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MAJORON EFFECTS IN RARE KAON DECAYS

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Abstract

We analyze, in the framework of the recently introduced doublet Majoron model, the contribution from the emission of a pair of light scalars to the decay $K^- \to \pi^- +$ *nothing*. We find that, for reasonable choices of the parameters, the new scalar contribution may be as large as one additional neutrino—antineutrino mode and provide a substantial modification of the pion spectrum. The effect may be a few times larger in the triplet Majoron model.

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1 Introduction

In a recent paper [1] we addressed the problem of the apparent solar neutrino deficit [2] in the context of a new majoron model in which total lepton number is broken spontaneously by a Higgs *doublet.* By assuming a standard fermion field content, an immediate consequence is that Majorana mass entries for the neutrinos are generated radiatively. This leads to a quite simple extension of the standard electroweak theory, where a solution of the solar neutrino problem through matter enhanced neutrino oscillations (Michejev-Smirnov-Wolfenstein mechanism [3]) is naturally implemented. Indeed, the combined presence of the strong astrophysical bound [4] on the lepton breaking vacuum expectation value (VEV) and the radiative origin of the neutrino mass matrix lead to neutrino masses in the regime required by the MSW mechanism. In addition, the peculiar structure of the neutrino mass matrix (the same as in the original Zee model [5]) provides a simple description of oscillations in terms of only one mixing angle, thus leading to a predictive model for matter enhanced neutrino oscillations. Only four extra scalar fields are present in addition to the content of the standard model (SM) : the majoron J and its *light* neutral partner ρ_L (always present in non-singlet majoron models [6]) and two singly charged scalars whose mass ranges are quite constrained by present phenomenology. The presence of these new scalar degrees of freedom provides interesting phenomenology for the model and a way of testing neutrino oscillation parameters in experiments not directly related to solar neutrino physics. Indeed, the combined effects of the presence of the majoron (and therefore the small bound of the order of 10 KeV on the related VEV) and the required consistency with the MSW explanation for the solar neutrino depletion, bound from above the masses of the charged scalars in such a way that the loop induced $\mu \rightarrow e\,\gamma$ decay is, for a wide range of the paramenters, within *two* orders of magnitude from the present experimental limit. The two charged scalars may provide also important corrections to the Veltman ρ parameter and the W and Z mass interdependence. The constraints coming from the present data are quite stringent and were discussed in ref. [1].

If the phenomenology related to the coupling of the majoron (and ρ_L) with the neutrinos is here largely suppressed due to the radiative origin of the coupling (the interaction with other matter, through the mixing with the ordinary Higgs doublet, is suppressed as well due to the small VEV), quite interesting turns out to be the analysis of the majoron phenomenology related to the gauge couplings. In particular, the fact that the majoron belongs to an $SU(2)$ doublet implies that the scalar contribution to the width of the Z, through $Z \rightarrow J\ \rho_L$, is equivalent in our model to 1/2 a neutrino-antineutrino mode. This contribution is four times smaller than the analogous one in the triplet model of Gelmini and Roncadelli [7], thus providing a way of discriminating the two models through neutrino counting at the new $e^+ - e^-$ colliders. Indeed, neutrino counting at LEP is expected to reach a sensitivity of $0.3 - 0.2$ neutrino generations [8]. In addition, it is worthwhile to remark that $Z \rightarrow J\ \rho_L$ represents the *only* non-standard contribution to neutrino-like decays present in the model; this may be relevant when comparing with the prediction for the $Z-$width in the *supersymmetric* majoron model [9] (which shares the feature of a doublet majoron), where light neutralinos and/or sneutrinos may provide indeed further contributions.

Among other decays of gauge bosons to scalars which may lead to an experimental signature of the model, particularly interesting is $W^+ \rightarrow h^+\ J$, where h^+ is the lighter of the two charged scalar present in the doublet majoron model, which allows for a test of the charged scalar mass up to the W scale [10]. The presence of ρ_L, a light neutral scalar which in our model interacts very weakly with matter, may also have various cosmological and astrophysical implications [11].

In this paper we are extending the analysis of the phenomenological implications of the majoron gauge couplings to a rare kaon decay which may provide in the near future a sensitive test for new physics, namely the decay $K^- \to \pi^- + \textit{nothing}$ [12,13], where *nothing* stands in SM for a neutrino-antineutrino pair. The branching ratio predicted in SM, by summing over three neutrino generations, varies between 3×10^{-11} and 6×10^{-10} [14], where the constraints on the mixing angles and top mass coming from the observed related decay $K_L^0 \to \mu^+\mu^-$ are taken into account. This is about three orders of magnitude below the present experimental bound of 1.4×10^{-7} [15].

Various authors have discussed the implications of the presence of "new" physics for this decay. In particular, it has been argued that the presence of a fourth generation of fermions, may allow for a enhancement of more than one order of magnitude over the SM prediction [16]. The presence of supersymmetry may show up in a twofold way: by contributing to the standard $\nu\bar{\nu}$ channel through the virtual exchange of supersymmetric particles or by directly adding new channels through the emission of light neutral weak interacting superpartners. The first case has been studied in ref. [17] where it is shown that virtual SUSY loops may at most give a contribution of the same order as the SM one. Analogous results are obtained for the one-loop emission of a pair of photinos [18,19,20], higgsinos [18], sneutrinos [21]. One remarkable exception is given by the possible presence, in a class of low-energy minimal N=1 supergravity models, of flavour non-diagonal photino-quark-squark vertices. In this case there may exist a "tree" level contribution to the decay $K^- \to \pi^-\tilde{\gamma}\tilde{\gamma}$ which, for squark masses below m_W, leads to a decay rate close to the present experimental limit [17,22]. We have however to remark that such a small values for the neutralino masses, required to render the process kinematically allowed, are disfavoured in most of the models so far proposed.

From this short revue of the $K^- \rightarrow \pi^- + \textit{nothing}$ physics, it is clear that the possibility that in the very near future the experimental bound may be improved by about *three* orders of magnitude [23], bringing it at the SM treshold, makes further investigation of this process, in view of the search for signals of new physics, very interesting.

In non-singlet majoron models the presence of the massless majoron and its very light neutral partner ρ_L ($m_{\rho_L} < 10\ KeV$) provides extra channels for the aforemontioned decay, namely JJ, $\rho_L\rho_L$ and $\rho_L J$. Whereas the latter turns out to be generally not relevant, the first two modes may give a contribution comparable to one neutrino generation and provide a substantial modification of the energy spectrum of the emitted pion. In addition to the graphs which take advantage of the majoron (and ρ_L) gauge couplings there is a further contribution provided by the effective flavour changing vertex of the "ordinary" neutral Higgs. Its coupling with the light scalars depends on a ratio of coupling constants in the Higgs potential and may therefore vary in principle over a wide range. Indeed, for a light Higgs this contribution could be by far the dominant one. However, the study of the effect of the renormalization of the Higgs potential on the VEV's hierarchy present in the model allows us to give a definite estimate of the standard Higgs contribution, which turns out to be of the same order of the "gauge" ones. It is also to be remarked that there is an ongoing controversy about the perturbative calculation of the effective $qq'H$ vertex. We will comment on that in sects. 3 and 5.

