

Bounding Effective Operators at the One-Loop Level: The Case of Four-Fermion Neutrino Interactions

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Abstract

The contributions of non-standard four-neutrino contact interactions to electroweak observables are considered at the one-loop level by using the effective quantum field theory. The analysis is done in terms of three unknown parameters: the strength of the non-standard neutrino interactions, \tilde{F} , an additional derivative coupling needed to renormalize the divergent contributions that appear when the four-neutrino interactions are used at the loop level and a non-standard non-derivative $Z\text{-}\bar{\nu}\nu$ coupling. Then, the precise measurements of the invisible width of the Z -boson at LEP and the data on the neutrino deep-inelastic scattering yield the result $\tilde{F} = (-100 \pm 140)G_F$. Assuming that there are no unnatural cancellations between the contributions of the three effective couplings a much stronger bound is obtained: $|\tilde{F}| \lesssim 2G_F$, which is a factor 200 better than the one obtained in previous analyses based on tree level calculations.

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1 Introduction

Until now the Standard Model [1] of electroweak interactions (SM) has passed very successfully all precision experimental tests. Especially intensive have been the studies of the different four-fermion processes between leptons and quarks. In the SM such reactions are mediated by the electroweak vector bosons. The large body of experimental data collected at different energy scales (from practically zero-energy up to the mass of the Z -boson) confirms that, indeed, four-fermion interactions are mediated by gauge bosons.

Nevertheless, there is a widespread belief among theoreticians that the current theory of electroweak interactions is only an effective low-energy limit of a more fundamental theory. If there is some new dynamics beyond the SM, it might result in some deviations from the SM predictions for four-fermion processes. For example, standard fermions could take part in processes with the exchange of some non-standard intermediate state. If the mass of the intermediate particle is larger than the Fermi scale, then, at the Fermi scale (and below) the new interaction can be described by effective four-fermion operators suppressed by $1/M^2$, where M is a scale of the order of the mass of the heavy intermediate particle.

Obviously one of the most elusive among the non-standard four-fermion interactions is that which involves only neutrinos. This type of interactions can naturally appear in models with extra neutral gauge bosons or new scalars. Here we will assume simply that this interaction exists without asking about its particular origin.

It is clear that if some “secret” neutrino interaction (SNI) exists, it can only be tested indirectly.

The first studies of possible non-standard ν - ν interactions were performed many years ago [2, 3]. In ref. [3] different weak processes sensitive to such an interaction were investigated for a SNI with pure vector form

$$\mathcal{L}^{\nu-\nu} = F(\bar{\nu}\gamma_\alpha\nu)(\bar{\nu}\gamma^\alpha\nu). \quad (1)$$

In particular the SNI, which contributes to the decays $\pi^+ \rightarrow e^+\nu_e\bar{\nu}\nu$ and $K^+ \rightarrow l^+\nu_l\bar{\nu}\nu$ ($l = e, \mu$), could modify the lepton energy spectra in K^+ and π^+ decays. From an analysis of these spectra the following bounds on the coupling F were obtained [3]

$$|F| \leq 10^7 G_F, \quad |F| \leq 2 \times 10^6 G_F, \quad (2)$$

where G_F denotes the weak Fermi constant.

Similar bounds were found [3] from the absence of leptons with “wrong” charge in the process $\nu_\mu + N \rightarrow \mu^+ + \nu_\mu + \nu_\mu + X$.

Later on these bounds were improved in a special experiment [4] searching for the decay $K^+ \rightarrow \mu^+\nu_\mu\bar{\nu}\nu$. From the negative result of this experiment the following limit was set:

$$F \leq 1.7 \times 10^5 G_F. \quad (3)$$

The reason why bounds on the non-standard neutrino interaction coming from low-energy experiments are so loose is evident. The SNI contributes only to the decays with four particles in the final state, and such processes are strongly suppressed by phase space compared with the standard leptonic π and K decays.

