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The Hyperchargeless Triplet Majoron Model

A. Santamaria*

Department of Physics, Carnegie Mellon University,

Pittsburgh, PA 15213, USA

Abstract

We study the general conditions to maintain the scale of the lepton number breaking VEV at the electroweak scale. It is shown that the only possibilities are if the main component of the resulting majoron is a hyperchargeless complex triplet or a neutral singlet. Models with a hyperchargeless triplet, even though phenomenologically more interesting, seem to be very difficult to build because they like to break charge conservation. However we have found a particular extension, by adding an additional neutral singlet, that solves this problem. The model can give a Majorana mass to the neutrinos in the eV range, $\mu \to e\ \gamma$ can proceed with branching ratios at the verge of the present experimental limit and there are no additional decay modes of the Z^0 into invisible particles.

*Also at Departament de Física Teòrica, Universitat de València and IFIC, Universitat de València-CSIC.

1. Introduction

The standard model of electro-weak interactions predicts massless neutrinos. This is because i) in the model there are no right-handed neutrinos to combine with left-handed neutrinos to generate a Dirac mass term and ii) the minimal Higgs content of the model leads to the automatic conservation of lepton number.

Although there is no definite experimental evidence for massive neutrinos, there are some indications for non-vanishing neutrino masses. If neutrinos are massive at all it is necessary to extend the standard theory by adding new scalars, new fermions and/or new gauge bosons. If nature chooses the first possibility the neutrino mass terms must be necessarily Majorana mass terms, thus the lepton number must be broken either explicitly or spontaneously. The second case is the most interesting for it implies the existence of a new Goldstone boson, associated with the spontaneous breakdown of lepton number, called the majoron [1]. The majoron presents, in general, a very interesting phenomenology that can be used to test the models apart from the direct neutrino experiments which are difficult to perform if the neutrino masses are very small. Thus, for example, the Gelmini-Roncadelli model (GR)[2,3] will be tested at LEP because the decay of the Z^0 gauge boson to neutral scalars contributes to the decay width like two more generations of light neutrinos [3].

Majoron models, in general, can be classified in two groups : i) models in which the lepton number is broken by the vacuum expectation value (VEV) of a SU(2) singlet. Here the scale of lepton number breaking is quite arbitrary but in most of the cases it is required to be very large [1], moreover the phenomenology is very limited. ii) models with the lepton number broken by a nonsinglet VEV, like the model of Gelmini-Roncadelli [2] or the doublet majoron model[4]. In these kind of models the lepton number breaking VEV cannot be larger than the electroweak scale because it contributes to the gauge boson masses. In addition, as the majoron has gauge couplings, the phenomenology is much more interesting [3,5,6].

In the existing nonsinglet majoron models [2,4,7] the majoron has a tree-level coupling to electrons directly proportional to the lepton number breaking VEV. This coupling makes the reaction $\gamma + e \to e\, J$ proceed at a very high rate in the cores of red giant stars. As the majoron scarcely interacts with ordinary matter it escapes freely producing a too fast cooling of the star. This puts a bound on the coupling, which allows us to obtain a very strong bound on the lepton number breaking VEV $v < 10 - 100\ KeV$ [3,8] six orders of magnitude below the electroweak scale. This poses a problem of naturalness. Indeed it has been shown [9,10] that at the one loop level there will appear in general a correction to the small VEV proportional to the large VEV. Although, at this level, this correction can be avoided by fine tuning the parameters in the Higgs potential, at higher loops it is not clear what will happen.

In any case it would be nice if we can find a majoron model in which the scale of lepton number breakdown is not so different from the electro-weak scale. This is the main purpose of this paper. In section 2 we study the necessary conditions to avoid the astrophysical bound on the lepton number breaking VEV. In section 3 we propose a simple extension of the standard model containing a hyperchargeless complex triplet of scalars to implement these conditions and we comment on some of the phenomenological issues of such a model. In section 4 we analyze the Higgs potential and the pattern of symmetry breaking. It turns out that the model, in its minimal configuration, is inconsistent because the true minimum of the potential breaks charge conservation. The problem can be solved by adding a neutral scalar singlet carrying lepton number, but even in that case the solution depends crucially on the singlet-doublet-triplet coupling. The singlet VEV can be of the same order of magnitude as the triplet VEV and the majoron a roughly equal combination of triplet and singlet. Thus, the phenomenological implications of the model can be maintained. Finally in section 5 we collect the main results of the paper.

