

# New constraints on $R$ -parity violation from $\mu$ - $e$ conversion in nuclei

K. Huitu <sup>a,1</sup> J. Maalampi <sup>b,2</sup> M. Raidal <sup>c,3</sup> and A. Santamaria <sup>c,4</sup>

<sup>a</sup>*Helsinki Institute of Physics, P.O. Box 9, FIN-00014 University of Helsinki,  
Finland*

<sup>b</sup>*Theoretical Physics Division, Department of Physics, P.O. Box 9, FIN-00014  
University of Helsinki, Finland*

<sup>c</sup>*Departament de Física Teòrica, IFIC, CSIC-Universitat de València,  
E-46100 Burjassot, València, Spain*

---

## Abstract

We derive new constraints on the products of explicitly  $R$ -parity violating couplings  $\lambda$  and  $\lambda'$  in MSSM from searches for  $\mu$ - $e$  conversion in nuclei. We concentrate on the loop induced photonic coherent conversion mode. For the combinations  $|\lambda\lambda|$  which in  $\mu$ - $e$  conversion can be probed only at loop level our constraints are in many cases more stringent than the previous ones due to the enhancement of the process by large  $\ln(m_{\tilde{f}}^2/m_f^2)$ . For the combinations of  $|\lambda'\lambda'|$  the tree-level  $\mu$ - $e$  conversion constraints are usually more restrictive than the loop ones except for two cases which involve the third generation. With the expected improvements in the experimental sensitivity, the  $\mu$ - $e$  conversion will become the most stringent test for *all* the involved combinations of couplings.

---

## 1 Introduction

The minimal supersymmetric standard model (MSSM) extended by explicit breaking of  $R$ -parity,  $R = (-1)^{3(B-L)+2S}$  [1], has recently received a lot of attention. In MSSM, the conservation of  $R$ -parity is put in by hand. Therefore,

---

<sup>1</sup> E-mail: katri.huitu@helsinki.fi

<sup>2</sup> E-mail: maalampi@rock.helsinki.fi

<sup>3</sup> E-mail: raidal@titan.ific.uv.es

<sup>4</sup> E-mail: Arcadi.Santamaria@uv.es

it is theoretically not well motivated. On the other hand, models exist in which the violation of  $R$ -parity is a necessity, e.g., the supersymmetric left-right model [2]. Furthermore, the observed excess of high  $Q^2$  events in HERA, if interpreted in the framework of supersymmetry, seems to indicate non-zero  $R$ -parity violating couplings of squarks [3].

Limits on  $R$ -parity violating couplings have been derived from a large variety of observables. Since the presence of  $R$ -parity violation leads automatically to the presence of lepton and/or baryon number violating processes, searches for them are especially suitable for constraining these couplings. In this Letter we derive new upper limits on the products of the couplings  $\lambda_{ijk}$  and  $\lambda'_{ijk}$  from the non-observation of  $\mu$ - $e$  conversion in nuclei occurring at one-loop level. We compare our results with the previously obtained constraints including the ones derived from the tree-level  $\mu$ - $e$  conversion. The latter process provides particularly restrictive bounds on some products of the couplings [4], but not for all of them. The loop induced conversion although suppressed by the loop factors is partly sensitive to different couplings than the tree-level process. Moreover, it was pointed out in Ref. [5] that in a wide class of models with effective interactions of four charged fermions, thus including the MSSM extended by  $R$ -parity violation, the  $\mu$ - $e$  conversion at one-loop is enhanced by large logarithms. Therefore, as will be seen, the  $\mu$ - $e$  conversion at loop level is more sensitive to certain combinations of couplings than the tree-level process.

The motivation for the present work is twofold. Firstly, our study enables us to put more restrictive bounds on some combinations of the couplings than do the previous works. Secondly, the expected improvements in the sensitivity of the  $\mu$ - $e$  conversion experiments running presently at PSI will make  $\mu$ - $e$  conversion the main probe of muon flavour conservation for most of the extensions of the standard model allowing for an order of magnitude more stringent tests of  $R$ -parity violation already in forthcoming months. Indeed, while the present bound on the conversion branching ratio in  ${}^{48}_{22}\text{Ti}$  is  $R_{\mu e}^{Ti} \lesssim 4.3 \cdot 10^{-12}$  [6], the SINDRUM II experiment taking currently data on gold,  ${}^{179}_{79}\text{Au}$ , should reach  $R_{\mu e}^{Au} \lesssim 5 \cdot 10^{-13}$  and next year on titanium  $R_{\mu e}^{Ti} \lesssim 3 \cdot 10^{-14}$  [7]. Furthermore, very recently a proposal for an experiment called MECO has been submitted to BNL [8] which is planned to achieve the sensitivity better than  $10^{-16}$ . The experimental prospects for searches of muon number violation have been recently reviewed by A. Czarnecki [9].

## 2 Previous constraints

To compare our analyses with the previous ones, we first collect and list the updated upper bounds on all products of  $R$ -parity violating couplings which are testable in  $\mu$ - $e$  conversion. Within the MSSM particle content the gauge

	previous bounds	$\mu-e$ at loop level/ $\sqrt{B}$	
	$m_{\tilde{f}} =$ 100 GeV	$m_{\tilde{f}} =$ 100 GeV	1 TeV
$ \lambda_{121} \lambda_{122} $	$6.6 \cdot 10^{-7}$ [30]	$4.2 \cdot 10^{-6}$	$3.2 \cdot 10^{-4}$
$ \lambda_{131} \lambda_{132} $	$6.6 \cdot 10^{-7}$ [30]	$5.3 \cdot 10^{-6}$	$3.9 \cdot 10^{-4}$
$ \lambda_{231} \lambda_{232} $	$5.7 \cdot 10^{-5}$ [29]	$5.3 \cdot 10^{-6}$	$3.9 \cdot 10^{-4}$
$ \lambda_{231} \lambda_{131} $	$6.6 \cdot 10^{-7}$ [30]	$8.4 \cdot 10^{-6}$	$6.4 \cdot 10^{-4}$
$ \lambda_{232} \lambda_{132} $	$1.1 \cdot 10^{-4}$ [29]	$8.4 \cdot 10^{-6}$	$6.4 \cdot 10^{-4}$
$ \lambda_{233} \lambda_{133} $	$1.1 \cdot 10^{-4}$ [29]	$1.7 \cdot 10^{-5}$	$1.0 \cdot 10^{-3}$