In ref. [5] the width of the decay $Z \rightarrow \nu\bar{\nu}\nu\bar{\nu}$ was calculated in the presence of a non-standard ν - ν interaction of the general vector and axial-vector form:

$$\mathcal{L}^{\nu-\nu} = F \sum_{i,j=e,\mu,\tau} (\bar{\nu}_i O_{i\alpha} \nu_i) (\bar{\nu}_j O_j^\alpha \nu_j), \quad (4)$$

where

$$O_i^\alpha = a_i \gamma^\alpha P_L + b_i \gamma^\alpha P_R, \quad (5)$$

$P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$ are the left and right chirality projectors and F, a_i, b_i are real parameters. The coupling F has dimension $[M]^{-2}$. From the invisible width of the Z -boson, which has been precisely measured at LEP, and assuming three generations of light neutrinos and lepton universality, the following bounds were obtained [5] ($\tilde{F} = F a_i^2$):

$$|\tilde{F}| \leq 390 G_F, \quad (6)$$

in the case of the $V - A$ structure of the non-standard interaction and

$$|\tilde{F}| \leq 710 G_F \quad (7)$$

for the pure vector case. As the phase-space suppression is much smaller in the case of the Z -boson decay, these last bounds are much better than those obtained from π - and K -meson decays.

Taking all previous bounds, however, the “secret” effective four-neutrino interaction could still be much stronger than the one predicted by the minimal SM¹, $\tilde{F} = G_F/\sqrt{2}$. All previous bounds are extracted from processes in which the new interaction is the only relevant one and, therefore, observables depend quadratically on \tilde{F} . Obviously if the new interaction enters in loop corrections to a SM process, modifications come through its interference with the SM amplitude and, then, the deviations from the SM predictions will depend linearly on the coupling \tilde{F} .

For example, ν - ν interactions will contribute to the decay $Z \rightarrow \bar{\nu}\nu$ at the one-loop level (see Figs. 1a and b) and consequently to the invisible width of the Z -boson. It is very simple to estimate the order of magnitude of the corresponding one-loop corrections:

$$\frac{\Delta\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{\nu}\nu}} \approx \frac{\tilde{F} M_Z^2}{(4\pi)^2}. \quad (8)$$

As the invisible width of the Z -boson is now measured with an accuracy better than 1% [10], one finds the following bound on the non-standard coupling \tilde{F} :

$$\tilde{F} \leq (1-10) \cdot G_F. \quad (9)$$

Clearly, from this rough estimate and from our previous discussion one expects stronger bounds on the “secret” ν - ν interaction coming from the one-loop analysis than those which follow from its contribution to the invisible width of the Z -boson at tree-level, eq. (6) and eq. (7).

The above estimate of the loop effects of the four-neutrino interactions is rather naïve, because the one-loop calculation is actually divergent: four-neutrino interactions are not renormalizable. This does not preclude us obtaining some information on them from loops, as long as the appropriate framework is used to obtain finite non-ambiguous results. This framework is the effective quantum field theory (EQFT) [11, 12]. In this language all operators allowed by the symmetries of the problem are already present in the effective Lagrangian from the beginning, therefore, there always exists a counterterm available to

¹Some information about ν - ν interactions was obtained also from astrophysical data[6, 7, 8]. The corresponding bounds are weaker than those of eq. (6) and eq. (7). Bounds coming from primordial nucleosynthesis can be much stronger [9] when the four-neutrino interaction involves both left- and right-handed neutrinos.

absorb any divergence that could appear in loop calculations. The number of the effective operators is generally infinite and experimental observables depend on an infinite number of unknown couplings². However, the effects of higher-dimension operators are suppressed in low-energy processes and one can truncate the Lagrangian by keeping only a finite number of operators. Using the EQFT language one has a well-defined prescription to calculate loop effects of non-renormalizable operators. The price that has to be paid is that it is not possible to analyse effects of one operator independently of other operators that mix with it under renormalization. Under certain assumptions one can reduce the basis of operators that mix. The more assumptions one makes the stronger will be the bounds one obtains on the couplings of the effective operators. The less assumptions one makes the more reliable will be the bounds obtained.

For example, if we want to analyse an operator that contributes to experimental observables at the one-loop level we can use a “minimal” set of operators (which, in general, does not form a closed basis) that contains the operator in question plus all the operators that mix “directly” with it at the one-loop level[13].

In this letter we discuss the procedure of bounding effective operators by considering the case of the four-neutrino contact interaction and obtain constraints on this elusive interaction from the processes in which it contributes at the one-loop level.