2. The majoron coupling to electrons

In all the majoron models we will have, at least, two broken $U(1)$ symmetries, $U(1)_Y$ of hypercharge and $U(1)_l$ of lepton number. The Noether's currents associated with these symmetries are

$$J^l_\mu = iL\chi^\dagger \overleftrightarrow{\partial}_\mu \chi + J^l_\mu(f) + \cdots$$
$$J^Y_\mu = \frac{i}{2}\varphi^\dagger \overleftrightarrow{\partial}_\mu \varphi + iy\chi^\dagger \overleftrightarrow{\partial}_\mu \chi + J^Y_\mu(f) + \cdots \tag{1}$$

where φ is the standard doublet, χ is the new Higgs multiplet which carries lepton number L and weak hypercharge y. $J^l_\mu(f)$ and $J^Y_\mu(f)$ are the fermionic parts of the currents. If the symmetry is broken spontaneously we must perform a shift in the neutral components of the fields

$$\varphi^{(0)} = u + \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)\,, \quad \chi^{(0)} = v + \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2) \tag{2}$$

Thus

$$J^l_\mu = -L\sqrt{2}v\partial_\mu\chi_2 + J^l_\mu(f) + \cdots$$
$$J^Y_\mu = -\partial_\mu\left(\frac{u}{\sqrt{2}}\varphi_2 + \frac{2yv}{\sqrt{2}}\chi_2\right) + J^Y_\mu(f) + \cdots \tag{3}$$

The Goldstone bosons associated with the breaking of each symmetry are given by the linear combinations of the fields that appear in the derivative terms. However the two linear combinations are not orthogonal. As we know that the Goldstone boson associated to the breaking of hypercharge is the one "eaten" by the Z^0 gauge boson, the majoron will be the orthogonal linear combination

$$\begin{aligned} G^0 &= \cos\theta\ \varphi_2 + \sin\theta\ \chi_2 \\ J &= -\sin\theta\ \varphi_2 + \cos\theta\ \chi_2 \end{aligned} \tag{4}$$

with

$$\sin\theta = \frac{2yv}{\sqrt{u^2 + 4y^2v^2}} \tag{5}$$

From eq. (3) substituting eq. (4), taking derivatives and solving for $\partial^\mu\partial_\mu J$ we obtain the equation of motion for the majoron.

$$\partial^\mu\partial_\mu J - \frac{1}{\cos\theta}\partial^\mu\left[\frac{1}{L\sqrt{2}v} J^l_\mu(f) - \sqrt{\frac{2}{u^2 + 4y^2v^2}}\sin\theta\ J^Y_\mu(f)\right] + \cdots = 0 \tag{6}$$

We are interested only in majoron couplings to electrons, so we can take $J^l_\mu(f) = -\overline{e_{aL}}\gamma_\mu e_{aL} - \overline{e_{aR}}\gamma_\mu e_{aR}$ and $J^Y_\mu(f) = -\frac{1}{2}\overline{e_{aL}}\gamma_\mu e_{aL} - \overline{e_{aR}}\gamma_\mu e_{aR}$. Obviously the lepton current for electrons is vectorial (e_L and e_R carry the same lepton number), thus if there is no mixing among charged leptons with different lepton number, and using the free Dirac equation for electrons, we obtain $\partial_\mu(\bar{e}_R\gamma^\mu e_R + \bar{e}_L\gamma^\mu e_L) = 0$. Doing the same with the hypercharge current we obtain $\partial_\mu(-\bar{e}_R\gamma^\mu e_R - \frac{1}{2}\bar{e}_L\gamma^\mu e_L) = -\frac{1}{2}\partial_\mu(\bar{e}_R\gamma^\mu e_R) = -i\frac{m_e}{2}\bar{e}\gamma_5 e$ Thus, after integration of the equation of motion (eq. (6)) the linear coupling of the majoron to electrons is

$$\mathcal{L}_{eJ} = i\frac{m_a}{\sqrt{2}u}\frac{2yv}{\sqrt{u^2+4y^2v^2}}J\ \overline{e_a}\gamma_5 e_a \tag{7}$$

To obtain this coupling, we have used several times the equations of motion, thus one would expect it to be valid only on mass-shell. However it can be shown that it is valid off-mass-shell, as well. This is because the coupling of eq. (7) is a Yukawa coupling and in the parameterization of eq. (2) Yukawa couplings are the only ones allowed in the fundamental lagrangian.

