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Extra dimensions at the one loop level: $Z \to b\bar{b}$ and B-$\bar{B}$ mixing

Joannis Papavassiliou and Arcadi Santamaria

Departament de Física Teòrica and IFIC, Universitat de València – CSIC
Dr. Moliner 50, E-46100 Burjassot (València), Spain

Abstract

We study, at the one loop level, the dominant new physics contributions from extra dimensions to $Z \to b\bar{b}$, as well as B-$\bar{B}$ and K-$\bar{K}$ mixing. We use a model with one extra dimension containing fermions which live in four dimensions, and gauge bosons and one scalar doublet propagating in five dimensions. We find that the effect of the infinite tower of Kaluza-Klein modes in $Z \to b\bar{b}$ is finite and gives a negative correction to $R_b = \Gamma_b/\Gamma_h$, which is used to set a lower bound of 1 TeV on the compactification scale M_c. On the other hand, we show that the box diagrams contributing to B-$\bar{B}$ and K-$\bar{K}$ mixing are divergent and, after proper regularization, we find that they increase the value of the function $S(x_t)$ which governs this mixing. The obtained value is perfectly compatible with available data.

11.10.Kk, 12.60.-i, 14.65.Fy, 14.65.Ha

I. INTRODUCTION

In the last years there has been a revival of interest in new physics scenarios in which the ordinary four dimensional standard model (SM) arises as a low energy effective theory of models defined in five or more dimensions. Apart from the fact that this type of models arise naturally in string scenarios, there are various reasons for this renewed interest. Probably the most exciting one is the realization that the size of the extra dimensions can be amazingly large without contradicting present experimental data [1–4]. This opens the door to the possibility of testing these models in the near future. In fact, a general feature of models with large extra dimensions is the presence of a tower of Kaluza-Klein (KK) states which, if light enough, could be produced in the next generation of accelerators (see for instance [5–7]). In addition, models based on large extra dimensions can be used to shed light on a variety of problems. First of all, by introducing a new scale close to the electroweak scale the hierarchy problem is pushed by a few orders of magnitude [2, 8]. Furthermore, by resorting to extra dimensions one might gain new insights on the size of the cosmological constant [9, 10]. In addition, supersymmetry breaking could be explained in the context of such theories [11]. Moreover, the linear running of gauge couplings obtained in models with extra dimensions can be used to lower the scale of gauge coupling unification (see for instance [12, 13]). Finally, by assigning fermions to different configurations of the extra dimensions one hopes to reproduce the hierarchical pattern of fermion masses (see for instance [14–18]).

Models with compact extra dimensions are in general not renormalizable, and one should regard them as low energy manifestations of some more fundamental theory, perhaps string theory. The effects of the extra dimensions are communicated to the four dimensional world through the presence of infinite towers of KK modes, which modify qualitatively the behavior of the low energy theory. In particular, the non-renormalizability of the theory is found when summing the infinite tower of KK states. Indeed, already when computing tree level processes, one encounters sums of the type

$$\sum_{n_1,n_2,\cdots=-\infty}^{+\infty} \frac{1}{n_1^2+n_2^2+\cdots+n_\delta^2}\,, \tag{1.1}$$

where δ is the number of extra dimensions. The above sum is divergent if $\delta > 1$. Notice that this type of behavior is different from conventional non-renormalizable theories where, at least at tree level, all processes are finite. Then, if $\delta > 1$ one readily assumes that the theory should be cut off at some scale above the compactification scale. In practice this is implemented by truncating the tower of KK modes at $n_i \sim 100$. Such a truncation mechanism is dynamically realized in the context of some string theories, where an exponential dumping factor suppresses the couplings of the KK modes to ordinary matter [4]. Models with only one extra dimension ($\delta = 1$) are especially interesting because the above sum is convergent. Therefore, the tree level predictions of five dimensional models are particularly stable with respect to the scale of any new physics beyond the compactification scale. However, as commented before, even such models are not renormalizable, and one expects that their bad high energy behavior will eventually manifest itself also at the level of the four dimensional theory with an infinity of KK modes. Thus, it is interesting to study the behavior of this type of models at the one loop level and investigate to what extent their good

tree level behavior is maintained. We will show in section III that the effect of summing the infinite tower of KK modes amounts to changing the propagator of the particle having KK modes by a propagator which behaves like $1/k$ for large k, instead of the canonical $1/k^2$ behavior. This ultimately will trigger the non-renormalizability even of models with only one extra dimension. In spite of that the integrals involving only one summation over KK modes are as well behaved as their counterparts in the original (zero-mode) renormalizable four dimensional theory; they too will therefore give rise to finite results.

Models with extra dimensions are also interesting from the phenomenological point of view because they are very predictive once the spectrum and the symmetries have been specified (e.g. which fields live in four dimensions and which fields live in the extra dimensions). For instance, five dimensional extensions of the SM or the minimal supersymmetric standard model (MSSM) contain only one additional parameter, the compactification radius, R, or its inverse, the compactification scale $M_c = 1/R$. In principle the theory also depends on the cutoff scale of the theory $M_s < 100 M_c$; however, for models with a single extra dimension this scale does not appear at tree level and, as we will see, many one-loop results are also rather insensitive to it. On the other hand, models with more than one extra dimension can depend heavily on this additional parameter.