2 Effective $Z\bar{\nu}\nu$ vertex in the presence of the four-neutrino interaction

We first calculate one-loop corrections to the Z -neutrino coupling due to a SNI of the general V, A form given in eq. (4). Note that interactions mediated by scalars can also be written in this form after a Fierz transformation. The flavour structure could be, however, more general. For simplicity we will only consider the flavour structure of eq. (4). On the other hand, it has been shown very recently [9] that four-neutrino interactions that involve both, left-handed and right-handed neutrinos are strongly bounded by cosmological arguments: if these kind of interactions are strong enough they would keep the three right-handed neutrinos in thermal equilibrium at the time of nucleosynthesis, therefore, disturbing it. But it is important to realise, that nucleosynthesis gives no bound at all if either a_i or b_i are zero in eq. (5), because in both cases right-handed neutrinos are completely decoupled. Interactions of only right-handed neutrinos are not interesting, therefore, in the analysis we could restrict ourselves to interactions among only left-handed neutrinos, that is $a_i \neq 0$ and $b_i = 0$. However, for the sake of generality we will keep for the moment the two interactions as given by eq. (5).

Finally we would like to remark that interactions of the form (4) are not $SU(2)$ gauge invariant by themselves. To write them in an explicitly gauge invariant form we would need to include an additional interaction among charged leptons (and neutrino-leptons as well) with exactly the same coupling. But such interactions, at least those involving electrons, are strongly bounded by different experiments. Therefore, if four-neutrino interactions are part of an $SU(2)$ gauge invariant interactions they can be bounded indirectly through the bounds on the four-fermion interactions involving charged leptons. On the other hand, the general effective Lagrangian, eq. (4), can be obtained from a gauge-invariant effective Lagrangian after spontaneous symmetry breaking. From a phenomenological point of view,

²Note, however, that if the effective theory is a low-energy limit of some known renormalizable theory (see e.g. [13]) all effective couplings can be expressed in terms of the few parameters of the underlying theory.

however, we could ask how strong four-neutrino interactions can be independently of any additional assumption.

We will use the naïve dimensional regularization scheme (with anticommuting γ_5). Then, there are two one-loop diagrams, shown in Figs. 1a and b, contributing to the $Z\bar{\nu}\nu$ vertex³. The corresponding amplitudes are given by

$$T_a(Z \rightarrow \nu_i \bar{\nu}_i) = -\frac{g}{2c_W} \mu^\epsilon \frac{F}{(4\pi)^2} \frac{4}{3} q^2 \left(\Delta_\epsilon(q^2) + \frac{5}{3} \right) \sum_{j=e,\mu,\tau} a_j \bar{u}(k') O_i^\alpha u(k) \epsilon(q)_\alpha, \quad (10)$$

$$T_b(Z \rightarrow \nu_i \bar{\nu}_i) = -\frac{g}{2c_W} \mu^\epsilon \frac{F}{(4\pi)^2} \frac{4}{3} q^2 \left(\Delta_\epsilon(q^2) + \frac{2}{3} \right) a_i^2 \bar{u}(k') \gamma^\alpha P_L u(k) \epsilon(q)_\alpha. \quad (11)$$

Here $q = k' - k$ is the four-momentum of the Z -boson, $c_W = \cos\theta_W$ is the cosine of the weak mixing angle, and the summation in T_a runs over the different neutrino-types in the loop; $u(k)$ denotes a Dirac spinor, ϵ_α is the wave function of the Z -boson. Finally, μ is the dimensional regularization mass parameter and $\epsilon = 2 - D/2$ with D the space-time dimension. Both diagrams are divergent. In dimensional regularization this divergence appears as a simple pole, $1/\epsilon$, in the function

$$\Delta_\epsilon = \frac{1}{\epsilon} - \gamma + \log(4\pi) - \log\left(\frac{-q^2 - i\eta}{\mu^2}\right) \equiv \frac{1}{\hat{\epsilon}} - \log\left(\frac{-q^2 - i\eta}{\mu^2}\right). \quad (12)$$