From eq. (7) it is obvious that, in general, the majoron only couples to electrons at tree level through the mixing with the standard doublet. This can be understood because the majoron multiplet cannot couple to two electrons directly ($\bar{e}\tilde{O}e\chi^{(0)}$ does not conserve lepton number). Therefore, all the coupling comes through the mixing with the standard doublet.

In eq. (7) we see that the coupling is proportional to the hypercharge of the majoron multiplet, thus the only way to suppress it is by choosing a multiplet without hypercharge. If we want to avoid fractional charges and high $SU(2)$ multiplets the only possibilities are a singlet or a triplet. If we choose the singlet the scale of the lepton number breaking is somehow arbitrary and the phenomenology will be rather poor because it does not enjoy the gauge couplings, so we will try to build a model using a triplet to break lepton number.

3. The hyperchargeless triplet majoron model

A scalar triplet without hypercharge cannot couple directly to fermions, hence, if the standard model is enlarged with only a hyperchargeless triplet there is no way to assign lepton number to it.The model must be enlarged with more scalars. The simplest possibility† is by adding a new charged singlet scalar h^+ with *two* units of lepton number assigned through the coupling

$$\mathcal{L}_Y = f_{ab}\overline{\widetilde{l_{aL}}}l_{bL}h^+ \tag{8}$$

where $\widetilde{l_{aL}} = i\tau_2 l^c_{aL}$ are the conjugate left-handed doublets and the sum over $a, b = e, \mu, \tau$ is understood.The coupling constants f_{ab} must be antisymmetric in flavour ($f_{ab} = -f_{ba}$).

In addition we need a scalar coupling of this singlet to the triplet in order to assign lepton number to the triplet. This coupling actually exists,

$$\mathcal{L}_{h\varphi\chi} = \lambda_9 h^-\varphi^T i\tau_2\chi\varphi + \ h.c. \tag{9}$$

where the triplet χ is represented by a 2×2 matrix. Thus, the model is defined by the couplings of eq. (8) and eq. (9) and the field content

$$h^+ \sim (0,1,2) \quad \varphi \sim (\tfrac{1}{2},\tfrac{1}{2},0) \quad \chi \sim (1,0,2) \tag{10}$$

The numbers in brackets refer to the $SU(2)_L$, $U(1)_Y$ and $U(1)_l$ transformation properties. The spectrum will contain, in addition to the spectrum of the standard model, three singly charged particles, the majoron and its scalar partner ρ_L.

We can now extract the main phenomenological characteristics of the model:

1. No tree level majoron-electron coupling.
This is because, by construction, the triplet does not carry hypercharge. Thus, the astrophysical bound on the lepton number breaking VEV no longer exists. However, a majoron-electron coupling can be generated at higher loops. In that case the coupling is

† It is easy to see that with only an additional doubly charged scalar singlet it is impossible to assign lepton number to the triplet.

suppressed by the loop and the large masses running around it, hence, we do not expect any problem due to these couplings.

2. Bound on the triplet VEV.

One of the crucial predictions of the standard model has been the tree level relation $\rho = M_W^2/M_Z^2 \cos^2\theta_W = 1$, which is satisfied to a high degree of accuracy. It is related to the fact that in the standard model the spontaneous breaking of symmetries is achieved through a scalar doublet. By adding to the standard model a triplet that develops a VEV this relation will be spoiled, because the triplet contributes to the masses of the gauge bosons in a different manner than doublets. The experimental degree of accuracy in the determination of ρ automatically puts a bound on the VEV of the new multiplet. From the general formulae for the contribution to the masses of the gauge bosons from several multiplets, $M_W^2 = \frac{1}{2}g^2 \sum_i v_i^2(I_i^2 + I_i - I_{3i}^2)$ and $M_Z^2 \cos^2\theta_W = g^2 \sum v_i^2 I_{3i}^2$, we obtain the value of ρ

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + 4\frac{v^2}{u^2} \tag{11}$$

The experimental value is[11] $0.998 < \rho < 1.014$, thus the bound on the triplet VEV is

$$v < 0.06\ u \simeq 10\ GeV \tag{12}$$

After one loop corrections the square of the small VEV will receive corrections proportional to the square of the large VEV. The constant of proportionality is of the order of [9,10] $g^4/(4\pi)^2$, thus a ratio of the order of the one given in eq. (12) does not represent any hierarchy problem.