Table 1

Upper limits on the products  $|\lambda\lambda|$  testable in  $\mu-e$  conversion for two different scalar masses  $m_{\tilde{f}}=100$  GeV and  $m_{\tilde{f}}=1$  TeV. The previous bounds scale quadratically with the sfermion mass. The scaling factor  $B$  is defined in the text and currently  $B=1$ .

invariance and supersymmetry allow for the following  $R$ -parity violating superpotential [10]

$$W_{\mathcal{R}} = \lambda_{ijk} \widehat{L}_i \widehat{L}_j \widehat{E}_k^c + \lambda'_{ijk} \widehat{L}_i \widehat{Q}_j \widehat{D}_k^c + \lambda''_{ijk} \widehat{U}_i^c \widehat{D}_j^c \widehat{D}_k^c - \mu_i L_i H_2, \quad (1)$$

where  $\lambda_{ijk} = -\lambda_{jik}$  and  $\lambda''_{ijk} = -\lambda''_{ikj}$ . The  $\lambda$ ,  $\lambda'$  and  $\mu$  terms violate the lepton number, whereas the  $\lambda''$  terms violate the baryon number by one unit. The last bilinear term in Eq. (1) gives rise to interesting physics which has been studied elsewhere [11]. We will not consider these effects in the present paper and we take as our working model the MSSM extended by the trilinear  $R$ -parity violating terms in Eq. (1).

Simultaneous presence of  $L$  violating ( $\lambda_{ijk}, \lambda'_{ijk}$ ) and  $B$  violating ( $\lambda''_{ijk}$ ) couplings would lead to too fast proton decay unless  $|\lambda' \cdot \lambda''| < 10^{-9}$  [12] for squark masses below 1 TeV. Several examples exist of models, in which there are huge differences between the strengths of lepton and baryon number violating couplings. In models, which break  $R$ -parity spontaneously, only the lepton number is violated. There are also examples of GUT models, in which quarks and leptons are treated differently [13,14]. In the following we assume that only lepton number violating couplings are non-vanishing.

Severe limitations come also from baryogenesis considerations if one requires that the GUT scale baryon asymmetry is preserved. It has been shown in [15] that any bound from cosmology can be avoided by demanding that one of the lepton numbers is conserved over cosmological time scales. Since we wish to study the  $\mu-e$  conversion, we could have conservation of  $\tau$  lepton number. On the other hand, if electroweak baryogenesis is assumed (see, e.g. [16]), the

restrictions on lepton number are removed. Therefore we do not impose any extra constraints on lepton number violating couplings.

Upper bounds on the magnitudes of individual couplings in Eq. (1) have been determined from a number of different sources. In particular one has considered charged current universality,  $e - \mu - \tau$  universality, forward-backward asymmetry,  $\nu_\mu e$  scattering, atomic parity violation [17], neutrinoless double beta decay [18],  $\nu$  masses [13,19],  $Z$ -boson partial width [20],  $D^+$ ,  $\tau$ -decays [21],  $D^0 - \bar{D}^0$  mixing,  $K^+$ ,  $t$ -quark decays [22],  $b\bar{b}$  production at LEP II [23] and heavy nuclei decay and  $n - \bar{n}$  oscillation [24]. The bounds from these works have been collected and updated in recent reviews [25].

We should also note that very strong constraints on individual couplings  $\lambda'$  can follow from the bounds on electric dipole moments of fermions [26]. However, these bounds are qualitatively different from the previously mentioned ones, since they depend strongly on the existence of additional sources of CP-violation in the MSSM and the strength of the additional CP-violation. In fact, the upper bounds obtained from electric dipole moments apply for  $|\lambda'|^2 |A| \sin \phi$ , where  $|A|$  is the so-called  $A$ -term and  $\phi$  is its phase, rather than for  $\lambda'$ . If one makes the assumption that  $A$  is real and the only source of CP-violation in the MSSM is the CKM matrix then these limits are identically washed away. Since these bounds are very model dependent and easily avoidable we shall not use them in our work.

More recently there has been a lot of interest in deriving bounds on the products of two such couplings which are stronger than the products of known bounds on the individual couplings. In addition to giving rise to unobserved rare processes these products turn out to be interesting also from the model building point of view. For example, bounds on these products constrain considerably models which attribute the origin of  $R$ -parity violation to approximate  $U(1)$  [27] or  $U(2)$  [28] flavour symmetry. Products of the type  $\lambda\lambda'$  have been constrained from searches for muonium-antimuonium conversion [4],  $\mu \rightarrow e\gamma$  [29] and heavier lepton decays to three lighter ones [30]. Combinations of  $\lambda\lambda'$  have been bounded from the tree-level  $\mu$ - $e$  conversion,  $\tau$ - and  $\pi$ -decays [4], from  $K \rightarrow l_1 \bar{l}_2$  [30] and from  $B \rightarrow l_1 \bar{l}_2$  [31]. Upper limits on  $\lambda\lambda'$ -s are found from neutrinoless double beta decay [32],  $\mu \rightarrow e\gamma$  [29]<sup>5</sup>, tree-level  $\mu$ - $e$  conversion [4],  $\Delta m_K$ ,  $\Delta m_B$  [30], exotic top decays [33] and non-leptonic [34] and semi-leptonic [35] decays of heavy quark mesons.