The extension of these results to the Gelmini-Roncadelli model is straightforward and we find that, once analogous values for the masses of the charged scalars running in the loop are considered, the triplet majoron model gives contributions about four times higher, analogously to what happens for the Z^0 width. This is a consequence of the different hypercharge associated with the majoron, which appears whenever gauge couplings are involved.

Although the potential increment in the $K^- \rightarrow \pi^- +$ *nothing* branching ratio overlaps with other possible sources, we think that the combination of the Brookhaven experiment with the information coming from high-energy experiments (as neutrino counting at LEP and SLC), may provide some insight on the presence of physics beyond the standard model. In addition, if in the future it is possible to obtain information on the shape of the pion spectrum, this could discriminate among different interpretations as well. We consider therefore this decay as a possible interesting test for the physics related to the spontaneous breaking of lepton number.

The paper is organized as follows. In sect. 2 we report the relevant majoron couplings in the doublet and triplet models. In sect. 3 we derive the effective Lagrangean for the $K^- \rightarrow \pi^- +$ *nothing* decay in majoron models. The resulting pion energy distributions and relative branching ratios are obtained in sect. 4. In sect. 5 we discuss the numerical results and present some final considerations.

2 Majoron Couplings

In order to compute the contributions to the $K^- \rightarrow \pi^- +$ *nothing* decay related to the emission of the two "light" scalars, we recall here the couplings of the Majoron and ρ_L to the gauge bosons and the "standard" neutral Higgs in the doublet majoron model. Unless otherwise stated we follow the notation of ref. [1].

Due to the derivative nature of the coupling, the Z gauge boson couples to both the majoron and ρ_L :

$$\mathcal{L}_{DZ} \simeq -\frac{g}{2\cos\theta_W} Z^\mu J \overleftrightarrow{\partial}_\mu \rho_L \qquad (1)$$

where we neglect the small mixings $O(v/u)$, $u \simeq 174\ GeV$ being the standard electroweak breaking VEV. On the other hand, the derivative couplings of the W are given by

$$\mathcal{L}_{DW} \simeq g\ W^{\mu +} i(\frac{1}{2}\phi^- \overleftrightarrow{\partial_\mu} \rho_L + i\frac{1}{2}\phi^- \overleftrightarrow{\partial_\mu} J)\ +\ h.c. \tag{2}$$

where ϕ^- is the charged component of the new doublet and is expressed in terms of the physical states as $\phi^- \simeq \cos\theta_{12} h_2^- + \sin\theta_{12} h_1^-$. We need also the couplings of J and ρ_L to two W's,

$$\mathcal{L}_{WW} \simeq \frac{g^2}{4} W_\mu^+ W^{\mu -}(J^2 + \rho_L^2) \tag{3}$$

Finally, we will consider the diagrams generated by the effective flavour changing vertex of the "standard" Higgs boson (ρ_H), whose coupling with the Majoron and ρ_L depend on the details of the Higgs potential. For this purpose and for further reference we report here the complete expression of the potential from ref. [1]

$$\begin{aligned} V\ &= \lambda_1(\varphi^\dagger\varphi - u^2)^2 + \lambda_2(\phi^\dagger\phi - v^2)^2 \\ &+\lambda_3(\varphi^\dagger\varphi - u^2)(\phi^\dagger\phi - v^2) \\ &+\lambda_4((\varphi^\dagger\varphi)(\phi^\dagger\phi) - (\varphi^\dagger\phi)(\phi^\dagger\varphi)) \\ &+\lambda_5 \mid \mu h^+ - \varphi^T i\tau_2\phi \mid^2 + \lambda_6 \mid \mu h^+ + \varphi^T i\tau_2\phi \mid^2 \\ &+\lambda_7 \mid h^+ \mid^4 + \lambda_8 \mid h^+ \mid^2 \mid \phi \mid^2 + \lambda_9 \mid h^+ \mid^2 \mid \varphi \mid^2 \end{aligned} \tag{4}$$

In eq. (4) φ and ϕ are respectively the standard and new Higgs doublet, whereas h^+ is a charged scalar singlet. The extra doublet and the singlet each carry two units of lepton number. We recall that hermiticity requires all the λ_i's to be real and the dimensional coupling μ may be taken, without loss of generality, real by absorbing its phase in the definition of the singlet h^+. For $\lambda_i > 0$, $i \neq 3$, and $|\lambda_3| < 2\sqrt{\lambda_1\lambda_2}$ the potential is semi-positive definite ($V \geq 0$) and assumes its minimun on the broken vacuum. The coupling we are interested in is readily extracted from eq. (4):

$$\mathcal{L}_H \simeq -\frac{1}{\sqrt{2}}\lambda_3 u(J^2 + \rho_L^2)\rho_H \tag{5}$$

It is important to notice that all the gauge couplings of the Majoron and ρ_L in the doublet Majoron model differ, with respect to the couplings in the triplet model, only by factors related to the different quantum numbers of the multiplets. It is therefore straightforward to rewrite the couplings in eqs. (1)–(3) for the GR model:

$$\mathcal{L}_{DZ}^{(T)} = 2\mathcal{L}_{DZ} \ , \tag{6}$$

$$\mathcal{L}_{DW}^{(T)} = \sqrt{2}\mathcal{L}_{DW} \ , \tag{7}$$

and

$$\mathcal{L}_{WW}^{(T)} = 2\mathcal{L}_{WW} \tag{8}$$

where in eq. (7) the charged component ϕ^- of the Higgs doublet has to be replaced by the singly charged component ω^- of the triplet Higgs. The coupling with the standard Higgs is analogous to the one in eq. (5). We will consider however in more detail the question of the evaluation of the Higgs couplings in the two models in sect. 5. For the time being, it is enough to remark that from eqs. (6)–(8) we expect the contributions to $K^- \to \pi^- +$ *nothing* in the Gelmini-Roncadelli model to be generally larger than in the doublet model (by about a factor 4 if one considers only the "gauge" contributions and analogous masses in the loops).

3 Effective Lagrangean for $d\ \bar{s} \to$ *nothing* in Majoron Models

While the amplitude for $d\ \bar{s} \to \nu\ \bar{\nu}$ has been computed by different authors [12,13,18], we need here to calculate the effective lagrangean for the transitions $d\ \bar{s} \to J\ J$, $\rho_L\ \rho_L$ and $J\ \rho_L$. These processes occur at the one loop level through four classes of diagrams. The first class is given by the Z-exchange graphs, which take advantage of the induced flavour changing vertex $d\bar{s}Z$ (fig. 1). As the coupling of the gauge boson Z to scalars is derivative, these diagrams only contribute to $d\ \bar{s} \to \rho_L\ J$. The second class uses the gauge coupling of eq. (3). The diagram is depicted in Fig. 2, and only contributes to $d\ \bar{s} \to J\ J$, $\rho_L\ \rho_L$. The third one consists of box diagrams where the physical charged scalars run in the loop (fig. 3). These diagrams give a contribution to all three scalar channels. We have neglected diagrams where charged higgses are coupled to the fermions because they are largely suppressed by factors v/u. Finally, the fourth contribution is due to the standard-Higgs exchange (fig. 4) through the coupling of eq. (4), similarly to the Z exchange. This graph only gives a contribution to the decay in identical scalars and its size depends critically on the parameters of the Higgs potential.