3. No decay $Z^0 \to J\ \rho_L$.

Possibly one of the crucial tests of nonsinglet majoron models will be neutrino counting experiments in Z^0 decay. Indeed, it can be shown that together with the majoron there is always another scalar particle ρ_L with a mass of the order of (or smaller than) the lepton number breaking VEV [12]. In addition, if the majoron belongs to a nonsinglet multiplet it will have a gauge coupling $Z_\mu^\nu J \overleftrightarrow{\partial}^\mu \rho_L$ giving a new unobserved Z^0 decay mode as well as the standard decay to neutrinos. In fact, in the GR model it gives an additional

contribution equivalent to two more generations of neutrinos[3]. In the doublet majoron model the contribution is 4 times smaller[4]. Neutrino counting at LEP is expected to reach a sensitivity equivalent to 0.3-0.2 neutrino generations[13] which is enough to probe these models. In the hyperchargeless triplet model even though this decay is kinematically allowed due to the bound of eq. (12), it cannot proceed because the coupling does not exist. This is obvious if we take into account that the hypercharge of the majoron is exactly zero (it does not contain any mixing with the standard doublet). This makes a significant difference between this model and the other nonsinglet majoron models [2,4,7]; it cannot be excluded by neutrino counting experiments at LEP.

4. The neutrino mass.

The main purpose of majoron models is to justify the smallness of the neutrino masses. Here, the Majorana neutrino mass is generated through radiative corrections. Thus, even though the lepton number breaking VEV can be relatively large, the masses are kept small. They are generated through the diagram of Fig. 1. It is very similar to the diagram that generates the neutrino mass in the Zee model [14] and, in fact, the mass matrix has exactly the same structure, symmetric without diagonal components. The diagonalization of this mass matrix has been studied in Ref. 15 and it leads to a pseudo-Dirac neutrino and to a Majorana neutrino with a mass proportional to the splitting between the two components of the pseudo-Dirac neutrino. The pseudo-Dirac neutrino mass can be estimated to be

$$m_\nu \simeq \frac{8}{(4\pi)^2}\lambda_8 \frac{u^2}{m_h^2} \ln\frac{m_h^2}{\bar{m}^2} G_F m_\tau^2 v f \tag{13}$$

where m_h is the mass of the charged singlet, $\bar{m}$ is some averaged mass of the charged components of the triplet and f is a function of the Yukawa couplings in eq. (8). It is important to notice that, in comparison to the Zee model, here the equivalent of the dimensional trilinear scalar coupling is fixed by the electroweak scale. Thus, unlike the Zee model, a large neutrino mass automatically implies a relatively small charged singlet mass. Using the bound (eq. (12)) on the triplet VEV, $\lambda_8 \simeq 0.1$, $f \simeq 0.1$ and putting a factor 1 for the logarithm we can easily obtain neutrino masses of the order of $1 - 100\ eV$

for a mass of the charged scalar singlet between 100-1000 GeV. The mass of the Majorana neutrino is two orders of magnitude smaller because it is suppressed by a factor $(m_\mu/m_\tau)^2$ [15].

5. $\mu \rightarrow e\ \gamma$ decay.

Exactly like in the Zee model, the presence of the coupling of eq. (8) induces the radiative decay $\mu \rightarrow e\ \gamma$. The diagram is depicted in Fig. 2 and the branching ratio is given by [16,4]

$$BR(\mu \rightarrow e\ \gamma) \simeq \frac{\alpha}{48\pi G_F^2 m_h^4} \mid f_{e\tau} f_{\mu\tau} \mid^2 \tag{14}$$

which can be at the verge of the present experimental limit [17] $BR(\mu \rightarrow e\ \gamma)_{exp} < 4.9 \times 10^{-11}$ if the mass of the charged singlet is not too heavy ($m_h < 10\ TeV$) and the Yukawa couplings are not too small. As commented previously, in this model, to have sizable neutrino masses we need a relatively small charged singlet mass, and the process can be relevant.

6. Bound on the masses of the triplet charged scalars.

Even though at tree level in the standard model, $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$, at one loop level there are finite radiative corrections which depend quadratically on the mass splitting in a multiplet[‡]. Using this effect it is possible to put a bound on the top quark mass [19] $m_t < 180\ GeV$. The same effect appears if extra Higgs multiplets are present [20] and can be used to constrain the masses of charged scalar particles. In the GR model these considerations give an upper bound on the mass of the charged particles of about $300\ GeV$ [6]. We expect a similar bound on the nonsinglet charged scalar masses.