We have presented the combinations of  $\lambda\lambda$  and  $\lambda'\lambda'$  contributing to the  $\mu$ - $e$  conversion in Table 1 and Table 2, respectively. The upper limits on these combinations of couplings from the previous works are presented in the ta-

---

<sup>5</sup> The bounds on  $\lambda'\lambda'$ -s in Ref. [29] should all be divided by 3 due to the omission of the colour factor in the calculation.

	previous bounds	$\mu$ - $e$ at tree-level/ $\sqrt{B}$	$\mu$ - $e$ at loop level/ $\sqrt{B}$	
	$m_{\tilde{f}} =$ 100 GeV	$m_{\tilde{f}} =$ 100 GeV	$m_{\tilde{f}} =$ 100 GeV	1 TeV
$ \lambda'_{211} \lambda'_{111} $	$5.0 \cdot 10^{-6}$	$8.0 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$8.3 \cdot 10^{-4}$
$ \lambda'_{212} \lambda'_{112} $	$1.4 \cdot 10^{-4}$	$8.5 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$8.3 \cdot 10^{-4}$
$ \lambda'_{213} \lambda'_{113} $	$1.4 \cdot 10^{-4}$	$8.5 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$8.3 \cdot 10^{-4}$
$ \lambda'_{221} \lambda'_{111} $	$5.0 \cdot 10^{-6}$	$4.0 \cdot 10^{-7}$	$3.7 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{222} \lambda'_{112} $	$4.8 \cdot 10^{-5}$ [30]	$4.3 \cdot 10^{-7}$	$3.7 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{223} \lambda'_{113} $	$4.8 \cdot 10^{-5}$ [30]	$4.3 \cdot 10^{-7}$	$3.7 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{231} \lambda'_{111} $	$8.8 \cdot 10^{-5}$	$1.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{232} \lambda'_{112} $	$4.8 \cdot 10^{-3}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{233} \lambda'_{113} $	$4.8 \cdot 10^{-3}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{211} \lambda'_{121} $	$4.8 \cdot 10^{-5}$ [30]	$4.0 \cdot 10^{-7}$	$7.3 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{212} \lambda'_{122} $	$4.8 \cdot 10^{-5}$ [30]	$4.3 \cdot 10^{-7}$	$7.3 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{213} \lambda'_{123} $	$4.8 \cdot 10^{-5}$ [30]	$4.3 \cdot 10^{-7}$	$7.3 \cdot 10^{-5}$	$2.5 \cdot 10^{-3}$
$ \lambda'_{221} \lambda'_{121} $	$1.4 \cdot 10^{-4}$	$8.0 \cdot 10^{-8}$	$2.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$
$ \lambda'_{222} \lambda'_{122} $	$1.4 \cdot 10^{-4}$	$2.1 \cdot 10^{-6}$	$2.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$
$ \lambda'_{223} \lambda'_{123} $	$1.4 \cdot 10^{-4}$	$2.1 \cdot 10^{-6}$	$2.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$
$ \lambda'_{231} \lambda'_{121} $	$2.6 \cdot 10^{-3}$	$2.0 \cdot 10^{-6}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{232} \lambda'_{122} $	$4.8 \cdot 10^{-3}$	$5.3 \cdot 10^{-5}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{233} \lambda'_{123} $	$4.8 \cdot 10^{-3}$	$5.3 \cdot 10^{-5}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{211} \lambda'_{131} $	$4.2 \cdot 10^{-4}$	$1.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{212} \lambda'_{132} $	$4.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{213} \lambda'_{133} $	$1.2 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-3}$	$8.3 \cdot 10^{-2}$
$ \lambda'_{221} \lambda'_{131} $	$4.2 \cdot 10^{-4}$	$2.0 \cdot 10^{-6}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{222} \lambda'_{132} $	$4.0 \cdot 10^{-3}$	$5.3 \cdot 10^{-5}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{223} \lambda'_{133} $	$1.2 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$	$4.0 \cdot 10^{-4}$	$2.2 \cdot 10^{-2}$
$ \lambda'_{231} \lambda'_{131} $	$2.6 \cdot 10^{-3}$ [29]	$8.0 \cdot 10^{-8}$	$8.7 \cdot 10^{-5}$	$3.2 \cdot 10^{-3}$
$ \lambda'_{232} \lambda'_{132} $	$2.6 \cdot 10^{-3}$ [29]	$1.3 \cdot 10^{-3}$	$8.7 \cdot 10^{-5}$	$3.2 \cdot 10^{-3}$
$ \lambda'_{233} \lambda'_{133} $	$4.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-3}$	$8.7 \cdot 10^{-5}$	$3.2 \cdot 10^{-3}$

Table 2

Upper limits on the products  $|\lambda'\lambda'|$  for two different scalar masses  $m_{\tilde{f}}=100$  GeV and  $m_{\tilde{f}}=1$  TeV. The previous bounds and the tree-level bounds scale quadratically with the sfermion mass. The scaling factor  $B$  is defined in the text and currently  $B = 1$ .

bles in the first columns. For  $\lambda'\lambda'$ -s the tree-level  $\mu$ - $e$  conversion bounds are presented separately in the second column of Table 2. All the couplings are assumed to be real. If the previously obtained bound on a particular product of couplings is stronger than the product of the bounds on the individual couplings then we have also presented in the tables the references from which the bound has been taken. In the opposite case the references are not given and can be found for example in Ref. [25]. The tree-level  $\mu$ - $e$  bounds are all taken from Ref. [4] assuming that there are no cancellations between different contributing terms. The bounds are given for two values, 100 GeV and 1 TeV, of the relevant supersymmetric scalar mass. If the listed bounds depend on the mass quadratically, which is the case for the bounds derived from tree-level processes, we have presented them for 100 GeV only. Extrapolation to 1 TeV can be done trivially by scaling the bound by two orders of magnitude. The quadratic dependence on the mass is also approximately true for the  $\lambda\lambda$ -type couplings in the loop-induced  $\mu \rightarrow e\gamma$ , but not for the  $\lambda'\lambda'$ -type couplings involving the third family. In this case the upper bound for  $m_{\tilde{f}} = 1$  TeV becomes  $|\lambda'_{23i}\lambda'_{13i}| < 1.9 \cdot 10^{-2}$ ,  $i = 1, 2$  [29].