We can conveniently describe the amplitudes for $d\ \bar{s} \to$ *nothing* in our model by the following effective lagrangean

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} 4\chi \left[\ \bar{s}\gamma^\mu L d \left(\sum_{i=1}^{3} \tilde{D}_i \bar{\nu}_i \gamma_\mu L \nu_i - \tilde{A} J \overleftrightarrow{\partial}_\mu \rho_L\right) + \frac{1}{2}\bar{s}(m_s L + m_d R) d\ \tilde{B}\ (J^2 + \rho_L^2)\right] \quad (9)$$

where L and R are the left and right-handed projector operators respectively and following the notation of ref. [13] we define

$$\chi \equiv \frac{g^2}{(4\pi)^2} = \frac{\alpha}{4\pi \sin^2\theta_W} \quad (10)$$

θ_W being the Weinberg angle.

All the coefficients in $\mathcal{L}_{eff}$ represent a sum over the contributions of the different quarks weighted by the elements of the Kobayashi-Maskawa (KM) matrix. Using the unitarity of the mixing matrix, the various coefficients appearing in eq. (9) may be generally written as

$$\tilde{A}(\{x_j\},\{w_k\}) = \sum_{j=c,t} U^*_{js} U_{jd}[\cos^2\theta_{12}\bar{A}(x_j,w_2) + \sin^2\theta_{12}\bar{A}(x_j,w_1)] \tag{11}$$

and analogously for $\tilde{B}$. The form factors $\bar{A}(x_j,w_k)$ and $\bar{B}(x_j,w_k)$ represent the difference between the contribution of the quark j and the quark u (this leads to the cancellation of the divergences independent on the quark masses). The variables x_j and w_k are defined as the square of the masses of u-quarks and charged scalars respectively in units of the W mass, namely $x_j \equiv m_j^2/m_W^2$ and $w_k \equiv m_{h_k}^2/m_W^2$. In the case of the triplet model, where there is only one singly charged scalar, or for those contributions which do not depend on the charged scalar mass, the two terms in eq. (11) reduce to one. In the next subsections we will evaluate the form factors $\bar{A}$ and $\bar{B}$ in the unitary gauge. As expected, the ultraviolet divergences proportional to the quark masses cancel when all the contributions are summed.

3.1 Z exchange diagrams

The one loop diagrams contributing to the effective coupling $\bar{s}\ d\ Z$, in the unitary gauge, are depicted in Fig. 1. We can write the vertex as

$$\Gamma^\mu_Z = \frac{1}{(4\pi)^2}\frac{g^3}{\cos\theta_W}\sum_{j=1}^{3} U^*_{js}U_{jd}\Gamma(x_j)\bar{s}\gamma^\mu L d \tag{12}$$

where the evaluation of the diagrams in Fig. 1 gives

$$\Gamma(x_j) = \frac{x_j}{8}\left(D_\epsilon + \frac{1}{2} + \ln x_j + \frac{3}{1-x_j} + \frac{3}{(1-x_j)^2}\ln x_j\right) \tag{13}$$

The term D_ϵ is defined as follows

$$D_\epsilon \equiv \frac{2}{n-4} + \gamma_E - \ln(4\pi) + \ln(\frac{m_W^2}{\mu^2}) \tag{14}$$

where $4 - n = \epsilon$ represents the usual pole of dimensional regularization, γ_E is the Euler's constant and μ is the mass parameter introduced by the dimensional regularization ('t Hooft mass).

From the effective vertex in eq. (12) and the coupling of the Z to J and ρ_L in eq. (1), by comparing with eq. (9) one obtains

$$\bar{A}_Z(x_j) \equiv \Gamma(x_j). \tag{15}$$

3.2 The "seagull" diagram

Through the quartic gauge coupling of eq. (3) we obtain the graph of Fig. 2 ("seagull" diagram). This diagram contributes to the effective lagrangean for $d\bar{s} \to \rho_L \rho_L$ and $d\bar{s} \to JJ$, thus giving a contribution to $\tilde{B}$. The evaluation in the unitary gauge gives

$$\tilde{B}_{WW} = \frac{3x_j}{8}\left(D_\epsilon - \frac{5}{6} + \ln x_j - \frac{2}{(1-x_j)^2} - \frac{1+x_j}{(1-x_j)^3}\ln x_j\right) \tag{16}$$

Again this term contains a divergence proportional to the internal quark masses which will cancel in the sum.

3.3 Box diagrams

The consideration of the diagrams in fig. 3 is necessary to insure the gauge invariance of the calculation and the final cancellation of the ultraviolet divergences. The sum of the direct and crossed diagrams in the case of JJ or $\rho_L\rho_L$ is equivalent to the symmetrization necessary for identical bosons. This gives origin to the same structure of the amplitude obtained previously for these modes. In the $\rho_L J$ case, a minus sign due to the couplings arise between the "direct" and "crossed" contributions, thus selecting the component of the amplitude antisymmetric with respect of the momenta of the final scalars. This leads to an effective derivative coupling to the scalars in analogy with the Z-exchange diagrams. The computation of the contributions to the two form factors gives

$$\bar{B}_{box} = \frac{x_j}{8}\left[-3(D_\epsilon - \frac{5}{6} + \ln x_j) + \frac{w_k}{w_k - x_j} + \frac{3w_k - 2x_j}{(w_k - x_j)^2} w_k \ln\frac{x_j}{w_k}\right] \tag{17}$$

and

$$\bar{A}_{box} = \frac{x_j}{8}\left[-(D_\epsilon + \frac{1}{2} + \ln x_j) + 2 + \frac{w_k}{w_k - x_j} + \frac{w_k^2}{(w_k - x_j)^2}\ln\frac{x_j}{w_k} - \frac{6}{x_j - w_k}\left(\frac{x_j \ln x_j}{1 - x_j} - \frac{w_k \ln w_k}{1 - w_k}\right)\right] \tag{18}$$

respectively. We remark that in eqs. (17) and (18) all the terms independent on the quark masses x_j do not appear because of the GIM cancellation (we take $x_1 = 0$). It is also wortwhile to notice that in the limit $w_k \to \infty$ these contributions do not vanish, but increase with $\ln w_k$.

3.4 Higgs exchange diagrams

The short distance contributions to the flavour changing transition $s \to d\ \rho_H$ have been computed by different authors [24,25]. They find* [24]

$$\mathcal{L}_{sdH} = \frac{g^2}{(4\pi)^2}\frac{g}{m_W}\sum_{i=1}^{3} U^*_{js}U_{jd}\ \frac{3}{8}x_j\ \bar{s}(m_s L + m_d R)d\ \rho_H \tag{19}$$

Using this effective vertex and the coupling in eq. (4), we can write the contribution to $\bar{B}$ induced by the diagram in fig. 4 as

$$\bar{B}_H = \frac{3}{2}\frac{\lambda_3}{g^2}\frac{m_W^2}{m_{\rho_H}^2}x_j = \frac{3\lambda_3}{16\lambda_1}x_j \tag{20}$$

where in the last expression we have used the relation $m^2_{\rho_H} = 4\lambda_1 u^2$. We see that the size (and sign) of this contribution is critically model dependent.

