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Masses, Mixings, Yukawa Couplings and their Symmetries

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Abstract

We present a method to find the number of real and imaginary observable parameters coming from the Yukawa sector in an arbitrary gauge theory. The method leads naturally to a classification of Yukawa couplings according to their symmetries and suggests a new parametrization of masses and mixings that is useful to study the behaviour of Yukawa couplings under the renormalization group. We apply it to some examples based on the Standard Model with Yukawa couplings obeying various chiral symmetries. We also show how our method of parameter counting can be used in some models with an enlarged leptonic sector.

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The most difficult part to understand of theories with spontaneously broken gauge theories is, together with the Higgs potential, the Yukawa sector. Its origin is unclear, and probably related to the origin of the spontaneous breaking of the symmetries itself. It contains a large number of free parameters that must be adjusted by hand to obtain a realistic spectrum of particles and mixings. From the 17 parameters needed to describe the electro-weak theory, 13 come from the Yukawa sector. In addition, the Yukawa couplings contain a lot of spurious parameters that cannot be observed because they can be removed from the Lagrangian with an appropriate choice of phases and mixings for the fermion fields. Thus, a simple question such as how many parameters are needed to describe a theory with Yukawa couplings cannot be answered at first sight. We cannot even say whether the couplings are real or not and so if the theory conserves CP or not. This represents a great complication when one tries to understand mass matrices in the Standard Model. In the last years this subject has received a lot of attention (for some popular mass-matrix models, see for example [1, 2, 3, 4]). Usually people make ansätze about a particular form of mass matrices and try to extract from them a reasonable spectrum of masses and mixings. However, completely different forms of mass matrices may be equivalent in their physical content. This has led to some confusion in the past. Moreover, some symmetries actually present in the Yukawa sector may be hidden when one does not work in the appropriate basis. For those reasons, an analysis based on symmetry should be superior and could clarify some of the issues present in particular forms of mass matrices. It would also allow a clear understanding of the number of parameters needed to describe the model.

The standard method to calculate the number of parameters needed to describe the Yukawa sector of a spontaneously broken gauge theory is just by construction. After symmetry breaking one diagonalizes the mass matrices of all fields and after that one uses the freedom in the phase definition of the fields to reabsorb as many phases as possible. The remaining parameters in the Lagrangian are the physical parameters. As we know, the method works very well for the Standard Model. However, as the complication of the Yukawa sector increases, the method becomes intricate and can lead very easily to error; the question of CP-violation, for example, can then be difficult to answer. In addition, in many cases, phases and mixings can be moved from the gauge couplings to the Yukawa sector or even included in a redefinition of the fields [5, 6], which leads to further confusion.

Here we propose a method, based on symmetry, to perform the counting before spontaneous symmetry breaking and without performing the explicit diagonalization of the mass matrices. The method clarifies the structure of the manifold of the parameters that are needed to describe the Yukawa sector of a gauge theory and suggests a parametrization that is useful when one wants to study the behaviour of the Yukawa couplings under the renormalization group. Basically the method consists in the study of the symmetries of both the full and the Yukawa Lagrangians. The number of parameters needed to describe the Yukawa sector comes from the balance between the number of parameters contained in the Yukawa matrices and the number of symmetries broken by the Yukawa couplings. As an example, we will start with the Standard Model with the most general set of Yukawa couplings. We will then impose to the Standard Model a set of chiral symmetries that could reduce the number of parameters and see how the method can be used in those cases. Finally we will apply the method to more involved models based on the same gauge group but different particle content.

The number of parameters needed to describe the Yukawa sector of a theory is obviously the same before and after spontaneous symmetry breaking[1]. Thus, we will start with the hadronic part of the electro-weak Lagrangian before symmetry breaking

$$\mathcal{L} = i\overline{Q_L}\not{D}\, Q_L + i\overline{u_R}\not{D}\, u_R + i\overline{d_R}\not{D}\, d_R + (\overline{Q_L}Y_u u_R\varphi + \overline{Q_L}Y_d d_R\tilde{\varphi} + \text{h.c.}) \ . \qquad (1)$$

Here Q_L, u_R and d_R are the standard quark fields, left-handed doublet, u-singlet and d-singlet respectively. If we assume n generations they are n-column vectors in generation space; Y_u and Y_d are the Yukawa couplings represented by $n\times n$ complex arbitrary matrices; $\not{D}\equiv\gamma^\mu D_\mu$ where D_μ is the Standard Model electro-weak covariant derivative. Finally φ is the Higgs doublet and $\tilde{\varphi}\equiv i\tau_2\varphi^*$.