In this paper we study a model with only one extra dimension at the one loop level following the bottom-up approach. Specifically, we will build a four-dimensional quantum field theory (QFT) containing an infinite tower of KK modes, derived from a five dimensional model. In this framework we will study some of the theoretical issues that arise when keeping the infinite tower of KK modes, as well as some of their phenomenological consequences.

There are many different types of models with large extra dimensions depending on the fields they contain and the exact location of these fields [2]. For our purposes we will adopt the simplest generalization of the SM, namely the so-called 5DSM with fermions living in four dimensions and gauge bosons and a single scalar doublet propagating in five dimensions [19]. This simple model will allow us to explore the behavior of the theory at the one loop level and, at the same time, to extract some phenomenological constraints derived from one loop processes which are enhanced due to their strong dependence on the top-quark mass, m_t. Thus, in section II we derive the relevant four dimensional Lagrangian containing the tower of KK modes from the five dimensional one. At energies much smaller than the compactification scale the tower of KK modes can be integrated out. This gives rise to a four-fermion interaction, which is also derived in section II. In section III we use the process $Z \to b\bar{b}$ as a laboratory to study the effect of the KK tower of charged scalar fields at the one-loop level. This process is also phenomenologically interesting because it is very well measured and because it is sensitive to the presence of additional scalar fields with couplings proportional to m_t. We find that the scalar KK modes give rise to a finite contribution, and discuss the reason for that. The theoretical prediction thus obtained, combined with the existing precise experimental value of R_b, is used to set stringent bounds on the compactification scale. Section IV is devoted to the study of two related processes, namely K-$\bar{K}$ and B-$\bar{B}$ mixing, induced by box diagrams involving the exchange of two scalar towers of KK modes. These diagrams are also enhanced by the top quark mass and are interesting from the phenomenological point of view. Contrary to the case of $Z \to b\bar{b}$, the presence of two towers of KK modes renders these diagrams divergent. Introducing the cutoff of the theory, M_s, we estimate their contribution and compare it with the available

experimental data. Finally in section V we collect and discuss the results.

II. THE LAGRANGIAN

When studying the dominant radiative corrections induced by the exchange of KK modes, it is natural to focus on processes which are known to be sensitive to radiative corrections even in the absence of KK modes. In the SM the most important loop effects are those enhanced due to the dependence on the heavy top-quark mass: $Z \to b\bar{b}$ [20–23], B-$\bar{B}$-mixing [24], and the ρ parameter.

If fermions live in four dimensions, as is the case in the model we consider, there are no KK modes associated with the top quark; therefore, there are no additional one-loop corrections to the ρ parameter enhanced by the top-quark mass. On the other hand, in models with gauge bosons living in the extra dimensions the ρ parameter is already modified at tree level, because the KK modes of gauge bosons mix with the standard zero-mode gauge bosons, a fact which provides interesting constraints on the compactification scale [7, 25–29]. We will therefore focus on the remaining two processes mentioned above.

In the SM the dominant contributions to $Z \to b\bar{b}$ and B-$\bar{B}$ -mixing come from diagrams with the charged scalars (the would-be Goldstone bosons) running in the loop, because their couplings are proportional to the top-quark mass. One can easily establish this in the Feynman or in the Landau gauges. The contributions from the exchange of gauge bosons are suppressed by powers of $(m_W/m_t)^2$ and vanish in the gauge-less limit ($g \to 0$) or in the large top-quark mass limit. However, because the top quark mass is not so heavy, the convergence of the expansion is rather slow [21] and the complete calculation is needed in order to match the experimental accuracy against the SM prediction. In spite of that, the dominant large top-quark mass approximation is good enough for many purposes, in particular when estimating the size of contributions stemming from new physics.

If the scalar doublet lives in five dimensions it will give rise to a tower of KK modes with Yukawa couplings proportional to the top-quark mass. Therefore, we expect the contributions from diagrams containing these couplings to be numerically dominant. If the scalar doublet lives in four dimensions there could still be important contributions coming from the exchange of the KK modes of the gauge bosons, but they are not enhanced by the top-quark mass. Therefore, we will only consider the coupling of a scalar doublet living in five dimensions to fermions living in four dimensions.

The relevant pieces of the five dimensional Lagrangian are ($\mu = 0, 1, 2, 3$ are four dimensional indices and $M = 0, 1, 2, 3, 5$[1] are five dimensional ones)

$$L = \int d^5x \left(\partial_M \varphi^\dagger \partial^M \varphi - \left(\bar{Q}_L Y_u u_R \, \varphi \, \delta(x^5) + \text{h.c.} \right) + \cdots \right) , \tag{2.1}$$

where $\varphi(x^M)$ is the $SU(2)$ scalar doublet which lives in five dimensions. $Q_L(x^\mu)$ and $u_R(x^\mu)$ are the standard left-handed quark doublets and right-handed singlets, respectively, which live in four dimensions. They carry additional flavor and color indices which have been

[1]Following the standard notation we label the fifth component as 5, even though we started at 0.