3.5 The coefficients $\bar{A}$, $\bar{B}$ and $\bar{D}$

From the results of the previous subsections, we can now obtain the final expressions for the form factors $\bar{A}$ and $\bar{B}$. It is immediate to check that all the divergent terms, as well as the dependence on the t'Hooft parameter μ, cancel out, as it must be on the grounds of renormalizability of the model. We therefore obtain for the function $\bar{A}$

$$\bar{A} = \bar{A}_Z + \bar{A}_{box} = \frac{x_j}{8}\left[3\left(\frac{1}{1-x_j} + \frac{1}{(1-x_j)^2}\ln x_j\right) + 2\right.$$

*There is an ongoing controversy on the reliability of the existent short distance evaluations of the flavour changing effective Higgs vertex. The authors of ref. [26] claim that on general grounds the transition amplitude in eq. (19) should be further suppressed by a factor $O(m^2_{s,d}/m^2_W)$ [27] and that this substantial discrepancy is a consequence of a failure to incorporate correctly scale invariance in the perturbative calculation. A detailed analysis is currently unavailable.

$$+\frac{w_k}{w_k - x_j} + \frac{w_k^2}{(w_k - x_j)^2} \ln\frac{x_j}{w_k} - \frac{6}{x_j - w_k}\left(\frac{x_j \ln x_j}{1 - x_j} - \frac{w_k \ln w_k}{1 - w_k}\right)\Bigg] \tag{21}$$

which in the limit of light internal quarks reduces to

$$\bar{A}(x_j \ll 1, w_k) = \frac{x_j}{8}\left(6 + 4\ln x_j - \frac{7 - w_k}{1 - w_k}\ln w_k\right) \tag{22}$$

Analogously, we obtain for the function $\bar{B}$

$$\bar{B} = \bar{B}_{WW} + \bar{B}_{box} + \bar{B}_H = \frac{x_j}{8}\Bigg[-\frac{6}{(1 - x_j)^2} - 3\frac{1 + x_j}{(1 - x_j)^3}\ln x_j$$

$$+\frac{w_k}{w_k - x_j} + \frac{3w_k - 2x_j}{(w_k - x_j)^2} w_k \ln\frac{x_j}{w_k} + \frac{3}{2}\frac{\lambda_3}{\lambda_1}\Bigg] \tag{23}$$

and

$$\bar{B}(x_j \ll 1, w_k) = -\frac{x_j}{8}\left(5 + 3\ln w_k - \frac{3}{2}\frac{\lambda_3}{\lambda_1}\right) \tag{24}$$

It is important to notice that in this limit the dependence on $\ln x_j$ in $\bar{B}$ cancels exactly.

For the purpose of comparison with the amplitude for the standard decay in two neutrinos we report the expression of the form factor $\bar{D}$ from ref. [13]

$$\bar{D} = \frac{x_j}{8}\Bigg[3\left(\frac{1}{1 - x_j} + \frac{1}{(1 - x_j)^2}\ln x_j\right) + 2 + \ln x_j$$

$$-\frac{9}{(x_j - 1)(y_i - 1)} + \frac{1}{y_i - x_j}\left(\left(\frac{x_j - 4}{x_j - 1}\right)^2 x_j \ln x_j - \left(\frac{y_i - 4}{y_i - 1}\right)^2 y_i \ln y_i\right)\Bigg] \tag{25}$$

where $y_i = (m_{L_i}/m_W)^2$, and $L_i = e, \mu, \tau$ are the masses of the charged leptons runing in the box diagrams. In the limit of $y_i \ll x_j$ eq. (25) reduces to

$$\bar{D}(x_j, y_i = 0) = \frac{x_j}{4}\left(\frac{x_j + 2}{x_j - 1} + 3\frac{x_j - 2}{(x_j - 1)^2}\ln x_j\right) \tag{26}$$

Finally, for small quark masses we obtain

$$\bar{D}(x_j \ll 1, y_i = 0) = -\frac{x_j}{2}(1 + 3\ln x_j) \tag{27}$$

4 Pion Spectrum and Branching Ratios

From the effective lagrangean of eq. (9), we can easily obtain the physical amplitudes for the different modes of the decay of the charged kaon. For the decay to $\rho_L J$ we obtain

$$T(K^- \to \pi^- \rho_L J) = -i\frac{G_F}{\sqrt{2}} 2\chi \tilde{A} \left\langle \pi^-(p_\pi) \right| \bar{s}\gamma_\mu d \left| K^-(p_K) \right\rangle (p_1 - p_2)^\mu$$

$$= -i\frac{G_F}{\sqrt{2}} 2\chi \tilde{A} \left[f_+(q^2)\, t_\mu + f_-(q^2)\, q_\mu \right] (p_1 - p_2)^\mu \qquad (28)$$

where $t \equiv p_K + p_\pi$, $q \equiv p_K - p_\pi$ and p_1 and p_2 are the momenta of J and ρ_L respectively. The form factors $f_+(q^2)$ and $f_-(q^2)$ are known from $K \to \pi\mu\bar{\nu}$ [15].

The amplitudes for the decays to JJ and $\rho_L\rho_L$ are instead given by

$$T(K^- \to \pi^- JJ) = T(K^- \to \pi^- \rho_L \rho_L) = \frac{m_s + m_d}{m_s - m_d} \frac{G_F}{\sqrt{2}} 2\chi \tilde{B}\, (m_K^2 - m_\pi^2) f_0(q^2) \qquad (29)$$

where the form factor $f_0(q^2)$ is defined as

$$f_0(q^2) \equiv f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2) \qquad (30)$$

As $m_d \ll m_s$, we can replace, with good approximation, the ratio between the masses of the external quarks in eq. (29) by a factor one.

From these amplitudes we obtain, in the rest frame of the kaon, the following distributions for the energy of the emitted pion

$$\frac{d\Gamma_{JJ}}{dE_\pi} = \frac{d\Gamma_{\rho\rho}}{dE_\pi} = \frac{G_F^2 \chi^2}{8(2\pi)^3 m_K} (m_K^2 - m_\pi^2)^2 \left|\tilde{B}\right|^2 \sqrt{E_\pi^2 - m_\pi^2} f_0^2(q^2) \qquad (31)$$

$$\frac{d\Gamma_{J\rho}}{dE_\pi} = \frac{G_F^2 \chi^2}{3(2\pi)^3} m_K \left|\tilde{A}\right|^2 \sqrt{(E_\pi^2 - m_\pi^2)^3} f_+^2(q^2) \qquad (32)$$

while for the emission of each neutrino species the result is

$$\frac{d\Gamma_{\nu\bar{\nu}}}{dE_\pi} = \frac{2G_F^2 \chi^2}{3(2\pi)^3} m_K \left|\tilde{D}\right|^2 \sqrt{(E_\pi^2 - m_\pi^2)^3} f_+^2(q^2) \qquad (33)$$

In eqs. (31)$-$(33) E_π varies between m_π and $(m_K^2 + m_\pi^2)/2m_K$. It is worth noting that the pion spectrum for the $J\rho_L$ mode coincides with the spectrum for the standard $\nu\bar{\nu}$ decay, being proportional to $|\vec{p}_\pi|^3$. However, the pion energy distribution for the emission of two Majorons (or two ρ_L) only depends linearly on $|\vec{p}_\pi|$, thus leading to a substantially different spectrum. In fig. 5 we show the shape of both spectra normalized to their maxima.