If $Y_u = Y_d = 0$, the Lagrangian is obviously invariant under the following chiral symmetries:

$$Q_L\to V_Q Q_L \qquad u_R\to V_u u_R \qquad d_R\to V_d d_R \ , \qquad (2)$$

where, V_Q, V_u, and V_d are $n\times n$ unitary matrices acting on flavour space. The Yukawa couplings break explicitly these symmetries, but in a very particular way. In fact, if we let the Yukawa couplings transform as follows

$$Y_u\to V_Q Y_u V_u^\dagger \qquad Y_d\to V_Q Y_d V_d^\dagger \ , \qquad (3)$$

the Lagrangian of eq. (1) is still invariant under the combined action of the transformations in eqs. (2) and (3). It is also easy to see that not all the symmetries in eq. (2) are broken by the Yukawa couplings. Indeed if we choose $V_Q = V_u = V_d = e^{i\alpha}$, the Yukawa Lagrangian remains invariant. This is nothing else than baryon number conservation.

Equation (3) defines an equivalence relation

$$(Y_u, Y_d)\Leftrightarrow(Y'_u, Y'_d) = (V_Q Y_u V_u^\dagger,\ V_Q Y_d V_d^\dagger) \ . \qquad (4)$$

The Lagrangians with couplings (Y_u, Y_d) and couplings (Y'_u, Y'_d) are completely equivalent. Thus, counting how many parameters are needed to describe masses, mixings, and Yukawa couplings, is the same as counting how many equivalent classes there are with respect to the equivalence relation of eq. (4).

This problem has some similarity with the problem of spontaneous symmetry breaking by the Higgs mechanism. There, one has a group G and a representation φ of Higgses whose vacuum expectation value breaks the group G down into the group $G'\subset G$ and one wants to know the number of physical Higgses. As we know, the number of physical Higgs degrees of freedom $N_{\varphi_{phys}}$ is

$$N_{\varphi_{phys}} = N_\varphi - N_{Goldstone} \ . \qquad (5)$$

Here N_φ is the number of degrees of freedom in the Higgs representation φ and $N_{Goldstone}$ is the number of Goldstone bosons that appear after spontaneous symmetry breaking. It is equal to the number of broken generators of G, which is the number of generators of the full group G minus the number of generators of the unbroken subgroup G' ($N_{Goldstone} = N_G - N_{G'}$). We have a similar situation here. A chiral symmetry G is broken explicitly by the Yukawa sector Y to a group G'. Then, only the broken part of G can be used to absorb parameters from Y. Following eq. (5) we could write

$$N_{Y_{phys}} = N_Y - N_G + N_{G'} \ . \qquad (6)$$

[1]For the moment, we leave aside the case of spontaneous breakdown of CP, in which some parameters can be moved from the Higgs potential to the Yukawa sector.

In the Standard Model, as follows from eq. (2), the group is $G = U(n)_Q \otimes U(n)_u \otimes U(n)_d$ and the Yukawa couplings $Y = (Y_u, Y_d)$ are two general $n \times n$ complex matrices transforming under the group G as eq. (4). Here the factors $U(n)$ denote the full unitary group $U(n) = SU(n) \otimes U(1)$. As discussed previously, for the most general Yukawa couplings, the only subgroup of G that leaves the Yukawa couplings invariant is a $U(1)_B$ such that $V_Q = V_u = V_d = e^{i\alpha}$.