suppressed. Y_u are 3×3 matrices in the flavor space. We have not written the Yukawa interaction of the down quarks because it is proportional to the down quark masses which are small. Of course these interactions are present and necessary for generating down quark masses and mixings. We have also omitted the kinetic terms of fermions, as well as gauge bosons interactions, which will not be relevant in our approximation. The rôle of $\delta(x^5)$ is to force the fermions to live in four dimensions. As usual, one assumes that the fifth dimension x^5 is compactified on a circle of radius R with the points x^5 and $-x^5$ identified (that is, an orbifold $S^1/\mathbb{Z}_2$). Fields even under the $\mathbb{Z}_2$ symmetry will have zero modes which will be present in the low energy theory. Fields odd under $\mathbb{Z}_2$ will only have KK modes and will disappear from the low energy spectrum. One chooses the scalar doublet to be even under the $\mathbb{Z}_2$ symmetry in order to have a standard zero mode Higgs field. Following the standard Kaluza-Klein construction, we Fourier expand the scalar fields as follows (from now on x refers only to the four dimensional coordinates x^μ)

$$\varphi(x^\mu, x^5) = \sum_{n=0}^{\infty} \cos \frac{n x^5}{R} \varphi_n(x^\mu) \,. \tag{2.2}$$

Substituting this expression into the fifth dimensional Lagrangian, eq. (2.1), and integrating over the fifth component leads to the four dimensional Lagrangian for the KK modes $\varphi_n(x)$. The kinetic terms, however, are not canonical, and we need to perform the following redefinitions of fields and couplings in order to cast them into canonical form:

$$\varphi_0(x) \to \frac{1}{\sqrt{2\pi R}} \varphi_0(x) \,, \quad \varphi_n(x) \to \frac{1}{\sqrt{\pi R}} \varphi_n(x) \,, \; (n \neq 0) \,, \quad Y_u \to \sqrt{2\pi R}\, Y_u \,. \tag{2.3}$$

Then, we arrive at the following four dimensional Lagrange density

$$\begin{aligned} \mathcal{L} = {} & \partial_\mu \varphi_0^\dagger \partial^\mu \varphi_0 - \left(\bar{Q}_L Y_u u_R \varphi_0 + \text{h.c.} \right) \\ & + \sum_{n=1}^{\infty} \left(\partial_\mu \varphi_n^\dagger \partial^\mu \varphi_n - \frac{n^2}{R^2} \varphi_n^\dagger \varphi_n - \left(\bar{Q}_L Y_u u_R \sqrt{2} \varphi_n + \text{h.c.} \right) \right) \,, \end{aligned} \tag{2.4}$$

which will be used in our calculations. Fermions obtain their masses when the neutral component of the zero mode Higgs field, $\varphi_0^{(0)}$, acquires a vacuum expectation value $\left\langle \varphi_0^{(0)} \right\rangle \equiv v$. Mass matrices are diagonalized in the standard way, and, if we only keep terms proportional to the top quark mass we obrtain the following Yukawa interaction between the mass eigenstates and the KK modes of scalar fields

$$\mathcal{L}_Y = -\sqrt{2} \frac{m_t}{v} \sum_{n=1}^{\infty} \left(\overline{t}_L \, t_R \varphi_n^{(0)} + \sum_f^{d,s,b} \overline{f}_L V_{tf}^* \, t_R \varphi_n^{(-)} + \text{h.c.} \right) \,, \tag{2.5}$$

where V_{tf} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, while $\varphi_n^{(0)}$ and $\varphi_n^{(-)}$ are the neutral and charged components of the KK scalar doublets, respectively. Notice the additional factor $\sqrt{2}$ in the coupling of the KK modes, which comes from the normalization of the zero mode in the Fourier expansion.

In the low energy limit one can integrate out the KK modes (by using the equations of motion, for instance) and obtain the following four fermion interaction (in the weak basis)

$$\mathcal{L}_{\rm eff} = \frac{(\pi R)^2}{3}(\bar{Q}_L Y_u u_R)(\bar{u}_R Y_u^\dagger Q_L)\,, \tag{2.6}$$

which can be expressed in terms of the mass eigenstates (keeping only terms proportional to m_t).

$$\mathcal{L}_{\rm eff} = \frac{(\pi R)^2}{3}\frac{m_t^2}{v^2}\left[(\bar{t}_L\, t_R)\,(\bar{t}_R\, t_L) + \sum_{f,f'}^{d,s,b}\left(\overline{f'}_L\, t_R\right)(\bar{t}_R\, f_L)\; V_{tf}V_{tf'}^*\right]\,. \tag{2.7}$$

The above Lagrangian provides, for instance, a four fermion interaction $(\bar{b}_L\, t_R)(\bar{t}_R\, b_L)$ (and also $(\bar{s}_L\, t_R)(\bar{t}_R\, s_L)$ and $(\bar{d}_L\, t_R)(\bar{t}_R\, d_L)$) which much in the spirit of ref. [30] can contribute at one loop level to the decay $Z \to b\bar{b}$ as well to B-$\bar{B}$ and K-$\bar{K}$ mixing. However, if we use the effective four-fermion interaction, the loop integral in fig. 2 is divergent and following ref. [30] one can only compute in this way the dominant logarithmic contributions. To obtain the non-logarithmic parts one should calculate the one-loop matching with the complete theory. One of the advantages of models with large extra dimensions is that they provide this full theory, which will allow us, as we will immediately show, to compute not only the logarithmic corrections but also the finite parts. In order to accomplish this, one has to maintain all KK modes as dynamical particles. Therefore, in what follows we will use the interactions given in eq. (2.5).