An interesting quantity is the ratio between the maxima of the spectra, since it is independent on the form factors $f_0(q^2)$ and $f_+(q^2)$, as $f_0(0) = f_+(0)$. This is also the part of the spectrum which is experimentally relevant. Normalizing to one $\nu\bar{\nu}$ mode, we obtain

$$\left(\frac{d\Gamma_{JJ}}{dE_\pi}\bigg|_{max} + \frac{d\Gamma_{\rho\rho}}{dE_\pi}\bigg|_{max}\right) \Big/ \frac{d\Gamma_{\nu\bar{\nu}}}{dE_\pi}\bigg|_{max} = \frac{3}{2}\frac{\left|\tilde{B}\right|^2}{\left|\tilde{D}\right|^2} \tag{34}$$

whereas for the emission of $J\rho_L$

$$\frac{d\Gamma_{J\rho}}{dE_\pi}\bigg|_{max} \Big/ \frac{d\Gamma_{\nu\bar{\nu}}}{dE_\pi}\bigg|_{max} = \frac{BR(K^- \to \pi^- J\rho_L)}{BR(K^- \to \pi^-\nu\bar{\nu})} = \frac{1}{2}\frac{\left|\tilde{A}\right|^2}{\left|\tilde{D}\right|^2} \tag{35}$$

The relative rates for the JJ and $\rho_L\rho_L$ modes depends instead explicitly on the form factors $f_+(q^2)$ and $f_0(q^2)$. Using the standard parametrization [15]

$$f_+(q^2) = f_+(0)(1 + \lambda_+\frac{q^2}{m_\pi^2})\ , \qquad f_0(q^2) = f_0(0)(1 + \lambda_0\frac{q^2}{m_\pi^2}) \tag{36}$$

we obtain

$$\frac{BR(K^- \to \pi^- JJ) + BR(K^- \to \pi^-\rho_L\rho_L)}{BR(K^- \to \pi^-\nu\bar{\nu})} = 3\frac{0.92 + 7.1\lambda_0 + 20\lambda_0^2}{0.95 + 4.6\lambda_+ + 9\lambda_+^2}\frac{\left|\tilde{B}\right|^2}{\left|\tilde{D}\right|^2} \tag{37}$$

Taking the values $\lambda_+ = 0.032$ and $\lambda_0 = 0.004$ [15], we finally have

$$\frac{BR(K^- \to \pi^- JJ) + BR(K^- \to \pi^-\rho_L\rho_L)}{BR(K^- \to \pi^-\nu\bar{\nu})} = 2.6\frac{\left|\tilde{B}\right|^2}{\left|\tilde{D}\right|^2} \tag{38}$$

The analysis is now reduced to the discussion of the relative magnitude of the functions $\tilde{A}$, $\tilde{B}$ and $\tilde{D}$, which we will perform in the next section.

5 Numerical Analysis and Discussion

The form factors $\tilde{A}$, $\tilde{B}$ and $\tilde{D}$, defined by eq. (11), contain the sum over the charm and top quark contributions weighted by the corresponding mixing angles. To estimate the effects of the presence of J and ρ_L for the $K^- \to \pi^- +$ *nothing* decay, we will separatly consider the case of charm and top dominance. In particular, the case of top dominance, which is also supported by the recent experimental indication of a large $B_d^0 - \bar{B}_d^0$ by the ARGUS collaboration, turns out to be the most interesting one.

We will begin by considering charm dominance. The dominant part in the form factors $\bar{A}$, $\bar{B}$ and $\bar{D}$ comes in this case from the $\ln x_j$ terms. From eq. (22), eq. (24) and eq. (27) we have in the limit $x_j \ll 1$

$$\bar{A} \simeq \frac{1}{2} x_j \ln x_j, \qquad \bar{B} \simeq O(1)\, x_j, \qquad \bar{D} \simeq -\frac{3}{2} x_j \ln x_j \tag{39}$$

The important point is that in the function $\bar{B}$, which produces the modification of the pion spectrum, the logarithmic terms have, in this limit, cancelled exactly. Thus, for $m_c \simeq 1.8\ GeV$, $\bar{B}$ turns out to be one order of magnitude smaller than $\bar{D}$, which implies a suppression of about two orders of magnitude in the rate. Also the contribution to the branching ratio due to the decay $K^- \to \pi^- J\ \rho_L$ is very small

$$R_c^{(D)} \equiv \frac{BR(K^- \to \pi^- J\ \rho_L)}{BR(K^- \to \pi^- \nu \bar{\nu})} \simeq \frac{1}{18} \simeq 0.05 \tag{40}$$

In the triplet Majoron model, the effect is enhanced but still minimal

$$R_c^{(T)} = 4 R_c^{(D)} \simeq 0.2 \tag{41}$$

In conclusion, charm dominance does not offer interesting perspectives for a test of majoron physics in this decay.

On the other hand, if the dominant contribution comes from top exchange, the modes related to the emission of the new scalars may give a substantial enhancement. In figs. 6 and 7 we show the dependence of the form factors on the mass of the top-quark, for different values of the mass of the charged scalars. By comparing the two plots we see that the decays $K^- \to \pi^- J\, J$ and $K^- \to \pi^- \rho_L \rho_L$ may play a relevant role. For instance for $m_t = 80\ GeV$ and $m_h = 300\ GeV$ we have $\left|\bar{B}/\bar{D}\right|^2 \simeq 0.4$ Using eq. (38) we finally obtain that the contribution of the decays to identical scalars may well be of the order of *one* neutrino generation.[†] In addition, we recall that in this case there is also an important modification of the shape of the spectrum (Fig. 5). The effect of $K^- \to \pi^- J\ \rho_L$ is smaller and, as follows by comparing eq. (35) and fig. 7, it can amount at most to 0.2 neutrino generations, without any modification of the pion spectrum.

In the case of the Gelmini-Roncadelli model, these estimates, considering only the "gauge" contributions and analogous values for the charged Higgs mass, have to be multiplied by a factor 4. Thus, in the triplet model, it is possible to obtain a contribution from the decay to identical scalars equivalent to *four* additional $\nu\bar{\nu}$ modes.

What about the contribution of the Higgs exchange diagrams? If in eq. (20) we take $\lambda_3 \simeq g^4$ and $m_{\rho_H} \simeq 20\ GeV$, reasonable and phenomenologically allowed values, we obtain that the contribution of the Higgs exchange diagrams to $\bar{B}$ is by more than one order of magnitude the dominant one in the amplitude. Translating this value into branching ratios, we obtain a decay rate to light scalars which is two orders of magnitude larger than the total

[†] In the case of the doublet majoron model the scalar mass indicated in the figures represents effectively the contribution of the two charged scalars as given in eq. (11). From the analysis in ref. [1], it follows then that even a quite large value of m_h, as the one used in the text, does not preclude the presence of a relatively light charged scalar ($\lesssim$100 GeV).

decay rate to neutrinos, approaching closely the present experimental limit.