After eq. (6), counting of parameters is trivial. Taking into account that an arbitrary complex matrix of dimension n contains n^2 moduli and n^2 phases and that a $U(n)$ matrix contains $n(n-1)/2$ moduli and $n(n+1)/2$ phases, we find that Y_{phys} can be expressed in terms of

couplings and symmetries	moduli	phases
(Y_u, Y_d)	$2n^2$	$2n^2$
$U(n)_Q \otimes U(n)_u \otimes U(n)_d$	$-3n(n-1)/2$	$-3n(n+1)/2$
$U(1)_B$	0	1
Y_{phys}	$2n + n(n-1)/2$	$(n-2)(n-1)/2$

. (7)

In particular if $n = 3$ we have $6+3$ moduli and 1 phase. After symmetry breaking, some of the moduli are related to masses and other to mixings or remaining Yukawa couplings. To distinguish among them we can repeat the argumentation but using the completely broken theory with all the heavy and Higgs bosons decoupled. In the case of the Standard Model, what remains is a model with the standard fermion content but only QED (and QCD if included in the analysis)[2]. The resulting Lagrangian is

$$\mathcal{L} = i\overline{u_L} \not{D} u_L + i\overline{u_R} \not{D} u_R + i\overline{d_L} \not{D} d_L + i\overline{d_R} \not{D} d_R + (\overline{u_L} M_u u_R + \overline{d_L} M_d d_R + \text{h.c}) \ . \quad (8)$$

Here $\not{D}$ is the standard QED covariant derivative and M_u and M_d are the quark mass matrices related to the Yukawa couplings by the vacuum expectation value of the Higgs scalar.

Except for the mass terms, the Lagrangian of eq. (8) is invariant under the following symmetries

$$u_L \to V_{u_L} u_L \qquad u_R \to V_{u_R} u_R \qquad d_L \to V_{d_L} d_L \qquad d_R \to V_{d_R} d_R \ . \quad (9)$$

Using similar arguments as above, but now taking into account that the full Lagrangian has separate flavour conservation for u-type and d-type quarks, i.e. $G = U(n)_{uR} \otimes U(n)_{uL} \otimes U(n)_{dR} \otimes U(n)_{dL}$ and $G' = (U(1)_u)^n \otimes (U(1)_d)^n$, we find that the Lagrangian (8) gives rise to only $2n$ masses. Of course this is obvious since the symmetries in eq. (9) are enough to diagonalize the two mass matrices completely. Thus, from the $2n + n(n-1)/2$ moduli present in the Yukawa sector of the Standard Model, $2n$ correspond to masses and $n(n-1)/2$ to mixings, as expected [7] (see also [8]).

Yukawa couplings related by eq. (4) are in the same class of equivalence. To find a parametrization of the physical Yukawa sector ve have to characterize the equivalent classes, and this can be done by taking one element of each equivalence class. We can use a V_Q and a V_u rotation to diagonalize the matrix Y_u, $V_Q Y_u V_u^\dagger = D_u$. After that, since any arbitrary complex matrix can be written as a Hermitian matrix times a unitary matrix, we can use a V_d transformation to write Y_d as a positive-definite Hermitian matrix. Thus $(Y_u, Y_d) \to (D_u, H_d)$, with D_u a positive-definite diagonal matrix and H_u a positive-definite Hermitian matrix. But, D_u is still invariant under transformations like $D_u \to K D_u K^\dagger$,

[2]We have to keep all the exact symmetries. Fields with different charges never mix.

with K a diagonal matrix of phases, while H_d is not. This means that we can use K to absorb phases from H_u. In fact we can chose K such that the next-to-diagonal diagonal elements of H_d be real and positive. The only remaining symmetry is just $U(1)_B$ of baryon number, as expected. Thus, without loss of generality we can represent the physical Yukawa couplings of the Standard Model by a positive-definite diagonal matrix D_u for the u-type quarks and a positive-definite Hermitian matrix $\hat{H}_d$ with the next-to-diagonal diagonal elements of $\hat{H}_d$ real and positive:

$$(Y_u, Y_d) \to Y_{phys} = (D_u, \hat{H}_d) \ . \tag{10}$$

One can easily check that the right-hand side of eq. (10) contains also $2n + n(n-1)/2$ moduli and $(n-1)(n-2)/2$ phases. Of course, one can further write $\hat{H}_d$ in terms of a *Kobayashi-Maskawa*-type matrix and a diagonal matrix of masses.