In our analysis we will assume lepton universality for the three generations of neutrinos. Then, we can rewrite the sum of T_a and T_b in the form

$$T = T_a + T_b = -\frac{g}{2c_W} \mu^\epsilon G_F q^2 \sum_{A=L,R} c_1^A (\gamma_{12}^A \Delta_\epsilon(q^2) + \kappa_{12}^A) \bar{u}(k') \gamma^\alpha P_A u(k) \epsilon(q)_\alpha, \quad (13)$$

which has been expressed in terms of the renormalized (scale-dependent) dimensionless couplings

$$c_1^L = \frac{F a_i^2}{G_F}, \quad c_1^R = \frac{F a_i b_i}{G_F} \quad (14)$$

and the following constants

$$\gamma_{12}^L = \frac{1}{3\pi^2}, \quad \gamma_{12}^R = \frac{1}{4\pi^2}, \quad \kappa_{12}^L = \gamma_{12}^L \frac{17}{12}, \quad \kappa_{12}^R = \gamma_{12}^R \frac{5}{3}. \quad (15)$$

As we see, the SNI generates, at the one-loop level, a derivative coupling of the Z -boson to neutrinos that is divergent. In order to obtain a finite amplitude for processes involving the $Z\bar{\nu}\nu$ vertex, the effective Lagrangian should contain a term able to absorb this divergence in its coupling. This term can be written as

$$\mathcal{L}_2 = -\frac{g}{2c_W} \mu^\epsilon G_F \sum_{i=e,\mu,\tau} \sum_{A=L,R} (c_2^A + \Delta c_2^A) (\bar{\nu}_i \gamma^\alpha P_A \nu_i) \partial^\beta Z_{\beta\alpha}, \quad (16)$$

where $Z_{\beta\alpha} = \partial_\beta Z_\alpha - \partial_\alpha Z_\beta$; $c_2^A(\mu)$ ($A = L, R$) are the $\overline{\text{MS}}$ renormalized couplings and the corresponding counterterms are

$$\Delta c_2^A = -c_1^A \gamma_{12}^A \frac{1}{\hat{\epsilon}}.$$

It is important to remark that since we are not assuming $SU(2)$ invariance for the effective Lagrangian, a direct (non-derivative) non-standard coupling of the Z boson to neutrinos

³There is no wave-function renormalization of the external neutrinos in our case. Such massless tadpole-like diagrams are zero in dimensional regularization.

is in principle possible. Such a coupling could be generated from a gauge-invariant effective Lagrangian after spontaneous symmetry breaking. Although in dimensional regularization and for massless neutrinos it is not needed, there is no symmetry that forbids it. In fact it will appear naturally if other regularization schemes are used. Therefore, in order to be completely general we will include it in the analysis and will see how it affects our bounds. This additional interaction has the form

$$\mathcal{L}_\delta = -\frac{g}{2c_W}\mu^\epsilon \sum_{i=e,\mu,\tau} \sum_{A=L,R} \delta^A Z_\alpha (\bar{\nu}_i \gamma^\alpha P_A \nu_i) . \quad (17)$$

The contribution of the operators (16) and (17) is schematically shown in Fig. 1c. Then, the full renormalized $Z\bar{\nu}\nu$ vertex will be given by the sum of the three diagrams of Fig. 1:

$$\hat{T} = -\frac{g}{2c_W} \sum_{A=L,R} g_A(q^2) \bar{u}(k') \gamma^\alpha P_A u(k) \epsilon(q)_\alpha , \quad (18)$$

where

$$g_A(q^2) = \delta^A(\mu) + G_F q^2 \left(c_2^A(\mu) + c_1^A(\mu) \left(\gamma_{12}^A \left(\log(\mu^2/|q^2|) + i\pi\theta(q^2) \right) + \kappa_{12}^A \right) \right) , \quad A = L, R . \quad (19)$$

Here, $\delta^A(\mu)$ gives the contribution of the direct non-derivative Z -neutrino interactions. The running couplings in our approximation (we neglect all contributions with gauge bosons running in the loops) are given by

$$c_1^A(\mu) \approx c_1^A(\mu_0) \quad (20)$$

$$c_2^A(\mu) \approx c_2^A(\mu_0) + c_1^A(\mu_0) \gamma_{12}^A \log \left(\frac{\mu_0^2}{\mu^2} \right) , \quad (21)$$

where μ_0 is some reference scale. Thus, the effective four-neutrino operator at the one-loop level contributes to the running of the coupling of the operator (16) and we have to consider mixing between at least these two operators⁴. The coupling $\delta(\mu)$ does not mix with the other couplings because it correspond to an operator of different dimension, then $\delta^A(\mu) \approx \delta^A(\mu_0)$.