The most stringent constraint on the products  $|\lambda_{122}\lambda'_{211}|$ ,  $|\lambda_{132}\lambda'_{311}|$ ,  $|\lambda_{121}\lambda'_{111}|$  and  $|\lambda_{231}\lambda'_{311}|$  follows from the tree-level  $\mu$ - $e$  conversion and is  $4.0 \cdot 10^{-8}$  for  $m_{\tilde{f}} = 100$  GeV.

### 3 Coherent $\mu$ - $e$ conversion

Due to the extraordinary sensitivity of ongoing experiments searching for  $\mu$ - $e$  conversion in nuclei, it can be expected that in the nearest future this process will become the main test of muon flavour conservation [5]. To take full advantage of the  $\mu$ - $e$  experiments in studying possible new physics we present a general formalism for the coherent  $\mu$ - $e$  conversion. In practice only the appropriate expressions for the coherent transition are needed, since the coherent process is enhanced by the large coherent nuclear charge and it dominates over the incoherent transition. Indeed, for the nuclei of interest the coherent rate of the conversion constitutes about 91-94% of the total rate [36].

The theory of  $\mu$ - $e$  conversion in nuclei was first studied by Weinberg and Feinberg [37] and the early developments in this field have been summarised in [38]. Since then various nuclear models and approximations are used in the literature to calculate the coherent  $\mu$ - $e$  conversion nuclear form factors. It is important to notice that the results from the shell model [39], local density approximation [36] as well as quasi-particle RPA approximation [40] do not differ significantly from each other for both  ${}^{48}_{22}Ti$  and  ${}^{208}_{82}Pb$  nuclei showing consistency in understanding of nuclear physics involved [40]. We follow the notation of Ref. [36] taking into account the corrections to the approximation

from the exact calculations performed in the same work. The corrections are negligible for  ${}^{48}_{22}\text{Ti}$  as the local density approximation works better for light nuclei but they are sizable, of the order of 40%, for  ${}^{208}_{82}\text{Pb}$  and  ${}^{179}_{79}\text{Au}$ .

The relevant  $\mu$ - $e$  conversion effective Lagrangian can be expressed as

$$\mathcal{L}_{eff} = \frac{4\pi\alpha}{q^2} j_{(ph)}^\lambda J_\lambda^{(ph)} + \frac{G}{\sqrt{2}} \sum_{i,m} \left( j_{i(non)}^\lambda J_\lambda^{i(non)} + j_{m(non)} J^{m(non)} \right), \quad (2)$$

where the first term describes the photonic conversion and the second set of terms describe different contributions to the non-photonic conversion mechanisms. Here  $q^2$  denotes the photon momentum transfer,  $G$  the effective coupling constant appropriate for the interactions considered,  $j^\lambda$  and  $j$  represent the leptonic and  $J^\lambda$  and  $J$  hadronic vector and scalar currents, respectively. The photonic mechanism is enhanced by small  $q^2$  but can occur only at loop level and is thus suppressed by loop factors. The non-photonic mode is of interest, if the conversion can occur at tree-level like in models with non-diagonal  $Z'$  [41] and Higgs [42] couplings, models with leptoquarks [43], or if the loop contributions are enhanced by some other mechanism, e.g., models with non-decoupling of massive neutrinos [44]. In models with broken  $R$ -parity, one may expect that both modes are important since the photonic mode is enhanced by large logarithms.

Most generally the leptonic current for the photonic mechanism can be parametrised as

$$j_{(ph)}^\lambda = \bar{e} \left[ (f_{E0} + \gamma_5 f_{M0}) \gamma_\nu \left( g^{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2} \right) + (f_{M1} + \gamma_5 f_{E1}) i \sigma^{\lambda\nu} \frac{q_\nu}{m_\mu} \right] \mu \quad (3)$$

where the form factors  $f_{E0}$ ,  $f_{E1}$ ,  $f_{M0}$  and  $f_{M1}$  can be computed from the underlying theory. Since the interaction of photon with fermions is well known, the hadronic current can in this case be written down immediately in terms of the nucleon spinor  $\bar{\Psi} = (\bar{p}, \bar{n})$  as [38]

$$J_\lambda^{(ph)} = \bar{\Psi} \gamma_\lambda \frac{1}{2} (1 + \tau_3) \Psi, \quad (4)$$

where  $\tau_3$  is the isospin Pauli matrix.

We do not consider non-photonic conversion mode in this Letter. However, since in the context of the  $\mu$ - $e$  conversion only the vector currents and the corresponding nuclear matrix elements have been discussed in the literature (see, e.g., Ref. [41]) while most of the models contain also new scalar particles, we feel that the following comment is in order. To calculate the tree-level  $\mu$ - $e$

conversion rate, one has to proceed from the quark level to the nucleon level by computing the quark current matrix elements  $\langle N | \bar{q} \gamma_\lambda q | N \rangle = G_V^{(q,N)} \bar{N} \gamma_\lambda N$  and  $\langle N | \bar{q} q | N \rangle = G_S^{(q,N)} \bar{N} N$  for the nucleon  $N = p, n$  (the coherent transition rate depends only on the quark vector or scalar couplings). Due to the conservation of the vector current and its coherent character, with the vector charge equal to the quark number, and assuming strong isospin symmetry, one can determine in the limit  $q^2 \approx 0$   $G_V^{(u,p)} = G_V^{(d,n)} \equiv G_V^{(u)} = 2$  and  $G_V^{(d,p)} = G_V^{(u,n)} \equiv G_V^{(d)} = 1$ . However, in the case of the scalar operator  $\bar{q} q$  one has to rely on nucleon models. Here we point out two extreme situations. In the limit of the fully non-relativistic quark model, for which  $\bar{q} q$  can be basically regarded as a number operator, it is known that  $G_S \approx G_V$ . This also determines the upper bounds for the scalar form factors. The largest differences between the form factors are expected to occur in the case of fully relativistic models like the MIT bag model. In the latter case one has estimated the scalar form factors for massless quarks to be [45]  $G_S^{(u)} = 4/3$  and  $G_S^{(d)} = 2/3$ . The actual values of the scalar form factors thus lay in between these two limiting cases. Therefore the constraints derived from the tree-level  $\mu$ - $e$  conversion in Ref. [4] can be somewhat relaxed due to this uncertainty but not more than a factor of 2/3 or so.