Since the full Lagrangian is invariant under the combined action of transformations (2) and (3) it is easy to show that the renormalization group equations for the Yukawa couplings must be covariant with respect to the set of transformations (3). This guarantees that the renormalization group equations can be written only in terms of the parameters in the right-hand side of eq. (10). This way we can obtain a set of renormalization group equations with all the unphysical parameters removed[3].

The method we just explained for counting the physical parameters works very well in the Standard Model with the most general Yukawa couplings. However, in the form we have presented it, it is not suitable when some of the masses are zero or there are some symmetries among them. We want to generalize the method to those cases. We will impose that the only allowed reductions of parameters are those that are protected by some symmetry. For instance, suppose we want to make the first $n-1$ d-type quarks (in n generations) exactly massless. We could impose an additional chiral symmetry such as

$$Q_L \to Q_L \qquad u_R \to u_R \qquad d_R \to \begin{pmatrix} U & 0 \\ 0 & 1 \end{pmatrix} d_R \ , \tag{11}$$

where U is a $(n-1) \times (n-1)$ general unitary matrix acting only on the first $n-1$ generations of d_R quarks. Clearly the only way to make the Yukawa Lagrangian invariant under this symmetry is for

$$Y_u = \text{arbitrary} \qquad Y_d = \begin{pmatrix} 0 & \cdot & \cdot & 0 & \alpha_1 \\ 0 & \cdot & \cdot & 0 & \alpha_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & 0 & \alpha_n \end{pmatrix} \ . \tag{12}$$

Then, the Yukawa Lagrangian is only invariant under $U(n-1) \otimes U(1)_B$. Naïve counting starting from the full group $G = U(n)_Q \otimes U(n)_u \otimes U(n)_d$ does not work in this case, because the couplings in eq. (12) are not a representation of the full group G. Said in another way, general G transformations do no preserve the form of the Yukawa matrices in eq. (12) and, thus, we cannot use the full group to absorb parameters from the Yukawa sector. It is not difficult to see that the only subgroup of G that preserves the form of the Yukawa couplings in eq. (12) is $\hat{G} = U(n)_Q \otimes U(n)_u \otimes U(n-1) \otimes U(1)_B$. Now we can use $\hat{G}$ to absorb parameters from the Yukawa sector and the counting of physical parameters

[3] A similar approach has been followed by [9, 10, 11, 12]

comes as follows

couplings and symmetries	moduli	phases
(Y_u, Y_d)	n^2+n	n^2+n
$U(n)_Q \otimes U(n)_u \otimes U(n-1)_d \otimes U(1)_B$	$-(n-1)(3n-2)/2$	$-n(3n+1)/2-1$
$U(n-1) \otimes U(1)_B$	$(n-1)(n-1)/2$	$(n-1)n/2+1$
Y_{phys}	$n+1+(n-1)$	0

. (13)

The model leads to n massive u-type quarks, 1 massive d-type quark and $n-1$ mixings. There is no CP-violating phase. This result can be explicitly checked by full diagonalization of the Yukawa sector.

Another extreme case can be obtained by imposing an axial $U(1)$ symmetry on the d-quark:

$$Q_L \to Q_L \qquad u_R \to u_R \qquad d_{iR} \to d_{iR} \qquad i=2,\cdots,n \qquad d_{1R} \to e^{i\alpha} d_{1R}\ . \tag{14}$$

Then, the Yukawa couplings take the following form

$$Y_u = \text{arbitrary} \qquad Y_d = \begin{pmatrix} 0 & \alpha_{1,1} & \cdot & \cdot & \alpha_{1,n-1} \\ 0 & \alpha_{2,1} & \cdot & \cdot & \alpha_{2,n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \alpha_{n,1} & \cdot & \cdot & \alpha_{n,n-1} \end{pmatrix}\ . \tag{15}$$