On the other hand, because the standard $Z\bar{\nu}\nu$ coupling only involves left-handed neutrinos, the lowest-order effect of the non-standard vertex eq. (18) comes via its interference with the standard coupling and therefore only the real part of the left-handed vertex in eq. (18) will be relevant for our analysis. Thus, “new physics” depends on three unknown parameters, $\delta^L(\mu)$, $c_1^L(\mu)$ and $c_2^L(\mu)$. In addition, as commented above, couplings to right-handed neutrinos are strongly bounded from cosmological data.

Obviously, to put independent bounds on these three parameters in processes which depend only on the induced effective Z -neutrino vertex one needs experimental information obtained at least at three different energy scales. We would like to note that the behaviour of the three terms is quite different, while the terms proportional to c_1^L and c_2^L are also proportional to q^2 the δ^L term is independent of q^2 and can be bounded at very low energies.

⁴Obviously, there are many other four-fermion operators like $(\bar{l}l)(\bar{\nu}\nu)$, etc., which also mix with the Z -neutrino coupling (16). But as we are neglecting loops with gauge bosons, they do not mix directly at the one-loop level with the four-neutrino operator and, as they can be strongly bounded from other processes, we will disregard them.

3 Neutrino neutral-current experiments and bounds on the four-neutrino contact interaction

In this section we will consider the bounds on the four-neutrino coupling (and other effective couplings entering in the $Z\nu\nu$ vertex (18)), which follow from neutrino neutral-current experiments.

As we discussed above, in our approximation, the effect of the non-standard operators is taken into account by three renormalized coupling constants $\delta^L(\mu)$, $c_1^L(\mu)$ and $c_2^L(\mu)$. Since physical results are independent from μ we can freely choose this scale. We will take $\mu = M_Z$ as the reference scale and define, for further use,

$$\delta = \delta^L(M_Z), \quad c_1 = c_1^L(M_Z), \quad c_2 = c_2^L(M_Z), \quad \gamma = \gamma_{12}^L = \frac{1}{3\pi^2}, \quad \kappa = \kappa_{12}^L = \frac{17}{36\pi^2}. \quad (22)$$

Then all our observables will depend on the quantity

$$\text{Re} \{g_L(q^2)\} = \delta + G_F q^2 (c_2 + c_1 \kappa + c_1 \gamma \log(M_Z^2/|q^2|)). \quad (23)$$

We will first consider bounds from the precise measurement of the invisible Z -width at LEP. From the effective vertex (18) we can easily obtain the partial decay width of the Z -boson into two neutrinos. It can be written in the following form:

$$\Gamma(Z \rightarrow \bar{\nu}\nu) = \Gamma^{SM}(Z \rightarrow \bar{\nu}\nu) + \Delta\Gamma_{\bar{\nu}\nu}, \quad (24)$$

where $\Gamma^{SM}(Z \rightarrow \bar{\nu}\nu)$ is the SM contribution (including radiative corrections) and $\Delta\Gamma_{\bar{\nu}\nu}$ contains the effects of the non-standard operators. At lowest order it comes from the interference of the non-standard amplitude with the SM amplitude and we have

$$\Delta\Gamma_{\bar{\nu}\nu} = \Gamma^{SM}(Z \rightarrow \bar{\nu}\nu) 2 \text{Re} \{g_L(M_Z^2)\}, \quad (25)$$

where the function $\text{Re} \{g_L(q^2)\}$ is given by eq. (23) and for $q^2 = M_Z^2$ it is

$$\text{Re} \{g_L(M_Z^2)\} = \delta + G_F M_Z^2 (c_2 + c_1 \kappa). \quad (26)$$

Assuming that there are three generations of neutrinos, the non-standard contribution to the invisible width of the Z -boson is

$$\Delta\Gamma_{invis} = 3\Delta\Gamma_{\bar{\nu}\nu}, \quad (27)$$

where $\Delta\Gamma_{\bar{\nu}\nu}$ is given by eq. (25). On the other hand this quantity can also be expressed as

$$\Delta\Gamma_{invis} = \Gamma_{invis} - 3 \left(\frac{\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{l}l}} \right)^{SM} \Gamma_{\bar{l}l}. \quad (28)$$