It is quite interesting however, that we can limit the "arbitrariness" of the Higgs contribution with a theoretical argument. Indeed, Grzadkowski and Pich [28] have studied, in the context of the triplet majoron model, the conditions for the stability of the VEV hierarchy under renormalization of the Higgs potential and found that this requirement leads to a simple relation among couplings in the potential. Considering only the renormalization induced by the gauge bosons they find

$$\lambda_3^{(T)} \simeq 8 \frac{1+\cos^4\theta_W}{1+2\cos^4\theta_W} \lambda_1^{(T)} \simeq 5.8\ \lambda_1^{(T)} \tag{42}$$

By comparing with eq. (20) we see that this relation allows us to remove the dependence of the Higgs mediated amplitude on the parameters of the potential. We have redone the analysis in our model and, with the same assumptions, we find

$$\lambda_3 \simeq 2\ \lambda_1 \tag{43}$$

The details of the complete analysis will be presented elsewhere [29]. By implementing the relations (42) and (43) in eq. (23), we obtain that the contribution of the standard-Higgs exchange becomes of the same order of the gauge ones, although opposite in sign. Once again the triplet contribution turns out to be larger than the doublet one. The inclusion of the Higgs contribution thus generally reduces the previous estimates, the size of the effect depending on the masses of the charged scalars (for the values considered above there is a reduction of about a factor two). It is important however to recall that the size of the effective flavour changing coupling of the standard Higgs to fermions has become recently controversial (see the footnote in sect. 3). If the claim of the authors of ref. [26] is confirmed, the Higgs contribution becomes negligeable and only the "gauge" diagrams contribute to the $K^- \rightarrow \pi^- +$ *nothing* decay. We like also to remark that, in the interesting case of top dominance, QCD corrections are not expected to play any appreciable role, since no large mass ratios appear in the loops for the scalar modes. The same holds, in the case of top dominance, for the standard $\nu\bar{\nu}$ channel [18].

In conclusion, we have shown that the decay $K^- \rightarrow \pi^- +$ *nothing* may constitute a sensitive test for majoron physics, specially if it is possible in the future to obtain information on the shape of the spectrum away from the maximum. In any case, the contribution to the branching ratio due to the new scalar modes may be as large as one additional neutrino mode in the doublet model and few times larger in the triplet model, thus incentivating the search for the presence of physics beyond the standard model in this class of rare kaon decays.

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Figure Captions

Fig. 1 One loop diagrams contributing to the effective flavour changing vertex of the Z^0 gauge boson in the unitary gauge.

Fig. 2 The WW contribution to the effective vertex $\bar{s}\ d \to J\ J$ and $\bar{s}\ d \to \rho_L\ \rho_L$ ("seagull" diagram).

Fig. 3 Box diagrams, with charged scalars in the loop, contributing to JJ, $\rho_L \rho_L$ (a) and $J\rho_L$ (b).

Fig. 4 Standard Higgs exchange diagrams contributing to $\bar{s}\ d \to J\ J$ and $\rho_L\ \rho_L$.

Fig. 5 Pion energy distributions in $K^- \to \pi^- + nothing$. The dashed line represents the shape of the spectrum for $J\ J$ and $\rho_L\ \rho_L$ emission whereas the solid line shows the shape for the $\nu\bar{\nu}$ and $J\ \rho_L$ modes. Both spectra are normalized to their maxima.

Fig. 6 The ratio of the form factors for $K^- \to \pi^- J\ J$ and $K^- \to \pi^- \nu\bar{\nu}$ is shown as a function of the top mass for different values of the effective charged scalar mass.

Fig. 7 Same as in fig. 6 for the $K^- \to \pi^- J\ \rho_L$ form factor.