7. Possibility of $Z^0 \rightarrow \chi^+\chi^-$ and $W^+ \rightarrow \chi^+ J$.

The majoron and the charged scalars that belong to the triplet interact with full strength with the gauge bosons, thus if they are light enough, they can give an important contribution to the decay width of the gauge bosons.

[‡]In the renormalisation scheme introduced by Sirlin [18] ρ is fixed to be one to all orders, but the correction appears in the effective $\nu - fermion$ neutral current interaction.

The couplings and the field content are not enough to define a theory. We must show that the vacuum of the model is, indeed, the true vacuum.

4. Higgs potential and pattern of symmetry breaking

With the field content of the model, the most general gauge invariant and lepton number conserving Higgs potential can be written as

$$V(\varphi,\chi,h) = V_1(\varphi,\chi) + V_2(\varphi,\chi,h) \tag{15}$$

where

$$\begin{aligned} V_1 \; &= \lambda_1(\varphi^\dagger\varphi - u^2)^2 + \lambda_2(Tr\{\chi\chi^\dagger\} - v^2)^2 \\ &+\lambda_3(\varphi^\dagger\varphi - u^2)(Tr\{\chi\chi^\dagger\} - v^2) \\ &+\lambda_4(2\varphi^\dagger\chi\chi^\dagger\varphi - \varphi^\dagger\varphi\ Tr\{\chi\chi^\dagger\}) \\ &+\lambda_5(2Tr\{\chi\chi^\dagger\chi\chi^\dagger\} - Tr\{\chi\chi^\dagger\}^2) \end{aligned} \tag{16}$$

The triplet has been represented by a 2×2 complex matrix,

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^{(0)} & \chi_1^{(+)} \\ \chi_2^{(-)} & -\frac{1}{\sqrt{2}}\chi^{(0)} \end{pmatrix} \tag{17}$$

while for the φ we have used the standard representation as a two component complex vector.

The piece of the potential that depends on the charged singlet scalar is

$$\begin{aligned} V_2 = \;& \mu^2\,|h|^2 + \lambda_6\,|h|^4 + \lambda_7\,|h|^2\,|\varphi|^2 + \lambda_8\,|h|^2\,Tr\{\chi\chi^\dagger\} \\ & -\lambda_9 h^-\varphi^T i\tau_2\chi\varphi + \lambda_9^* h^+\varphi^\dagger\chi^\dagger i\tau_2\varphi^* \end{aligned} \tag{18}$$

Other possible couplings like $\mu_1^2 Tr\{\chi^2\}$ or $\mu_2\varphi^\dagger\chi\varphi$ are forbidden because of lepton number conservation, and as a consequence the only dimensional parameters in the Higgs potential will be the two VEV's and the mass term of the charged singlet. Terms like $\lambda' h^-\varphi^T i\tau_2\chi^\dagger\varphi$ are also forbidden once one makes the assignment of lepton number given by the λ_9 term in eq. (18).

To minimize the whole potential is a difficult task. We will start by studying V_1 because it is independent of the charged singlet scalar and after that we will try to generalize the result to the whole potential.

The charge conserving minimum of the potential must be

$$<\varphi>=\begin{pmatrix} 0 \\ <\varphi^{(0)}> \end{pmatrix}, \quad <\chi>=\begin{pmatrix} \frac{1}{\sqrt{2}}<\chi^{(0)}> & 0 \\ 0 & -\frac{1}{\sqrt{2}}<\chi^{(0)}> \end{pmatrix} \tag{19}$$

These requirements can be written in an invariant way

$$J_{\varphi\chi}\equiv\frac{<\varphi^\dagger\chi\chi^\dagger\varphi>}{<\varphi^\dagger\varphi><Tr\{\chi\chi^\dagger\}>}=\frac{1}{2} \quad J_\chi\equiv\frac{<Tr\{\chi\chi^\dagger\chi\chi^\dagger\}>}{<Tr\{\chi\chi^\dagger\}>^2}=\frac{1}{2} \tag{20}$$

$J_{\varphi\chi}$ and J_χ are related to the relative angles formed between the doublet and the triplet and the real and imaginary components of the triplet. Performing a $SU(2)$ rotation we can diagonalize the hermitian positive semidefinite matrix $\chi\chi^\dagger$

$$U\chi\chi^\dagger U^\dagger=\begin{pmatrix} x_1^2 & 0 \\ 0 & x_2^2 \end{pmatrix} \tag{21}$$