The r.h.s. of eq. (28) is constructed only from observables measured at LEP [10]:

$$\Gamma_{invis} = 497.6 \pm 4.3 \text{ MeV}, \quad \Gamma_{\bar{l}l} = 83.87 \pm 0.27 \text{ MeV} \quad (29)$$

(we use the combined result from the four LEP experiments [10]) and the ratio of the neutrino and charged leptons partial widths calculated within the SM

$$\left(\frac{\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{l}l}} \right)^{SM} = 1.992 \pm 0.003. \quad (30)$$

The central value of the above quantity corresponds to $m_{top} = 150$ GeV and the small error is due to the variation of the mass of the top quark in the range $100 \text{ GeV} < m_{top} < 200 \text{ GeV}$. Using (29) and (30) we obtain

$$\Delta\Gamma_{invis} \simeq -4 \pm 5 \text{ MeV} . \quad (31)$$

From this experimental result and from our calculation of the extra contributions to the invisible Z decay width we obtain

$$-0.009 \leq \delta + G_F M_Z^2 (c_2 + c_1 \kappa) \leq 0.001 . \quad (32)$$

It is obvious that from this equation we cannot get bounds on all three couplings δ , c_1 and c_2 unless additional assumptions are considered. In equation (32) one can consider two situations:

1. There are no unnatural cancellations among the three terms δ , $\kappa G_F M_Z^2 c_1$ and $G_F M_Z^2 c_2$. Then each of them can be bounded independently of the others and we obtain:

$$|c_1| \leq \frac{0.009}{\kappa G_F M_Z^2} = 2, \quad |c_2| \leq \frac{0.009}{G_F M_Z^2} = 0.09, \quad |\delta| \leq 0.009 \quad (33)$$

2. $\delta \approx G_F M_Z^2 c_2 \approx \kappa G_F M_Z^2 c_1$. In this case there could be cancellations among the three terms. However, even if there are cancellations at this particular scale (M_Z) there will be no cancellations at other scales. In what follows we will show that, also in the case of cancellations at LEP energies, it is still possible to get some interesting bounds on the coupling c_1 if additional data obtained at different scales are used.

Several types of experiments are sensitive to the neutral current neutrino interactions at different q^2 scales. As the non-standard operators (4) and (16) contribute to the derivative $Z\bar{\nu}\nu$ coupling and this contribution is proportional to q^2 we can get some reasonable additional information on the couplings c_1 and c_2 only from DIS experiments at high energy ($-q^2 \simeq 100 - 1000 \text{ GeV}^2$). However, the direct Z -neutrino non-derivative interaction, given by δ , contributes with the same strength to any energy, therefore, it can be bounded also in low-energy experiments, e.g. in the elastic $\bar{\nu}$, ν -electron scattering ($-q^2 \simeq 10^{-2} \text{ GeV}^2$). Since DIS experiments are more precise and are performed at different energy scales, we will mainly use their results in our analysis.

There are several high-precision DIS measurements [15, 16, 17], CDHS, CHARM and CCFR. The first two experiments are performed with neutrino beam peek energies of about 50 GeV and the same energy spectrum, while CCFR operates with an average energy of 161 GeV. As we will see this gap in energies will be enough for our purposes.

The most interesting quantity measured in DIS experiments is the ratio of the neutral-current to charged-current cross sections for neutrino beams

$$R_\nu = \frac{\sigma_\nu^{NC}}{\sigma_\nu^{CC}} . \quad (34)$$

Again as in a case of $\Gamma(Z \rightarrow \bar{\nu}\nu)$ the theoretical prediction for this quantity can be written as a sum of the standard and the non-standard contributions:

$$R_\nu = R_\nu^{SM} + \Delta R_\nu . \quad (35)$$

Using our effective $Z\bar{\nu}\nu$ vertex, eq. (18), we obtain

$$\Delta R_\nu = \frac{2 \int dx \int dy \frac{d\sigma_\nu^{NC}}{dxdy} \operatorname{Re} \{g_L(q^2)\}}{\int dx \int dy \frac{d\sigma_\nu^{CC}}{dxdy}}, \quad (36)$$

where $d\sigma_\nu^{NC}/dxdy$ and $d\sigma_\nu^{CC}/dxdy$ are differential neutral- and charged-current cross sections calculated within the SM, and x and y are usual DIS variables. The momentum transfer squared is given by

$$q^2 = -2E_{beam}M_pxy \quad (37)$$

with E_{beam} the beam energy in the laboratory frame and M_p the proton mass. The function $\operatorname{Re} \{g_L(q^2)\}$ is defined in eq. (23).