It is easy to see that the group that preserves the form of these matrices is $\hat{G} = U(n)_Q \otimes U(n)_u \otimes U'(n-1)_d \otimes U(1)_B$ with the $U'(n-1)$ now acting on the $2,3,\cdots,n$ generations of d_R quarks, while the only symmetries of the full Lagrangian are $U(1)_{d_1} \otimes U(1)_B$. Again we can do the counting

couplings and symmetries	moduli	phases
(Y_u, Y_d)	$n^2+n(n-1)$	$n^2+n(n-1)$
$U(n)_Q \otimes U(n)_u \otimes U'(n-1)_d \otimes U(1)_B$	$-(n-1)(3n-2)/2$	$-n(3n+1)/2-1$
$U(1)_{d_1} \otimes U(1)_B$	0	2
Y_{phys}	$n+(n-1)+n(n-1)/2$	$(n-1)(n-2)/2$

(16)

This model just gives a massless d-quark. Phases and mixings are exactly the same as in the Standard Model for any number of generations. This result is interesting because the so-called $m_u=0$ solution of the Strong CP Problem (see for example [13, 14, 15] and references therein) is based in a symmetry like ours. In our discussion we have obviated the fact that chiral symmetries have in general anomalies. In the Standard Model, because of QCD instanton effects and because the chiral symmetries are anomalous, there is an additional phase that cannot be removed. Chiral transformations move it from the Yukawa sector to the QCD sector. This represents a problem because it gives a too large contribution to the electric dipole moment of the neutron unless it is made unnaturally small. If for instance $m_u=0$, there is an additional chiral symmetry (similar to the one we have just studied) and the extra phase can be rotated away. However, imposing an additional chiral symmetry is a very strong constraint on the mass matrices [see eq. (15)]. One has to redo all parameter counting and be sure that, there still is in the model an observable CP-violating phase that could account for all observed CP-violating effects. This is just what we did in our example.

The examples discussed above show that the use of symmetry is a natural and basis-independent way of choosing models for mass matrices. Of course, most of the symmetries lead to some massless quarks and/or some vanishing mixings, but the symmetries could be softly broken in order to generate a realistic pattern of masses and mixings. We find it interesting to explore and classify all possible symmetries that can be imposed on the Yukawa sector of the Standard Model according to the spectrum of masses and mixings they generate. This is nothing but the classification of all subgroups of $U(n)_Q \otimes U(n)_u \otimes U(n)_d$ according to the spectrum they produce. We think that this classification is essential for any mass-matrix modelling.

Let us apply our method of parameter counting to a more complicated example. Let us take an extension of the Standard Model including right-handed neutrinos with a Majorana mass term. The leptonic part of the Lagrangian is

$$\mathcal{L} = i\overline{\ell_L} \not{D} \ell_L + i\overline{\nu_R} \not{D} \nu_R + i\overline{e_R} \not{D} e_R + \left(\overline{\ell_L} Y_\nu \nu_R \varphi + \overline{\ell_L} Y_e e_R \tilde{\varphi} + \frac{1}{2}\overline{\nu_R^c} M \nu_R \ + \ h.c. \right) \tag{17}$$

where ℓ_L is the standard leptonic doublet, e_R is the right-handed charged lepton singlet, ν_R is the right-handed neutrino, and $\nu_R^c = \mathcal{C}\overline{\nu_R}^T$. The Yukawa couplings are the usual Y_e that give masses to the charged leptons, Y_ν that give a Dirac mass term to neutrinos and M is a complex symmetric Majorana mass matrix for right-handed neutrinos. It can be put in by hand, as we did, because right-handed neutrinos are singlet, however, it could be generated through the vacuum expectation value of some extra scalar singlet. The model just described is nothing else than the *see-saw* model[16, 17, 18, 19] of neutrino masses. If $Y_\nu = Y_e = M = 0$ the Lagrangian is invariant under the following symmetries

$$\ell_L \to V_\ell \ell_L \qquad \nu_R \to V_\nu \nu_R \qquad e_R \to V_e e_R \ , \tag{18}$$

V_ℓ, V_ν, and V_e are unitary matrices of dimension n for n generations of leptons. As in the Standard Model, the Yukawa couplings break these symmetries explicitly. However, they are broken in such a way that if we let the Yukawa couplings and the Majorana mass transform in the following way

$$Y_e \to V_\ell Y_e V_e^\dagger \qquad Y_\nu \to V_\ell Y_\nu V_\nu^\dagger \qquad M \to V_\nu^* M V_\nu^\dagger \ , \tag{19}$$

the Lagrangian remains invariant. At difference with the Standard Model, we see that the Yukawa couplings and the Majorana mass term completely break all the invariances of eq. (18), including lepton-number conservation.