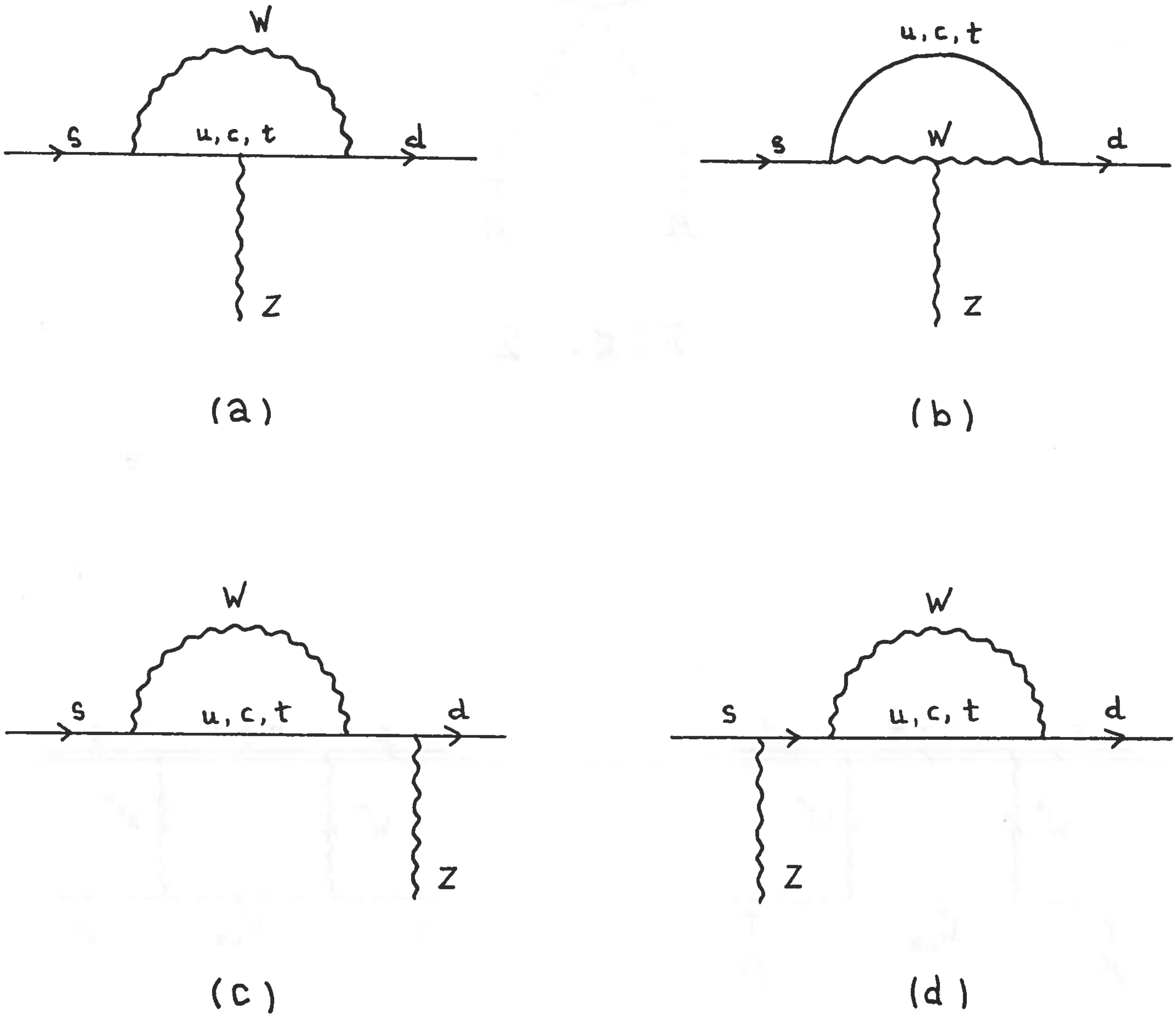

Fig. 1

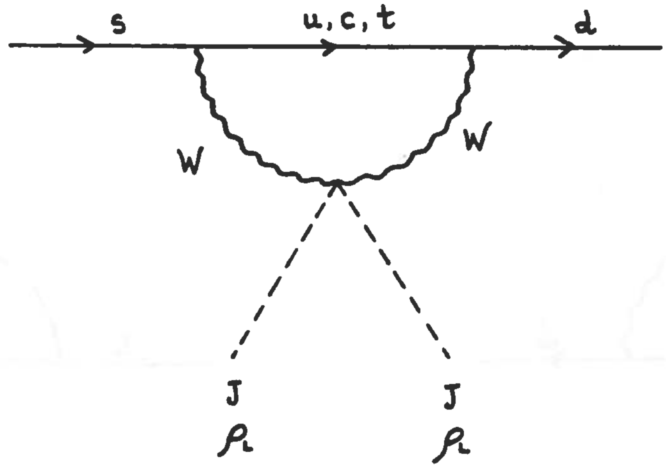

Fig. 2

(a) (b)