Due to the large mass of muon it is appropriate to take the customary non-relativistic limit for the motion of the muon in the muonic atom. In this limit the "large" component of the muon wave function factorizes out when calculating the coherent conversion rate. Therefore, for the photonic mechanism the coherent  $\mu$ - $e$  conversion branching ratio  $R_{\mu e}^{ph}$  can be expressed as

$$R_{\mu e}^{ph} = C \frac{8\pi\alpha^2}{q^4} p_e E_e \frac{|M(p_e)|^2}{\Gamma_{capt}} Z^2 \xi_0^2, \quad (5)$$

where  $E_e$  is the electron energy,  $E_e \approx p_e \approx m_\mu$  for this process,  $\Gamma_{capt}$  is the total muon capture rate,  $|M(p_e)|^2$  is the nuclear matrix element squared and

$$\xi_0^2 = |f_{E0} + f_{M1}|^2 + |f_{E1} + f_{M0}|^2 \quad (6)$$

shows the process dependence on the photonic form factors. The expression for  $|M(p_e)|^2$  in the local density approximation,

$$|M(q)|^2 = \frac{\alpha^3 m_\mu^3}{\pi} \frac{Z_{eff}^4}{Z} |\overline{F}_p(q)|^2, \quad (7)$$

the correction factors  $C$  to the approximation (compared with the exact calculation) as well as all numerical values of the above defined quantities for



${}^{48}_{22}Ti$  and  ${}^{208}_{82}Pb$  are presented in Ref. [36,46]. The result reads

$$R_{\mu e}^{ph} = C \frac{8\alpha^5 m_\mu^5 Z_{eff}^4 |\overline{F}_p(p_e)|^2 Z}{\Gamma_{capt}} \cdot \frac{\xi_0^2}{q^4}, \quad (8)$$

where  $C^{Ti} = 1.0$ ,  $C^{Pb} = 1.4$ ,  $Z_{eff}^{Ti} = 17.61$ ,  $Z_{eff}^{Pb} = 33.81$ ,  $\Gamma_{capt}^{Ti} = 2.59 \cdot 10^6$  s $^{-1}$ ,  $\Gamma_{capt}^{Pb} = 1.3 \cdot 10^7$  s $^{-1}$  and the proton nuclear form factors are  $\overline{F}_p^{Ti}(q) = 0.55$  and  $\overline{F}_p^{Pb}(q) = 0.25$ . Currently SINDRUM II experiment is running on gold,  ${}^{179}_{79}Au$ , but gold is not explicitly treated in Ref. [36]. However, since  $Z^{Au} = 79$  and  $Z^{Pb} = 82$  are so close to each other then, within errors, all the needed quantities for  ${}^{179}_{79}Au$  and  ${}^{208}_{82}Pb$  are approximately equal<sup>6</sup>. This result is strongly supported by theoretical calculations and experimental measurements of the total muon capture rate of  $Pb$  and  $Au$  [46]. In the numerical values we use the same experimental and theoretical input for both  ${}^{179}_{79}Au$  and  ${}^{208}_{82}Pb$ .

#### 4 Bounds from the photonic $\mu$ - $e$ conversion

The  $\mu$ - $e$  conversion in supersymmetric theories which conserve  $R$ -parity has been studied before in several works [47]. They conclude that other lepton flavour violating processes like  $\mu \rightarrow e\gamma$  are about an order of magnitude more sensitive to the new phenomena than  $\mu$ - $e$  conversion. However, in the case of broken  $R$ -parity one may expect the opposite result due to the logarithmic enhancement of  $\mu$ - $e$  conversion, which is about one order of magnitude at the amplitude level.

Addition of Eq. (1) to the MSSM superpotential leads to more interactions in the model, but the MSSM particle content remains. The relevant part of the Lagrangian is found by standard techniques [48]

$$\begin{aligned} L_{\cancel{U},\lambda,\lambda'} = & \lambda_{ijk} [\tilde{\nu}_{iL}\bar{e}_{kR}e_{jL} + \tilde{e}_{jL}\bar{e}_{kL}\nu_{iL} + \tilde{e}_{kR}^*\overline{\nu_{iL}^c}e_{jL} - (i \leftrightarrow j)] \\ & + \lambda'_{ijk} \left[ (\tilde{\nu}_{iL}\bar{d}_{kR}d_{jL} + \tilde{d}_{jL}\bar{d}_{kR}\nu_{iL} + \tilde{d}_{kR}^*\overline{\nu_{iL}^c}d_{jL}) \right. \\ & \left. - K_{jp}^\dagger (\tilde{e}_i\bar{d}_{kR}u_{pL} + \tilde{u}_{pL}\bar{d}_{kR}e_{iL} + \tilde{d}_{kR}^*\overline{e_{iL}^c}u_{pL}) \right] + h.c., \end{aligned} \quad (9)$$

where we have taken into account the flavour-mixing effects in the up-quark sector in terms of the CKM matrix elements  $K_{jp}$ . The effects of possible non-alignment of fermion and sfermion mass matrices are severely constrained [49] and therefore neglected.

<sup>6</sup> We thank H.C. Chiang and E. Oset for clarifying us this point.