Equation (19) defines the following equivalence relation

$$(Y_e, Y_\nu, M) \Leftrightarrow (Y_e', Y_\nu', M') = (V_\ell Y_e V_\nu^\dagger,\ V_\ell Y_\nu V_e^\dagger, V_\nu^* M V_\nu^\dagger) \ . \tag{20}$$

Again the number of physical parameters needed to describe the model is given by the difference between the number of parameters contained in the Yukawa coupling and the number of generators broken by the couplings.

Now the group of symmetries of the kinetic and gauge part is $G = U(n)_\ell \otimes U(n)_e \otimes U(n)_\nu$ and there is no unbroken subgroup. Counting of parameters is easy. Taking into account that an arbitrary complex matrix of dimension n contains n^2 moduli and n^2 phases, that a symmetric complex matrix contains $n(n+1)/2$ moduli and $n(n+1)/2$ phases, and that a $U(n)$ matrix contains $n(n-1)/2$ moduli and $n(n+1)/2$ phases, we find that Y_{phys} can be written in terms of

couplings and symmetries	moduli	phases
(Y_e, Y_ν, M)	$2n^2 + n(n+1)/2$	$n^2 + n(n+1)/2$
$U(n)_\ell \otimes U(n)_e \otimes U(n)_\nu$	$-3n(n-1)/2$	$-3n(n+1)/2$
	0	0
Y_{phys}	$3n + n(n-1)$	$n(n-1)$

. (21)

We obtain that even in the two-generation case there are two observable phases that could violate CP. As we did in the Standard Model, to know how many of the physical parameters are related to masses and how many to mixings, we have to decouple all kinds of massive bosons. Then, all the mass matrices can be diagonalized, leading in general to n massive charged leptons and $2n$ Majorana neutrinos. Thus, $3n$ of the moduli are masses, and the rest correspond to mixings.

To find a parametrization of the Y_{phys}, we can use the equivalence relation in eq. (20) to reduce the number of parameters in the Yukawa couplings. Using the fact that an arbitrary complex matrix can be written in a unique way as a unitary matrix times a positive-definite Hermitian matrix[4] and that a complex symmetric matrix can be written in a unique way in terms of a positive-definite diagonal matrix and a unitary matrix $S = U^T D U$ we can write the Yukawa couplings in the following form

$$(Y_e, Y_\nu, M) \rightarrow Y_{phys} = (H_e, H_\nu, D_m) \ . \tag{22}$$

Here H_e and H_ν are positive-definite Hermitian matrices and D_m is a positive-definite diagonal matrix. As we have exhausted all the symmetries in the group G to reduce the Yukawa couplings to this form and there is no subgroup of G that leaves Y_{phys} invariant, the set of parameters in the right-hand side of eq. (22) is the physical set of parameters needed to describe the theory. This means that we could start from the beginning without loss of generality by choosing Y_ν and Y_e as positive-definite Hermitian matrices and M as a positive definite diagonal matrix and *all* the parameters in those matrices would be observable as can be easily checked by counting the number of parameters contained in those couplings.