Fig. 3

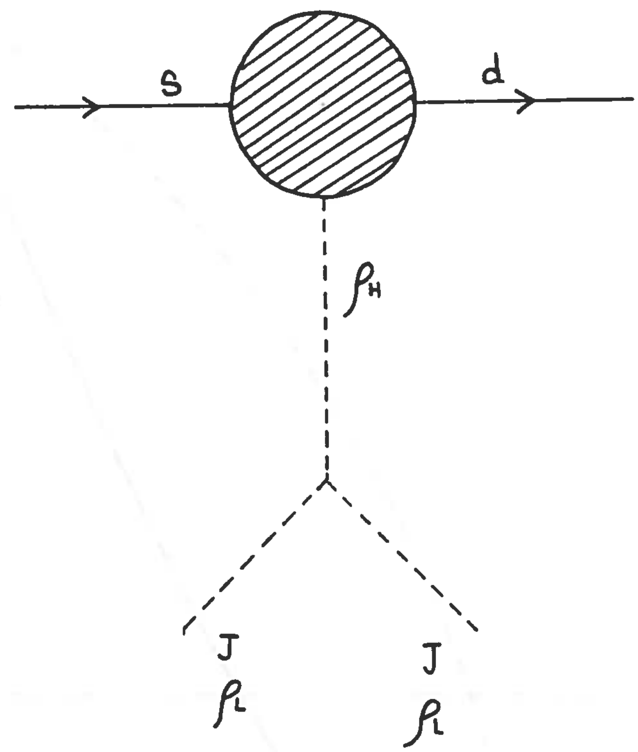

Fig. 4

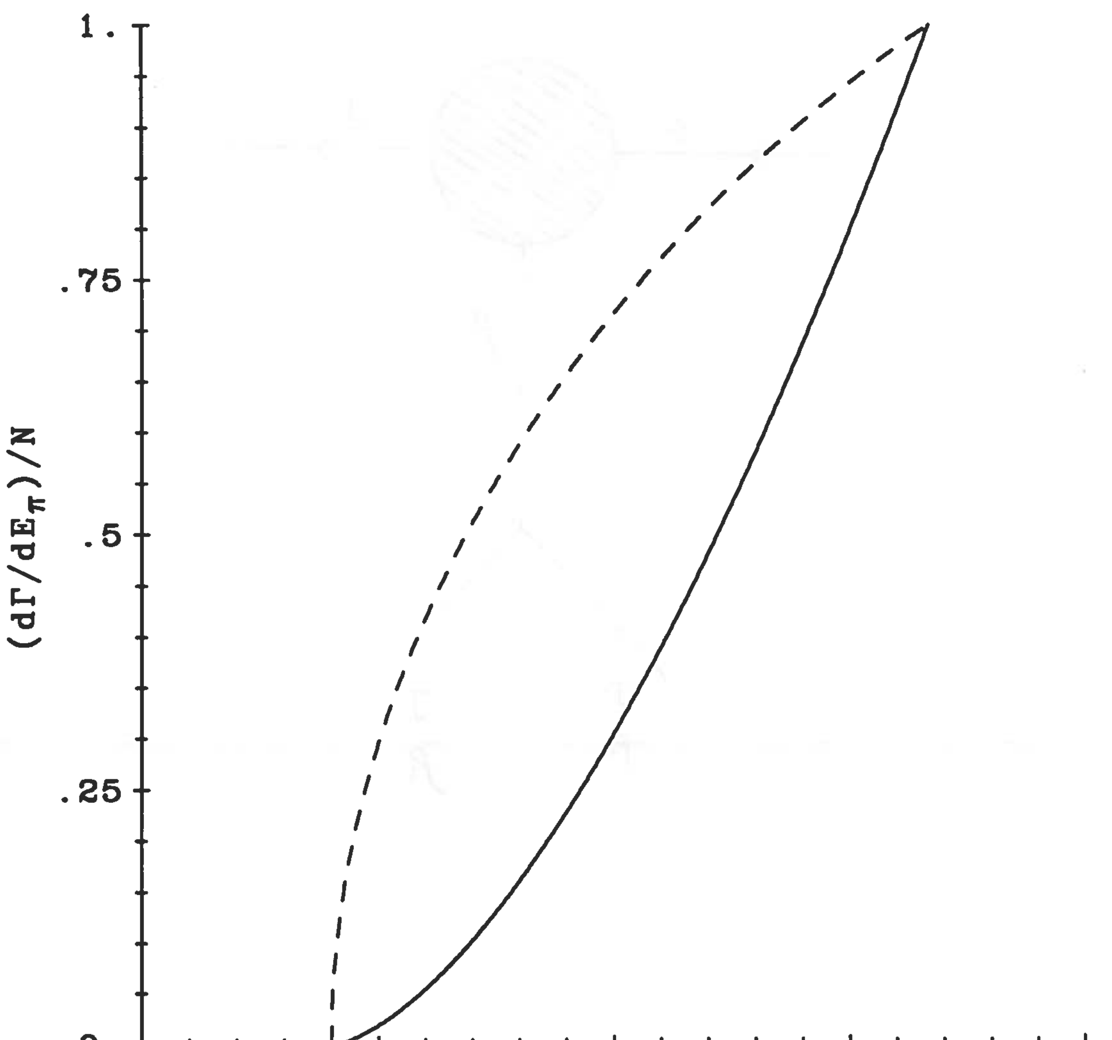

Fig. 5

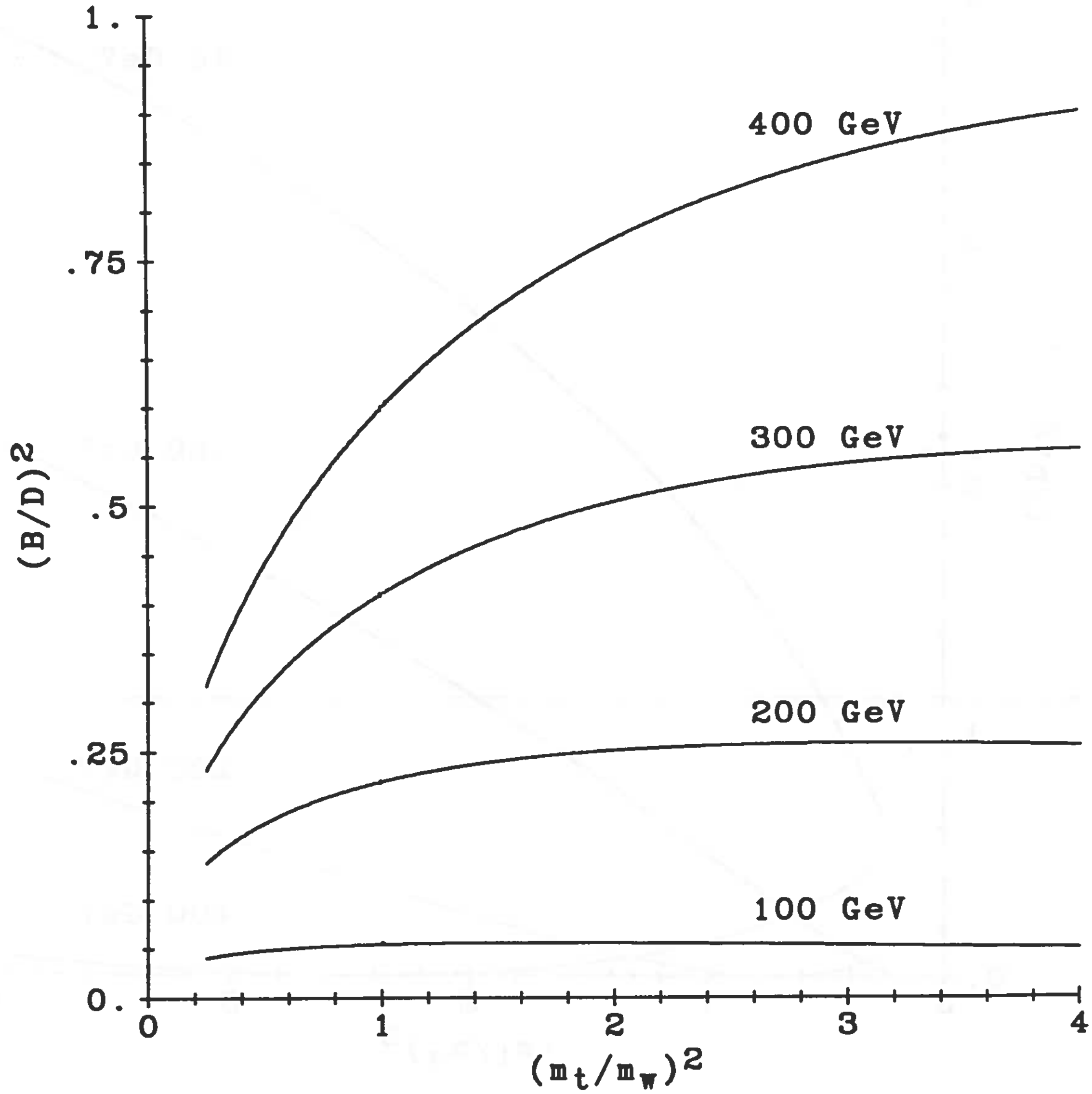

Fig. 6

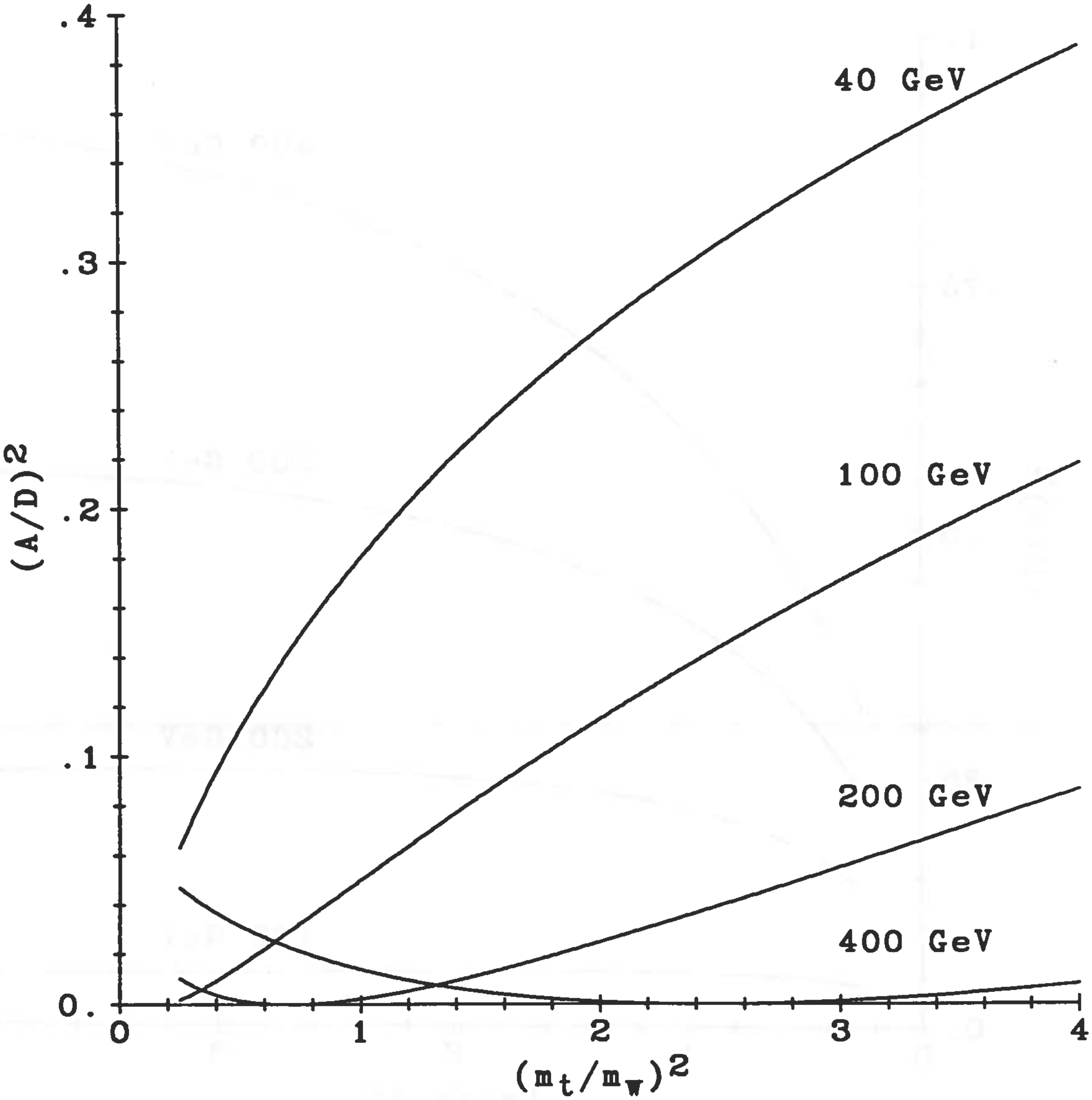

Fig. 7