Z^2/|q^2|)) , \quad (30)$$

gives the vertex $Z\nu\bar{\nu}$ induced by the four-fermion interaction at one loop. The constants γ and κ and c_1 are¹⁶

$$\gamma = \frac{1}{3\pi^2} , \quad \kappa = \frac{17}{36\pi^2} , \quad c_1 = \frac{F a^2}{G_F} = \frac{\bar{F}}{G_F} , \quad (31)$$

and c_2 is just the finite part of the counterterm needed to absorb the divergences encountered in the loop calculation (see fig. 2c).

Because the standard $Z\nu\bar{\nu}$ coupling only involves left-handed neutrinos, and because the lowest order effect of the non-standard interaction comes via its interference with the standard coupling, only interactions of left-handed neutrinos contribute.

Assuming that there are three generations of neutrinos, the non-standard contribution to the invisible width of the Z -boson is now

$$\Delta\Gamma_{invis} = 3\Delta\Gamma_{\bar{\nu}\nu} , \quad (32)$$

where $\Delta\Gamma_{\bar{\nu}\nu}$ is given above. Using the limits on $\Delta\Gamma_{invis}$ obtained in section 4 we get

$$-0.03 \leq c_2 + c_1 \kappa \leq 0.007 . \quad (33)$$

If there are no unnatural cancellations between the couplings each of the couplings can be bounded independently of the others and we obtain:

$$|c_1| = \frac{\bar{F}}{G_F} \leq 0.6, \quad |c_2| \leq 0.039, \quad (34)$$

However, even if there are cancellations at this particular scale ($q^2 = m_Z^2$) there will not be cancellations at other scales, because the logarithmic dependence of $g_L(q^2)$ on q^2 . Therefore, it will still be possible to get some interesting bounds on the coupling \bar{F} if additional data obtained at different scales are used¹⁶ (for instance DIS experiments at high energy ($-q^2 \simeq 100 - 1000 \text{ GeV}^2$)).

Thus, from the analysis of LEP data we can say that contact four-fermion neutrino interactions, involving only left-handed neutrinos, cannot be larger than standard neutral current interactions.

7 Primordial Nucleosynthesis

Data on primordial nucleosynthesis offers a limit on the number of the massless degrees of freedom contributing to the early universe expansion for temperatures $T \geq 1 \text{ MeV}$.

Three right-handed neutrinos in equilibrium with their left-handed partners at these temperatures are completely excluded. In fact, the three right-handed neutrinos should have decoupled at $T \simeq 200 \text{ MeV}$.

Four-neutrino interactions of the type considered involving both, left-handed and right-handed neutrinos, could keep right-handed neutrinos in thermal equilibrium¹⁸ through the reactions

$$\bar{\nu}_{Li} + \nu_{Ri} \Leftrightarrow \bar{\nu}_{Rj} + \nu_{Lj} \quad (35)$$

Requiring that there is decoupling at a temperature T , that is, enforcing that the interaction rate Γ is smaller than the expansion rate of the universe H , one obtains a rather stringent limit on four-neutrino couplings¹⁸:

$$F_V < 3 \times 10^{-3} G_F \quad (36)$$

for pure vector interactions.

If four-fermion interactions involve only left-handed (or only right-handed) neutrinos the limit does not apply at all.

Other constraints can be obtained from ultra-high energy AGN neutrinos¹⁹. However they are only relevant for relatively light mediators ($m_V < 0.5 \text{ GeV}$).

8 Conclusions

We have reviewed, from a historical point, the information obtained on the possible existence of a strong four-fermion contact neutrino interaction.

Bounds on four-neutrino interactions coming from K^+ and π^+ decays are very soft and still allow for interactions much stronger than standard model interactions.

From neutrino diffusion time in the supernova one can set better bounds but “Secret” interactions could still be large.

The invisible decay width of the Z gauge boson constrains strongly four-neutrino interactions, if they involve left-handed neutrinos. It gives contributions to four-neutrino decay at tree level and to two neutrino decay at one-loop. Using present data we find that SNI interactions involving left-handed neutrinos cannot be larger than neutral current standard model interactions.

“secret” interactions involving both, left-handed and right-handed neutrinos are severely constrained by primordial nucleosynthesis. Their strength must be at least two orders of magnitude smaller than in the standard model, although, these bounds do not apply to pure left-handed or pure right-handed couplings.

Taking all information together we find that there is no room for strong four-fermion contact neutrino interactions unless they involve *only* right-handed neutrinos.

Acknowledgements

We would like to acknowledge especially Samoil Bilenky for an enjoyable collaboration and for introducing us to the subject discussed in this paper. Many of the results presented here belong to Samoil. A.S. also likes to thank Wanda Alberico for giving him the opportunity to join Samoil and his friends in this special occasion. Finally A.S. is also indebted to the CERN Theory division, where this work has been written in part, for its hospitality. This work has been funded, in part, by CICYT under the Grant AEN-96-1718, by DGEIC under the Grant PB97-1261 and by the DGEUI of the Generalitat Valenciana under the Grant GV98-01-80.

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