Finally to give a flavour of how these techniques can be used to study more exotic models, we chose the model of refs. [20, 21, 22, 23] with the following Lagrangian,

$$\begin{aligned}\mathcal{L} &= i\overline{\ell_L}\, \not{D}\, \ell_L + i\overline{\nu_R}\, \not{D}\, \nu_R + i\overline{e_R}\, \not{D}\, e_R + i\overline{s_L}\, \not{D}\, s_L \\ &+ \left(\overline{\ell_L} Y_\nu \nu_R \varphi + \overline{\ell_L} Y_e e_R \tilde{\varphi} + \frac{1}{2}\overline{\nu_R} M_s s_L \ + \ h.c.\right) \ ,\end{aligned} \tag{23}$$

where s_L is a singlet that carries the same lepton-number as the leptons and M_s is an arbitrary complex matrix. The rest of the notation is the same as in the previous example. In the Lagrangian of eq. (23), total lepton-number has been imposed as a global symmetry. There are thus no Majorana mass terms for any of the singlet fermions. The group of invariances of the non-Yukawa part of the Lagrangian is $U(n)_\ell \otimes U(n)_e \otimes U(n)_\nu \otimes U(n)_s$, where $U(n)_s$ is the new invariance related to the field s_L. The only symmetry of the Yukawa couplings is just $U(1)_{lep}$ of lepton-number conservation. Doing now the usual

[4]We will assume that the matrices have non-zero determinant and there are no degeneracies. If there is some degeneracy or zero eigenvalues, this must be imposed with some symmetry and would change the counting of parameters.

counting we get

couplings and symmetries	moduli	phases
(Y_e, Y_ν, M_s)	$3n^2$	$3n^2$
$U(n)_\ell \otimes U(n)_e \otimes U(n)_\nu \otimes U(n)_s$	$-4n(n-1)/2$	$-4n(n+1)/2$
$U(1)_{lep}$	0	1
Y_{phys}	$2n+n^2$	$(n-1)^2$

. (24)

Once the gauge symmetry is broken, the Yukawa couplings generate mass terms for the leptons. It is easy to see that, as a consequence of lepton-number conservation, all neutrinos must be Dirac neutrinos or just massless. Then, from the full mass matrix of neutrinos, n massless neutrinos and n massive Dirac neutrinos arise, hence only n masses come from the neutrino mass matrix. The other n masses come from the charged lepton sector. Thus, from all the moduli, $2n$ of them correspond to masses and the rest n^2 correspond to mixings. In addition there are $(n-1)^2$ CP-violating phases that cannot be removed. This result is in complete agreement with the results obtained in [24] after full diagonalization of the mass matrices.

We have shown that a study of the chiral symmetries of the kinetic and gauge parts of the Lagrangian and the Yukawa part of the Lagrangian before spontaneous symmetry breaking allows us to compute, in a straightforward way, how many observable parameters come from the Yukawa couplings in a general gauge theory. Basically, we have to know which subgroup of chiral symmetries of the kinetic and gauge parts of the Lagrangian preserves the form of the Yukawa couplings and which group leaves them invariant. The number of parameters that can be absorbed from the Yukawa sector is just the difference between the parameters needed to describe those groups. Then, the number of observable parameters comes from the balance between the Yukawa couplings and the symmetries broken by them. The method is useful when the diagonalization of the full set of mass matrices becomes complicated. On the other hand it clarifies the analysis of observable parameters. We explain the technique with several examples based on the Standard Model, with Yukawa couplings obeying various chiral symmetries, and also with some extensions of the Standard Model with enlarged fermion sector.

The study of the mass matrices and the gauge couplings is not general enough, because some parameters such as phases, mixings, etc, can be moved from charged currents to Yukawa couplings or even be included in a field definition [5, 6]. Only the analysis of the full Lagrangian, and this is better done before symmetry breaking, can report the number of observable parameters in a non-ambiguous way. This point of view suggests a parametrization of Yukawa couplings in terms of invariants with respect to the chiral symmetries of the kinetic and gauge parts of the Lagrangian that can be useful in the study of the behaviour of Yukawa couplings under the renormalization group. We comment on this parametrization in the case of the Standard Model and the see-saw model for neutrino masses.

We find that a complete analysis and classification of all the chiral symmetries that can be imposed on the Yukawa sector, according to which spectrum of masses and mixings they originate is still missing. We think this analysis would be a good starting point for mass matrix modeling.

Finally we want to remark that we only considered symmetries acting on fermions. The analysis could be extended to symmetries involving fermions and scalars as well, like the Peccei-Quinn [25, 26] symmetry or horizontal symmetries [27, 28, 29, 30]. We also think that it can be extended to the spontaneous CP-violation case.

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