III. $Z \to b\bar{b}$

In the SM there are many diagrams contributing to the vertex corrections to $Z \to b\bar{b}$. In the Feynman or in the Landau gauges the dominant contribution for large m_t is captured by diagrams such as the one shown in fig. 1, with a charged would-be Goldstone boson running in the loop. In the unitary gauge these corrections originate from the longitudinal parts of the gauge boson propagators. In general there are strong cancellations among vertex diagrams (as the graph of fig. 1) and diagrams with self-energies in the external fermion legs in such a way that the dominant contribution is finite. By far the easiest way to compute these corrections is to resort to the equivalence theorem [31–34], i.e. to use the Ward identities [35–38] that relate the Z-$b\bar{b}$ vertex to the G^0-$b\bar{b}$ vertex , where G^0 denotes the would-be Goldstone boson associated to the Z gauge boson.

In the model we are considering, there are additional contributions enhanced by m_t that arise from the presence of the charged scalar KK modes, with interactions governed by eq. (2.5), which give rise to the diagram of fig. 1. If the gauge bosons also possess KK modes there will be additional diagrams, such as the one shown in fig. 1, in which the KK modes of the scalars will be replaced by the corresponding KK modes of the W-gauge bosons. Even though their contribution is formally suppressed by a factor $(m_W/m_t)^2$, we will estimate it at the end of this section. In such a case it is important to realize that the KK modes of the

charged scalars appearing inside the loop are not the would-be Goldstone bosons of the KK modes of the gauge bosons. In fact the mass of the KK modes associated to the gauge bosons is given by their fifth components[2]. This distinction becomes clear if one uses the unitary gauge for the KK modes of the gauge bosons. In this case the fifth components of the five dimensional gauge bosons are completely absorbed by the KK modes of the gauge bosons, i.e. there are not graphs containing would-be Goldstone bosons, while the KK modes of the scalars remain in the spectrum of physical particles, i.e. the diagram of fig. 1 persists.

Again, the easiest way to calculate the contribution of the scalar KK modes is to resort to the equivalence theorem for the external Z and compute the diagram of fig. 1 with the Z replaced by the $G^{(0)}$. Since the couplings of the KK modes to fermions are universal, summing all scalar contributions amounts to replacing the propagator of the SM would-be Goldstone boson by (for Euclidean momenta, which we will use in the momentum integrals after the Wick rotation)

$$\frac{1}{k_E^2} \to \frac{1}{k_E^2} + 2\sum_{n=1}^{\infty} \frac{1}{k_E^2 + n^2/R^2} = \sum_{n=-\infty}^{\infty} \frac{1}{k_E^2 + n^2/R^2} = \pi R \frac{\coth(k_E \pi R)}{k_E}, \tag{3.1}$$

where $k_E = \sqrt{k_E^2}$.

Notice the behavior of this propagator: for small k_E it reduces to the standard Goldstone propagator plus, if expanded at leading order, an additional constant which furnishes the contact interaction derived above, eq. (2.6). However, for large k_E it goes as $1/k_E$; as a result the ultraviolet (UV) behavior of this theory is worse than in the SM by one power of k_E, a fact which will eventually trigger the non-renormalizability of the theory. However, since in the large k_E limit only even powers of k_E contribute in standard QFT integrals, this worse UV behavior of the non-standard propagator does not create additional problems, as long as only one such propagator is inserted into a convergent graph. For instance, the dominant SM contribution to the $Zb\bar{b}$ vertex, fig. 1, is convergent because the integrand behaves as $1/k_E^6$; when we use the non-standard propagator this behavior will drop to $1/k_E^5$, which still leads to a convergent result.

To see how this works in detail we parametrize the effective $Z\bar{b}b$ vertex as

$$\frac{g}{c_W}\, \bar{b}\gamma^\mu (g_L P_L + g_R P_R) b Z_\mu \,, \tag{3.2}$$

where $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$ are, respectively, the left and right chirality projectors, $c_W^2 = 1 - s_W^2 = m_W^2/m_Z^2$, and

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[2]Fifth components of gauge bosons are odd under the $\mathbb{Z}_2$ symmetry, therefore they do not have zero modes. Masses for the zero-mode gauge bosons should be provided by the usual Higgs mechanism, while masses for non-zero mode gauge bosons are provided by their fifth components. Thus, only the zero mode scalars play the rôle of Goldstone bosons.

$$g_L = -\frac{1}{2} + \frac{1}{3}s_W^2 + \delta g_L^{\rm SM} + \delta g_L^{\rm NP}$$
$$g_R = \frac{1}{3}s_W^2 + \delta g_R^{\rm SM} + \delta g_R^{\rm NP}\,. \tag{3.3}$$