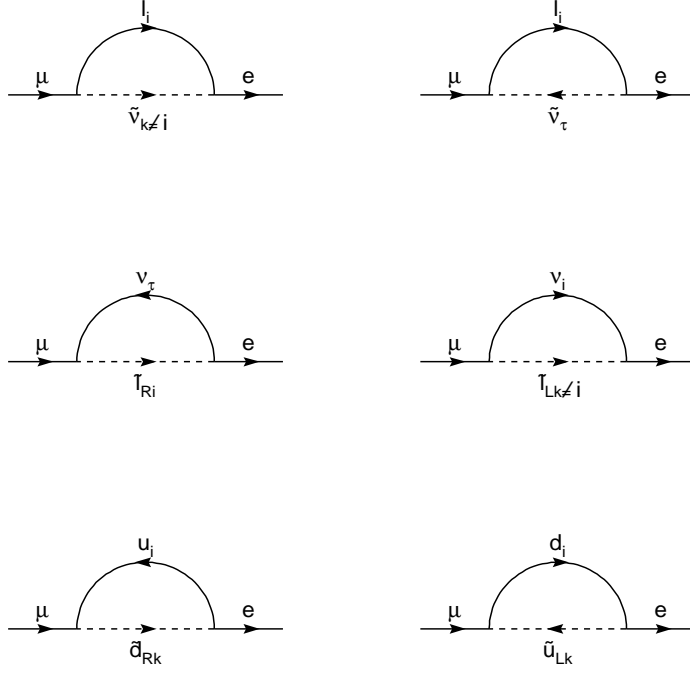


Fig. 1. Contributing loop diagrams for  $\mu$ - $e$  conversion. Unless otherwise indicated  $i, k = 1 \dots 3$ .

The Feynman diagrams contributing to the  $\mu$ - $e$  conversion at one-loop are shown in Fig. 1. The photon line is not shown, but it should be attached in all possible ways to the graphs. Following the general effective Lagrangian analyses of Ref. [5] we can say immediately that the first two diagrams with leptons in the loop give a logarithmically enhanced contribution to the  $\mu$ - $e$  conversion while the second two involving neutrino as an intermediate state are not enhanced. This is because in the latter set of diagrams one cannot attach the photon to the fermion line. For the typical fermion and sfermion masses we have  $|\ln(m_f^2/m_{\tilde{f}}^2)| \sim \mathcal{O}(10)$  and, therefore, the contribution of the second set of diagrams can be neglected without affecting our numerical results. For the rest of the diagrams we compute the photonic form factors defined in Eq. (3). Starting from the Lagrangian (9) and using dimensional regularization we obtain for the form factors proportional to  $\gamma_\lambda$

$$f_{E0}^{f=\ell} = \pm f_{M0}^{f=\ell} = -\frac{2(\lambda\lambda)}{3(4\pi)^2} \frac{-q^2}{m_{\tilde{f}}^2} \left( \ln \frac{-q^2}{m_{\tilde{f}}^2} + F_f(r) \right), \quad (10)$$

$$f_{E0}^{f=u} = -f_{M0}^{f=u} = -\frac{(\lambda'\lambda')N_c}{9(4\pi)^2} \frac{-q^2}{m_{\tilde{f}}^2} \left( \ln \frac{-q^2}{m_{\tilde{f}}^2} + F_f(r) - \frac{1}{12} \right), \quad (11)$$

$$f_{E0}^{f=d} = -f_{M0}^{f=d} = -\frac{(\lambda'\lambda')N_c}{18(4\pi)^2} \frac{-q^2}{m_{\bar{f}}^2} \left( \ln \frac{-q^2}{m_{\bar{f}}^2} + F_f(r) - \frac{1}{3} \right), \quad (12)$$

and for the form factors proportional to  $\sigma_{\mu\nu}$

$$f_{M1}^{f=\ell} = \mp f_{E1}^{f=\ell} = -\frac{(\lambda\lambda)}{3(4\pi)^2} \frac{m_\mu^2}{m_{\bar{f}}^2}, \quad (13)$$

$$f_{M1}^{f=u} = f_{E1}^{f=u} = -\frac{(\lambda'\lambda')N_c}{24(4\pi)^2} \frac{m_\mu^2}{m_{\bar{f}}^2}, \quad (14)$$

$$f_{M1}^{f=d} = f_{E1}^{f=d} = 0. \quad (15)$$

Here the superscripts  $f = \ell$ ,  $f = u$  and  $f = d$  denote contributions from the diagrams with leptons,  $u$ - and  $d$ -quarks in the loop, respectively,  $r = m_\mu^2/(-q^2)$ , where  $f$  stands for the virtual fermion in the loop,  $N_c = 3$  is the number of colours and

$$F_f(r) = -\frac{1}{3} + 4r + \ln r + (1-2r)\sqrt{1+4r} \ln \left( \frac{\sqrt{1+4r}+1}{\sqrt{1+4r}-1} \right). \quad (16)$$

There are three important limiting cases. If  $r \rightarrow 0$  then  $F_f = -1/3$ , if  $r \approx 1$  then  $F_f \approx 1.52$  and if  $r \gg 1$  then  $F_f = \ln r + 4/3$ .

As expected the form factors (10), (11), (12) contain large logarithms while the form factors (13), (14), (15) do not. The dependence of the form factors on particular combination of  $R$ -parity violating couplings as well as on the  $\pm$  signs in the form factors coming from the leptonic loops should be fixed when calculating the constraints on the products of couplings. Deriving these expressions we have taken into account that  $q^2 < 0$  and made a simplifying approximation  $x_f = m_\mu^2/m_{\bar{f}}^2 \ll 1$  which allows us to keep only the leading terms in  $x_f$ . This approximation is not adequate only in one case, i.e., if the fermion in the loop is top quark and  $m_{\bar{f}} = 100$  GeV. However, in this case the  $\mu$ - $e$  conversion is completely dominated by the diagram with the bottom quark in the loop and the top contribution can be neglected without making significant numerical error. Note also that in the first approximation Eq. (15) vanishes. For the expressions of  $f_{M1,E1}$  in higher orders of  $x_f$  we refer the reader to Ref. [29].

Now we are ready to constrain  $R$ -parity violating couplings. Substituting the numerical values of the parameters to Eq. (8) we find

$$R_{\mu e}^{ph} = 8.6 (12.3) \cdot 10^{-4} \text{TeV}^4 \frac{\xi_0^2}{q^4}, \quad (17)$$

where the first number in the expression corresponds to the conversion in  $Ti$  and the number in the brackets to the conversion in  $Pb$  and  $Au$  and  $\xi_0^2$  is defined in Eq. (6). Note that the conversion is somewhat enhanced in  $Pb$  and  $Au$  (in fact, maximised [36]) if compared with  $Ti$ . Substituting the current best bound on the conversion branching ratio  $R_{\mu e}^{Ti} \lesssim 4.3 \cdot 10^{-12}$  [6] and the form factors above to Eq. (17) we find the constraints on the products of  $R$ -parity violating couplings from  $\mu$ - $e$  conversion at one-loop level. The bounds on the contributing combinations of  $\lambda\lambda$  are listed in Table 1 and on the combinations of  $\lambda'\lambda'$  in Table 2 for two values of sfermion masses  $m_{\tilde{f}} = 100$  GeV and 1 TeV.