The final result of the analysis of the experimentally measured ratios R_ν is usually presented in terms of the value of weak mixing angle $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$. The quoted values are⁵:

$$\begin{aligned} \text{CDHS [15]} & : 0.2225 \pm 0.0066 \\ \text{CHARM [16]} & : 0.2319 \pm 0.0065 \\ \text{CCFR [17]} & : 0.2218 \pm 0.0059 . \end{aligned} \quad (38)$$

On the other hand the same quantity can be obtained from LEP and collider (UA2, CDF) data with very high precision. The average is [10]

$$\text{LEP} + M_W : \sin^2 \theta_W^{LEP} = 0.2255 \pm 0.0005 . \quad (39)$$

Using $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ as an input, one can obtain the predictions for R_ν , which in the case of neutrinos scattered off an approximately isoscalar target are given by the tree level expression [14]

$$R_\nu^{SM} = \frac{1}{2} - s_W^2 + \frac{5}{9}s_W^4 (1 + R^{cc}) , \quad (40)$$

where $R^{cc} = \sigma_\nu^{CC}/\sigma_\nu^{NC} \approx 0.4$ is the ratio of the antineutrino–neutrino charged-current cross sections.

In our analysis we neglect radiative corrections and parton-model corrections to the non-standard contribution ΔR_ν . Then, the deviations from the standard result are given by the difference between the values of R_ν obtained by using the $\sin^2 \theta_W^i$ measured in DIS experiments and those obtained by using $\sin^2 \theta_W^{LEP}$ measured at LEP. For every experiment we have:

$$\Delta R_\nu^i = R_\nu(\sin^2 \theta_W^i) - R_\nu(\sin^2 \theta_W^{LEP}) , \quad (41)$$

where ΔR_ν^i is given by eq. (36) and the index $i = \text{CDHS, CHARM, CCFR}$ refers to the different conditions of the different experiments (beam energy and the cut on the y -variable), which influence the calculation (36). In eq. (41) the predictions $R_\nu(\sin^2 \theta_W)$ are calculated according to the tree-level expression (40) and using values for $\sin^2 \theta_W$ given by (38) and (39). Then, for ΔR_ν^i we have:

$$\text{CDHS} : + 0.0022 \pm 0.0046 \quad (42)$$

$$\text{CHARM} : - 0.0039 \pm 0.0044 \quad (43)$$

$$\text{CCFR} : + 0.0026 \pm 0.0042 . \quad (44)$$

⁵The actual values we use are taken from a recent publication of the CCFR collaboration [17]; there, $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ is given for the same masses of the charm quark, $m_c = 1.31 \pm 0.24$ GeV, and the top quark $m_t = 150$ GeV for all experiments[15, 16, 17].

Using these experimental values we can obtain the bounds on the coupling c_1 , even if there are cancellations among the different couplings. Before doing the complete numerical analysis we will do a simple estimate of the bounds, which, as we will see, works very well. For this estimate we rewrite eq. (36) as

$$\Delta R_\nu = R_\nu 2 \operatorname{Re} \left\{ g_L(q_i^2) \right\} , \quad (45)$$

where q_i^2 is some effective average of q^2 for the experiment chosen in order to reproduce the complete result. We use that CDHS and CHARM experiments are performed with the same neutrino beam, then we average their results and obtain

$$CERN = CDHS + CHARM : \quad \Delta R_\nu = -0.00085 \pm 0.0032 , \quad \left| q_N^2 \right| \approx 14 \text{ GeV}^2 . \quad (46)$$

For CCFR we have

$$CCFR : \quad \Delta R_\nu = +0.0026 \pm 0.0042 , \quad \left| q_R^2 \right| \approx 45 \text{ GeV}^2 . \quad (47)$$