In the above equations the $-1/2+s_W^2/3$ and $s_W^2/3$ are the tree level contributions, $\delta g_L^{\rm SM}$ and $\delta g_L^{\rm SM}$ denote higher order corrections within the SM, whereas $\delta g_L^{\rm NP}$ and $\delta g_L^{\rm NP}$ parametrize the contributions coming from new physics. Notice that, in general, g_R only receives subdominant corrections (not proportional to the top quark mass) in both the SM and in most of new physics scenarios. In particular, the dominant SM contribution comes from the Goldstone boson diagrams running in the loop, fig. 1, and it is given by

$$\delta g_L^{\rm SM} \approx \sqrt{2}G_F m_t^4\, i \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_t^2)^2k^2} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\,. \tag{3.4}$$

Therefore, adding the KK modes we obtain

$$\delta g_L^{\rm NP} \approx \delta g_L^{\rm SM}\,(F(a)-1)\;, \tag{3.5}$$

where $a = \pi R m_t$, and

$$\begin{aligned} F(a) \;&=\; \pi R \int \frac{dk_E k_E^2 \coth(k_E \pi R)}{(k_E^2+m_t^2)^2} \Bigg/ \int \frac{dk_E k_E}{(k_E^2+m_t^2)^2} \\ &= 2a\int_0^\infty dx \frac{x^2}{(1+x^2)^2}\coth(ax) \end{aligned} \tag{3.6}$$

is the ratio of the non-standard to the standard integrals (in the Euclidean). $F(a)$ is, as expected, perfectly convergent. It can be expanded for small a, yielding

$$F(a) \approx 1 + a^2\left(-\frac{1}{3} - \frac{4}{\pi^2}\zeta'(2) - \frac{2}{3}\log(a/\pi)\right) \approx 1 + a^2\left(0.80979 - \frac{2}{3}\log(a)\right)\;, \tag{3.7}$$

where ζ' is the derivative of the Riemann zeta function. As commented before, the logarithmic contribution can be obtained easily by using the four-fermion interaction at the loop level and then cutting off the integrals at $k_E \approx 1/a$. This model, in addition to the logarithmic contribution provides also the non-logarithmic piece, and the result reported above is valid for any value of a. One important point about this result is that the additional contribution from the KK modes is always positive, a fact which will be of particular importance in the following phenomenological analysis.

A shift in the $Zb\bar{b}$ couplings gives a shift in $R_b = \Gamma_b/\Gamma_h$ (here $\Gamma_b = \Gamma(Z \to b\bar{b})$ and $\Gamma_h = \Gamma(Z \to \text{hadrons})$) given by

$$R_b = R_b^{\rm SM}\frac{1+\delta_{bV}^{\rm NP}}{1+R_b^{\rm SM}\delta_{bV}^{\rm NP}}, \tag{3.8}$$

where

$$\delta_{bV}^{\rm NP} = \frac{\delta\Gamma_b}{\Gamma_b^{\rm SM}} \approx 2\frac{g_L}{(g_L)^2+(g_R)^2}\delta g_L^{\rm NP} \approx -4.6\,\delta g_L^{\rm NP} \tag{3.9}$$

gives the relative change to Γ_b due to vertex corrections coming from new physics, $\Gamma_b = \Gamma_b^{\rm SM} + \delta\Gamma_b$. Here, quantities with superscript SM denote standard model values including complete radiative corrections. Note that non-vertex corrections are universal for all quarks and cancel in the ratio R_b.

In recent years there has been a significant controversy surrounding R_b, because for some time its measured value was more than two standard deviations away from the one predicted in the SM. However, the present experimental value is perfectly compatible with the SM [39]: $R_b^{\rm exp} = 0.2164 \pm 0.00073$, while $R_b^{\rm SM} = 0.2157 \pm 0.0002$, although the central value is still somewhat higher. Using these values together with equations (3.4), (3.5), (3.8) and (3.9), one immediately finds that $F(a) - 1 = -0.24 \pm 0.31$. However, as commented before, $F(a)$ is always larger than 1 since corrections from extra dimensions are always positive. In this case one should be especially careful when estimating confidence levels (CL) for the bounds on $F(a) - 1$. For this purpose we used the prescription of ref. [40], which provides more reliable limits than other approaches, and found the following 95% CL limit of $F(a) - 1 < 0.39$. After evaluation of the integral (3.6) the previous limit translates into an upper bound on a, $a < 0.56$, which amounts to the following lower bound on the compactification scale M_c,

$$M_c > 1\ \text{TeV}\,. \tag{3.10}$$

If only 68% CL limits are required we obtain $F(a) - 1 < 0.11$, $a < 0.26$, and $M_c > 2$ TeV. Quite interestingly, these one-loop bounds are comparable to those obtained from tree level processes [7, 25–29].

In the above discussion we have not taken into account the effects of the gauge boson KK modes because their contribution is suppressed by $(m_W/m_t)^2$. However, since $(m_W/m_t)^2 \sim 1/4$, such contributions, even though formally suppressed, could become numerically relevant and affect the obtained bounds. In addition, those contributions are present even if the scalar doublet lives in four dimensions and, as a consequence, has no KK modes. Therefore, we will provide an estimate of their size.