In future new experimental data on  $\mu$ - $e$  conversion will be available. To take into account the future improvements in the experimental sensitivity we define a scaling factor  $B$  as

$$B = \frac{R_{\mu e}^{Ti}(\text{present bound})}{R_{\mu e}^{Nu}(\text{future bound})} \times \begin{cases} 1.0, & \text{if } Nu = Ti, \\ 2.8, & \text{if } Nu = Au. \end{cases} \quad (18)$$

To get new bounds one just has to divide the bounds in Table 1 and Table 2 by the corresponding  $\sqrt{B}$ .

## 5 Discussion and conclusions

The photonic  $\mu$ - $e$  conversion which occurs at one-loop level probes the coupling products of the type  $\lambda\lambda$  and  $\lambda'\lambda'$ . For the combinations of  $\lambda\lambda$  comparison with the previously obtained bounds in Table 1 shows that in three cases out of six our new bounds are more stringent. This is because the conversion is enhanced by large logarithms  $\ln(m_{\tilde{f}}^2/m_f^2)$ . If SINDRUM II experiment will reach the expected sensitivity and show negative results one has to scale the bounds down by factors of  $\sqrt{B^{Au}} = 5$  and  $\sqrt{B^{Ti}} = 12$  which correspond to the expected limits  $R_{\mu e}^{Au} \lesssim 5 \cdot 10^{-13}$  and  $R_{\mu e}^{Ti} \lesssim 3 \cdot 10^{-14}$ , respectively. In this case the  $\mu$ - $e$  conversion bounds in Table 1 will be more stringent than the previous bounds for *all* testable combinations of  $\lambda\lambda$ . This is an important conclusion since no that significant improvements of the other experiments is expected. Note that in  $\mu$ - $e$  conversion  $\lambda\lambda$ -s can be probed only at loop level.

The  $\lambda'\lambda'$ -type couplings give also rise to the tree-level  $\mu$ - $e$  conversion. The bounds derived from this process are very strong for most products of the couplings but not for all of them. The reason is that for some combinations of the couplings, especially if the third family squarks are involved, their contribution to the  $\mu$ - $e$  conversion can be strongly suppressed by small off-diagonal CKM matrix elements. In the worst case the suppression factor can be as large as  $\lambda_W^6$ , where  $\lambda_W \sim 0.2$  is the Wolfenstein parameter. As a result the

tree-level  $\mu$ - $e$  conversion bounds are usually more stringent than the bounds from the other processes except for three cases. The bound on  $|\lambda'_{223}\lambda'_{133}|$  obtained by multiplying the bounds on the individual couplings is stronger than the  $\mu$ - $e$  conversion bound (note also that the previous bound on  $|\lambda'_{213}\lambda'_{133}|$  is only marginally weaker than the tree-level  $\mu$ - $e$  result). Importantly, as the result of our calculation, the bounds on  $|\lambda'_{232}\lambda'_{132}|$  and  $|\lambda'_{233}\lambda'_{133}|$  derived from the *loop* induced  $\mu$ - $e$  conversion are more stringent than the ones from the tree-level  $\mu$ - $e$  conversion. With the new data all the  $\mu$ - $e$  conversion bounds on  $|\lambda'\lambda'|$ -s will be more stringent than the others. Since the  $B$  factors for the loop induced and tree-level conversions are the same then the last two entries in Table 2 remain most stringently constrained by the loop results.

In conclusion, searches for the  $\mu$ - $e$  conversion in nuclei, taking into account both the photonic conversion mode studied in this Letter and the non-photonic mode, constrain most of the products of  $R$ -parity violating couplings testable in this process more stringently than do the other processes. In the nearest future this conclusion is expected to apply for all the products.

## Acknowledgements

M.R. thanks A. van der Schaaf for information about ongoing experiments at PSI and J. Bernab e, H.C. Chiang, E. Oset and V. Vento for discussions on nuclear physics involved in  $\mu$ - $e$  conversion. This work has been supported in part by CICYT (Spain) under the grant AEN-96-1718.

## References

- [1] G. Farrar and P. Fayet, Phys. Lett. **B76** (1978) 575.
- [2] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. **D48** (1993) 4352; K. Huitu and J. Maalampi, Phys. Lett. **B344** (1995) 217.
- [3] For extensive literature see, e.g., G. Altarelli, CERN-TH-97-195, hep-ph/9708437, and references therein.
- [4] J.E. Kim, P. Ko and D.-G. Lee, Phys. Rev. **D56** (1997) 100.
- [5] M. Raidal and A. Santamaria, FTUV/97-56, hep-ph/9710389.
- [6] SINDRUM II Collaboration, C. Dohmen *et al.*, Phys. Lett. **B317** (1993) 631.
- [7] A. van der Schaaf (spokesman of SINDRUM II), PSI proposal R-87-03; and private communication.