Then, the bounds from the different experiments can be expressed as

$$|\delta + G_F M_Z^2 (c_2 + c_1 \kappa)| \leq b_L \quad (48)$$

$$|\delta + G_F q_R^2 (c_2 + c_1 \kappa + c_1 \gamma \log(M_Z^2 / |q_R^2|))| \leq b_R \quad (49)$$

$$|\delta + G_F q_N^2 (c_2 + c_1 \kappa + c_1 \gamma \log(M_Z^2 / |q_N^2|))| \leq b_N , \quad (50)$$

From the estimates in eq. (32), eq. (46) and eq. (47), and taking into account that $R_\nu \approx 0.314$ we have

$$b_L = 0.009 , \quad b_R = 0.0110 , \quad b_N = 0.0051 , \quad (51)$$

Using error propagation, from eqs. (48)–(50) we can extract bounds for the different couplings δ , c_2 and c_1 . Taking into account that $|q_N^2|, |q_R^2| \ll M_Z^2$ we obtain

$$|\delta| \leq (b_N / (1 - z)) \sqrt{1 - z^2 b_R / b_N} = 0.012 \quad (52)$$

$$|c_2| \leq \kappa \sqrt{b_N^2 + b_R^2} / \left(\gamma G_F |q_R^2| \log(M_Z^2 / |q_R^2|) (1 - z) \right) = 10 \quad (53)$$

$$|c_1| \leq \sqrt{b_N^2 + b_R^2} / \left(\gamma G_F |q_R^2| \log(M_Z^2 / |q_R^2|) (1 - z) \right) = 222 . \quad (54)$$

Here $z = (|q_N^2| \log(M_Z^2 / |q_N^2|)) / (|q_R^2| \log(M_Z^2 / |q_R^2|)) \approx 0.4$. These are reliable bounds in the case of cancellations among the contributions of the different operators.

In the complete analysis, we did a three-parameter fit, in δ, c_2, c_1 , of the theoretical expressions eq. (25) and eq. (36) to the data. We would like to note that in the numerical calculation we used rather simple parametrizations of the parton distribution functions [18]. However, different choices of the structure functions do not change our results noticeably. The reason for this is that our non-standard contributions are mainly sensitive to the parton distributions at large values of the x and y variables.

As a result of the three-parameter fit to the full body of data we obtain the following bounds at 68% C.L.

$$\delta = 0.004 \pm 0.009 \implies |\delta| \lesssim 0.013 \quad (55)$$

$$c_2 = 4.7 \pm 7 \implies |c_2| \lesssim 12 \quad (56)$$

$$c_1 = -100 \pm 140 \implies |c_1| \lesssim 240 . \quad (57)$$

These constraints are in a good agreement with our estimate in eqs. (52)–(54). The extreme values of \tilde{F} , of order $\sim 240G_F$, are possible only because of large cancellations between the contributions of the three non-standard couplings. If one decides that such cancellations are unnatural, then one obtains a much better bound for the contact four-neutrino interaction. The complete analysis gives in this case

$$|\tilde{F}| \lesssim 2G_F . \quad (58)$$

The above bounds can be improved in the future. In the case of cancellation between the different couplings, the bounds are defined essentially by the errors of the experiments at lower energies; therefore, only better DIS data can improve the bound in eq. (57), especially if DIS experiments are performed at higher energies. If there are no cancellations between the different couplings, the higher scale experiment is the relevant one, and future improvements of the measurement of the invisible width of the Z at LEP will be very important.

4 Conclusions

In this letter we have obtained new constraints on non-standard four-neutrino interactions coming from their contribution at the one-loop level to the invisible width of the Z -boson and to deep inelastic scattering. The bounds obtained from a conservative model-independent analysis of the “secret” neutrino interactions at the one-loop level improve at least by a factor 2 previous constraints, (6) and (7), that were obtained from the study of the non-standard $Z \rightarrow 4\nu$ decay. If there are no unnatural cancellation between the contributions of the various non-standard couplings, a much stronger bound on the strenght of a four-neutrino interaction has been obtained. This bound is 200 times better than previous constraints.

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Figure captions

Figure 1a–c: Diagrams that give contributions to the $Z\bar{\nu}\nu$ vertex in the presence of the non-standard four-neutrino interaction. In diagram (a), neutrinos of different flavours are running in the loop.

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