At energies below the compactification scale one can integrate the KK modes of the gauge bosons and obtain the following four-fermion interaction for the third generation (in the eigenvalue basis and neglecting CKM mixings)

$$\mathcal{L}_{\rm gauge} = -\frac{(\pi R)^2}{3}\frac{g^2}{2}\left(\bar{b}_L\gamma_\mu t_L\right)\left(\bar{t}_L\gamma^\mu b_L\right)\,, \tag{3.11}$$

to be compared with the contribution from scalar modes obtained from (2.7) (again neglecting CKM mixings)

$$\mathcal{L}_{\rm Ytb} = \frac{(\pi R)^2}{3}\frac{m_t^2}{v^2}\left(\bar{b}_L\, t_R\right)\left(\bar{t}_R\, b_L\right)\,. \tag{3.12}$$

As commented above and discussed in ref. [30], one can use these effective Lagrangians to obtain the leading logarithmic corrections to $Z \to b\bar{b}$. In order to achieve that we compute the divergent part of the diagram shown in fig. 2, where the symbol $\otimes$ denotes the insertion of any of these four-fermion operators. It turns out that the different Lorentz structure of the two four fermion interactions in (3.11) and (3.12) gives an additional factor -2 in the

former case. Therefore, up to non-logarithm corrections, one can include the effect of the exchange of KK modes of gauge bosons by multiplying the effect of the scalar KK modes by a factor $1 + 2(m_W/m_t)^2$, which gives a non-negligible correction. Notice that due to the positive relative sign, inclusion of this correction would lead to a 20% improvement in the bound on M_c. Moreover, this correction will remain even in the absence of scalar KK modes; in that case one can still place a bound on M_c of about 0.7 TeV.

IV. BOX CONTRIBUTIONS TO K-$\bar{K}$ AND B-$\bar{B}$ MIXING AND THE DIVERGENCES

In the SM, the mixing between the B^0 meson and its anti-particle is also completely dominated by the top-quark contribution. The explicit m_t dependence of the box diagrams is given by the loop function [24]

$$S(x_t)_{\rm SM} = \frac{x_t}{4}\left[1 + \frac{9}{1-x_t} - \frac{6}{(1-x_t)^2} - \frac{6x_t^2\log(x_t)}{(1-x_t)^3}\right], \qquad x_t \equiv \frac{m_t^2}{M_W^2}, \tag{4.1}$$

which contains the hard m_t^2 term, i.e. $x_t/4$, induced by the longitudinal W exchanges. The same function controls the top–quark contribution to the K-$\bar{K}$ mixing parameter ε_K. The measured top-quark mass, $m_t = 175$ GeV, implies $S(x_t)_{\rm SM} \sim 2.5$.

The KK modes of the charged components of the doublet also contribute to this box diagram. The total dominant contribution, SM plus KK modes, can be obtained by substituting the propagator (3.1) in the box diagram, fig. 3. However, as discussed in the previous section, the modified propagator behaves as $1/k_E$ for large k_E, and therefore, the insertion of two propagators of this type turns this modified diagram into UV divergent. On the other hand, the insertion of only one modified propagator still yields a finite result.

We write the correction to $S(x_t)$ as

$$S(x_t) = S(x_t)_{\rm SM} + \delta S(x_t)\,, \qquad \delta S(x_t) = \frac{x_t}{4}\,(G(a) - 1) \quad , \tag{4.2}$$

where the function $G(a)$ is again the ratio of the non-standard to standard box integrals[3]

$$\begin{aligned} G(a) &= (\pi R)^2 \int \frac{dk_E k_E^3 \coth^2(k_E \pi R)}{(k_E^2 + m_t^2)^2} \Big/ \int \frac{dk_E k_E}{(k_E^2 + m_t^2)^2} \\ &= 2a^2 \int_0^\infty dx \frac{x^3}{(1+x^2)^2} \coth^2(ax) \end{aligned} \tag{4.3}$$

[3]Notice that, even though the SM box integral is given exactly by the same expression as that of the SM vertex integral in the previous section, their original structures are rather different . In particular, the box diagram contains two scalar propagators whereas the vertex diagram only contains one.

which is clearly divergent for $x \to \infty$. In order to estimate this integral, we split $\coth(ax) \to \frac{1}{ax}(1 + \underbrace{ax\coth(ax) - 1}_{})$ and rewrite $G(a)$ as

$$\begin{aligned} G(a) &= 2\int_0^\infty dx \frac{x}{(1+x^2)^2}\left(1 + 2\left(ax\coth(ax) - 1\right) + \left(ax\coth(ax) - 1\right)^2\right) \\ &= 1 + 2\left(F(a) - 1\right) + 2\int_0^\infty dx \frac{x}{(1+x^2)^2}\left(ax\coth(ax) - 1\right)^2 . \end{aligned} \tag{4.4}$$