- [8] M. Bachman *et al.*, BNL Proposal P940, 1997.
- [9] A. Czarnecki, hep-ph/9710425.
- [10] S. Weinberg, Phys. Rev. **D26** (1982) 287; N. Sakai and T. Yanagida, Nucl. Phys. **B197** (1982) 533; C.S. Aulakh and R.N. Mohapatra, Phys. Lett. **B119** (1982) 136; L.J. Hall and M. Suzuki, Nucl. Phys. **B231** (1984) 419; J. Ellis *et al.*, Phys. Lett. **B150** (1985) 142; S. Dawson, Nucl. Phys. **B261** (1985) 297; S. Dimopoulos and L.J. Hall, Phys. Lett. **B207** (1987) 210.
- [11] J.C. Romao, A. Ioannissyan and J.W.F. Valle, Phys. Rev. **D55** (1997) 427; M.A. Diaz, J.C. Romao and J.W.F. Valle, hep-ph/9706315, and references therein.
- [12] A. Smirnov and F. Vissani, Phys. Lett. **B380** (1996) 317.
- [13] L.J. Hall and M. Suzuki, in Ref. [10].
- [14] R. Hempfling, Nucl. Phys. **B478** (1996) 3.
- [15] H. Dreiner and G.G. Ross, Nucl. Phys. **B410** (1993) 188.
- [16] M. Fukugita and T. Yanagida, Phys. Lett. **B174** (1986) 45.
- [17] V. Barger, G.F. Giudice and T. Han, Phys. Rev. **D40** (1989) 2987.
- [18] R.N. Mohapatra, Phys. Rev. **D34** (1986) 3457; M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys. Rev. Lett. **75** (1995) 17, and Phys. Rev. **D53** (1996) 1329.
- [19] S. Dimopoulos and L.J. Hall, in Ref. [10] ; R.M. Godbole, P. Roy and X. Tata, Nucl. Phys. **B401** (1993) 67; A.S. Joshipura, V. Ravindran and S.K. Vempati, hep-ph/9706482.
- [20] G. Bhattacharyya, J. Ellis and K. Sridhar, Mod. Phys. Lett. **A10** (1995) 1583; G. Bhattacharyya, D. Choudhury and K. Sridhar, Phys. Lett. **B355** (1995) 193; J. Ellis, S. Lola and K. Sridhar, Phys. Lett. **B408** (1997) 252.
- [21] G. Bhattacharyya and D. Choudhury, Mod. Phys. Lett. **A10** (1995) 1699.
- [22] K. Agashe and M. Graesser, Phys. Rev. **D54** (1996) 4445.
- [23] J. Erler, J.L. Feng and N. Polonsky, Phys. Rev. Lett. **78** (1997) 3063.
- [24] R. Barbieri and A. Masiero, Nucl. Phys. **B267** (1986) 679; S. Dimopoulos and L.J. Hall, Phys. Lett. **B196** (1987) 135; J.L. Goity and M. Sher, Phys. Lett. **B346** (1995) 69, erratum Phys. Lett. **B385** (1996) 500.
- [25] G. Bhattacharyya, Nucl. Phys. Proc. Suppl. **52A** (1997) 83; and IFUP-TH 43/97, hep-ph/9709395; H. Dreiner, hep-ph/9707435.
- [26] M. Frank and H. Hamidian, CUMQ/HEP 97, hep-ph/9706510.
- [27] T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. **D52** (1995) 5319.

- [28] R. Barbieri, G. Dvali and L. Hall, Phys. Lett. **B377** (1996) 76; R. Barbieri, L. Hall, S. Raby and A. Romanino, Nucl. Phys. **B493** (1997) 3; R. Barbieri, L. Hall and A. Romanino, Phys. Lett. **B401** (1997) 47; G. Bhattacharyya, IFUP-TH 31/97, hep-ph/9707297.
- [29] M. Chaichian and K. Huitu, Phys. Lett. **B384** (1996) 157.
- [30] D. Choudhury and P. Roy, Phys. Lett. **B378** (1996) 153.
- [31] J.-H. Jang, Y.G. Kim and J.S. Lee, Phys. Rev. **D55** (1997) 367.
- [32] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **75** (1995) 2276.
- [33] J.M. Yang, B.-L. Young and X. Zhang, AMES-HET-97-5, hep-ph/9705341.
- [34] C.E. Carlson, P. Roy and M. Sher, Phys. Lett. **B357** (1995) 99; C.-D. Lü and D.-X. Zhang, Phys. Lett. **B392** (1997) 193.
- [35] J.-H. Jang, Y.G. Kim and J.S. Lee, Phys. Lett. **B408** (1997) 367; KAIST-TH 97/21, hep-ph/9711504.
- [36] H.C. Chiang *et al.*, Nucl. Phys. **A559** (1993) 526.
- [37] S. Weinberg and G. Feinberg, Phys. Rev. Lett. **3** (1959) 111, e: *ibid.*, p. 244.
- [38] J.D. Vergados, Phys. Rep. **133** (1986) 1.
- [39] T.S. Kosmas and J.D. Vergados, Nucl. Phys. **A510** (1990) 641.
- [40] T.S. Kosmas, Amand Faessler and J.D. Vergados, J. Phys. **G23** (1997) 693.
- [41] J. Bernabeu, E. Nardi and D. Tommasini, Nucl. Phys. **B409** (1993) 69.
- [42] D. Ng and J.N. Ng, Phys. Lett. **B320** (1994) 181.
- [43] S. Davidson, D. Bailey and B. Campell, Z. Phys. **C61** (1994) 613.
- [44] D. Ng and J.N. Ng, Phys. Lett. **B331** (1994) 371; D. Tommasini *et al.*, Nucl. Phys. **B444** (1995) 451; G. Barenboim and M. Raidal, Nucl. Phys. **B484** (1997) 63.
- [45] T.A. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12** (1975) 2060; F.E. Close, *An Introduction to Quarks and Partons*, Academic Press, 1979, p. 420.
- [46] H.C. Chiang, E. Oset and P. Fernandez de Cordoba, Nucl. Phys. **A510** (1990) 591.
- [47] See, e.g., R. Barbieri and L. Hall, Phys. Lett. **B338** (1994) 212; R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. **B445** (1995) 219; T.S. Kosmas and J.D. Vergados, Phys. Rep. **264** (1996) 251, and references therein.
- [48] H. Haber and G. Kane, Phys. Rep. **117** (1985) 75.
- [49] D. Choudhury, F. Eberlein, A. König, J. Louis, S. Pokorski, Phys. Lett. **B342** (1995) 180; B. de Carlos, J.A. Casas, J.M. Moreno, Phys. Rev. **D53** (1996) 6398.