The divergence is contained in the last term. To evaluate it we cut off the integral at $x \approx n_s/a$, where n_s is related to the scale at which new physics enters to regulate the five dimensional theory. In particular, $M_s \sim n_s M_c$ and $n_s \gg 1$. Then, after a change of variable $y = ax$ the last term can be re-written as

$$\begin{aligned} 2a^2\int_0^{n_s} dy \frac{y}{(a^2+y^2)^2}\left(y\coth(y) - 1\right)^2 &\approx 2a^2\int_0^{n_s} dy \frac{1}{y^3}\left(y\coth(y) - 1\right)^2 \\ &\approx 2a^2\left(-1.38136 + \log(n_s)\right) , \end{aligned} \tag{4.5}$$

where in the second expression we have assumed $a \ll 1$, and, in addition, in the last expression we have also taken $n_s \gg 1$. Combining this result with (4.4) and (3.7) we obtain

$$G(a) \approx 1 + a^2\left(-1.14314 - \frac{4}{3}\log(a) + 2\log(n_s)\right) . \tag{4.6}$$

We have checked that the coefficients of the two logarithms, $\log(a)$ and $\log(n_s)$, can also be obtained by performing first the convergent momentum integrals and subsequently truncating the divergent double series at $\sim n_s$. However, this latter method is technically far more complicated than the one presented here.

For moderate values of $a \sim 0.2$ and $n_s \sim 10$ the new physics correction is only about 0.2. For more extreme values (for instance $a \sim 0.6$ and $n_s \sim 100$), we find that the contribution from extra dimensions to the function $G(a)$ is about 3. Notice also that, as discussed at the end of sec. III, the presence of diagrams with gauge boson KK modes could modify the bounds on M_c by a factor of about 20%. However, given the uncertainty in the calculation of the box diagrams due to the dependence on the scale M_s, estimating such effects seems superfluous. The important point, however, is that the contribution from extra dimensions to the function $S(x_t)$ is always positive.

We can use the measured B_d^0-$\bar{B}_d^0$ mixing to infer the experimental value of $S(x_t)$ and, therefore, to set a limit on the $\delta S(x_t)$ contribution. The explicit dependence on the quark–mixing parameters can be resolved by combining the constraints from $\Delta M_{B_d^0}$, ε_K, and $\Gamma(b \to u)/\Gamma(b \to c)$. In ref. [30] a complete analysis of the allowed values for $S(x_t)$ was performed by varying all parameters in their allowed regions. The final outcome of such an analysis is that $S(x_t)$ could take values within a rather large interval, namely

$$1 < S(x_t) < 10 . \tag{4.7}$$

Since most of the errors come from uncertainties in theoretical calculations, it is rather difficult to assign confidence levels to the bounds quoted above. The lower limit is very

stable under changes of parameters, while the upper limit could be modified by a factor of 2 by simply doubling some of the errors.

Given that the standard model value for $S(x_t)$ is $S(x_t)_{\rm SM} = 2.5$, *positive* contributions can be comfortably accommodated, whereas negative contributions are more constrained. As we have seen, extra dimensions result in *positive* contributions to $S(x_t)$; in fact one can obtain values that could approach the upper limit of $S(x_t)$ only for rather small values of the compactification scale M_c and large values of the scale of new physics, M_s. It seems therefore that, at present, the above bounds do not provide good limits on M_c. On the other hand, if future experiments combined with theoretical improvements were to furnish a value for $S(x_t)$ exceeding that of the SM, our analysis shows that such a discrepancy could easily be accommodated in models with large extra dimensions.

V. CONCLUSIONS

We have studied, at the one loop level, the minimal extension of the SM with one extra dimension compactified in $S^1/\mathbb{Z}_2$. Fermions live in 4 dimensions, while gauge bosons and the scalar doublet live in 5 dimensions and therefore give rise to a tower of KK modes. In the case of a single extra dimension the contribution of the infinite tower of KK modes lead to finite tree level predictions. We have investigated whether this feature persists at the one loop level, by considering two amplitudes which are enhanced by the top-quark mass, namely $Z \to b\bar{b}$ and B-$\bar{B}$ mixing.

The infinite tower of KK modes enters in the calculation of $Z \to b\bar{b}$ by modifying the propagator of the charged scalars running in the vertex diagram. This can be effectively taken into account by using a modified propagator for the scalars which for large k behaves as $1/k$, instead of the canonical behavior of $1/k^2$. In spite of that the effect is finite and calculable. The result, when compared with precise experimental data on $R_b = \Gamma_b/\Gamma_h$, is used to place stringent limits on the compactification scale, M_c, $M_c > 1$ TeV at the 95% CL, which are comparable to the bounds obtained from tree-level processes.

The box diagrams contributing to B-$\bar{B}$ and K-$\bar{K}$ mixings contain two propagators of KK modes. The double sum over KK modes amounts to the replacement of both propagators by the aforementioned softer ones, a fact which increases the UV behavior of the diagram by two powers, and renders it divergent. Thus, due to such contributions the theory becomes non-renormalizable already at the one-loop level. To estimate their size one has to assume that the model is embedded in a more complete theory which would provide an effective cutoff at scales larger than M_c. In practice, this can be realized either by cutting off the infinite integrals at momenta of order of M_s, the scale where new physics enters to regularize the five dimensional theory, or by truncating the sum of KK modes at some value of n_s, $n_s \sim M_s/M_c$, with n_s expected to be order ~ 100 or less. This way we can estimate the correction induced by the extra dimension to the function $S(x_t)$ which parametrizes the short distance physics in B-$\bar{B}$ and K-$\bar{K}$ mixings. A phenomenological analysis shows that $1 < S(x_t) < 10$, while the SM value is $S(x_t) = 2.5$. This suggests that moderate *positive* extra contributions to $S(x_t)$ are still allowed. Since within the model we consider the contributions to $S(x_t)$ from KK modes is always positive and moderate in size, no interesting bounds can be obtained form this process. However, if in the future a value of $S(x_t)$ larger than the SM value is

found, extra dimensions could easily accommodate it.

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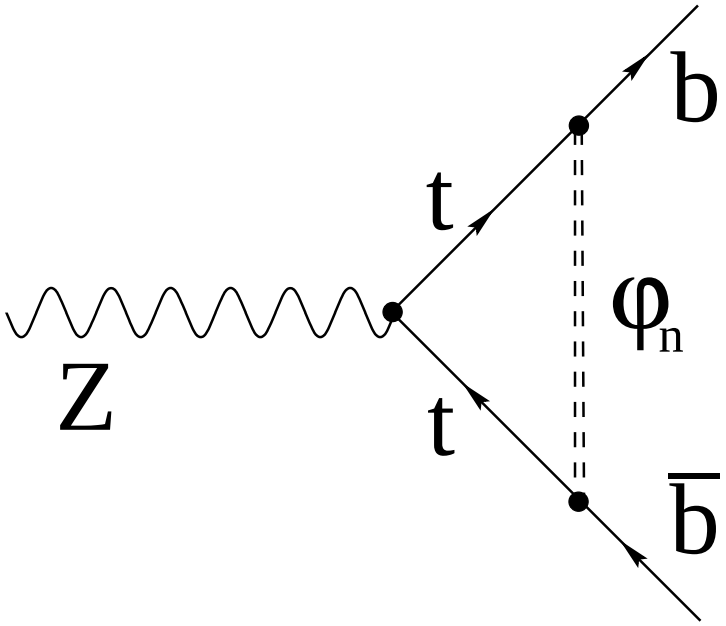

FIG. 1. Diagram contributing to $Z \to b\bar{b}$ if the scalar doublet lives in five dimensions. The tower of KK modes of charged scalars is represented by the dashed double line.

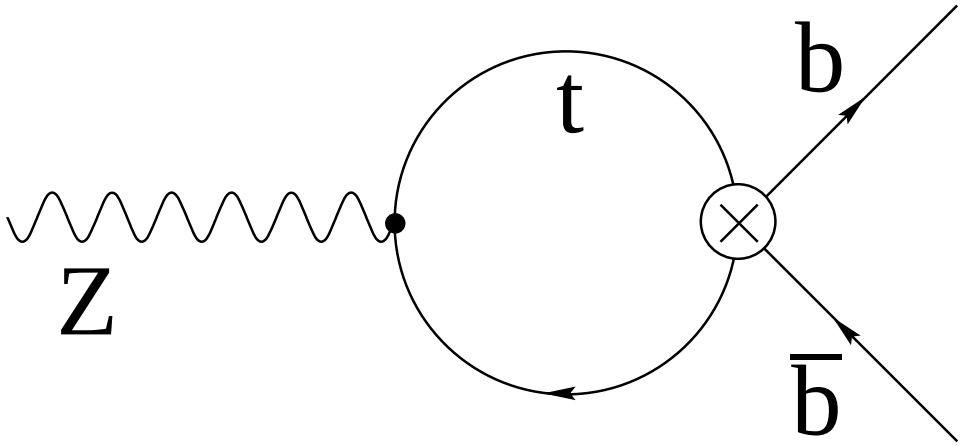

FIG. 2. Effective field theory diagram used in the computation of the leading logarithmic corrections induced by four fermion interactions.

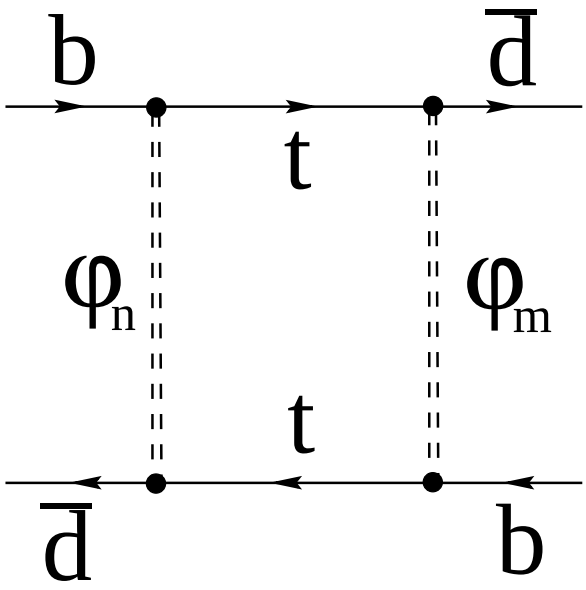

FIG. 3. Box diagram contributing to B-$\bar{B}$ and K-$\bar{K}$ mixings. The tower of KK modes is represented by the dashed double lines.