Thus, it is enough to study the following potential

$$\begin{aligned} V_1 &= \lambda_1(y_1^2+y_2^2-u^2)^2+\lambda_2(x_1^2+x_2^2-v^2)^2 \\ &+\lambda_3(y_1^2+y_2^2-u^2)(x_1^2+x_2^2-v^2) \\ &+\lambda_4(y_1^2-y_2^2)(x_1^2-x_2^2)+\lambda_5(x_1^2-x_2^2)^2 \end{aligned} \tag{22}$$

y_1 and y_2 are the moduli of the two components of the standard doublet in this basis. V_1 only depends on y_1 and y_2 and it is independent of the phases of the fields.

The requirements for the vacuum of eq. (20) are expressed by

$$x_1^2=x_2^2 \tag{23}$$

Thus, the only thing we have to do is to check that the absolute minimum of the potential in eq. (22) satisfies these requirements.

The cancellation of the first derivatives gives

$$\begin{aligned} &2y_1[2\lambda_1(y_1^2+y_2^2-u^2)+\lambda_3(x_1^2+x_2^2-v^2)+\lambda_4(x_1^2-x_2^2)]=0 \\ &2y_2[2\lambda_1(y_1^2+y_2^2-u^2)+\lambda_3(x_1^2+x_2^2-v^2)-\lambda_4(x_1^2-x_2^2)]=0 \\ &2x_1[2\lambda_2(x_1^2+x_2^2-v^2)+\lambda_3(y_1^2+y_2^2-u^2)+\lambda_4(y_1^2-y_2^2)+2\lambda_5(x_1^2-x_2^2)]=0 \\ &2x_2[2\lambda_2(x_1^2+x_2^2-v^2)+\lambda_3(y_1^2+y_2^2-u^2)-\lambda_4(y_1^2-y_2^2)-2\lambda_5(x_1^2-x_2^2)]=0 \end{aligned} \tag{24}$$

From these equations is clear that there exists a solution with $x_1^2=x_2^2$ if $y_1^2=y_2^2$, corresponding to

$$x_1^2=x_2^2=\frac{1}{2}v^2 \qquad y_1^2=y_2^2=\frac{1}{2}u^2 \tag{25}$$

The fact that in this basis $y_1^2=y_2^2$ does not represent any problem because we can always rotate this solution to obtain a doublet with only a neutral component; this is because $\langle\chi\chi^\dagger\rangle$, being proportional to the identity, is invariant. However we have to check that eq. (24) gives a true minimum. To do so we must show that the matrix of the second derivatives is positive definite.

Using eq. (25) the matrix of the second derivatives (in the basis (y_1,y_2,x_1,x_2)) is

$$\begin{pmatrix} a & a & b & c \\ a & a & c & b \\ b & c & d & e \\ c & b & e & d \end{pmatrix} \tag{26}$$

where

$$a\equiv 4\lambda_1u^2,\ b\equiv 2(\lambda_3+\lambda_4)uv,\ c\equiv 2(\lambda_3-\lambda_4)uv,\ d\equiv 4(\lambda_2+\lambda_5)v^2,\ e\equiv 2(\lambda_2-\lambda_5)uv \tag{27}$$

A matrix is positive semidefinite, if and only if, all its principal minors are positive or zero[21]. From eq. (26) we can extract the principal minor

$$\begin{vmatrix} a & a & b \\ a & a & c \\ b & c & d \end{vmatrix}=-a(b-c)^2=-64\lambda_1\lambda_4^2u^4v^2 \tag{28}$$

which is always negative because a must be positive. This means that the solution of eq. (24) we have found is not a true minimum, it must be a saddle point and thus it does not represent the true vacuum of the model. The true minimum of the potential of eq. (22) is obtained for

$$y_1^2 = 0, \quad y_2^2 \neq 0, \quad x_1^2 = 0, \quad x_2^2 \neq 0 \tag{29}$$

However this solution does not satisfy eq. (23) which means that it breaks charge conservation with the assignments of the weak hypercharge we have given[§].

Now we have to consider the whole Higgs potential to see if the inclusion of terms depending on the charged singlet scalar can change the situation. It can be shown that the field configuration

$$<\varphi>=\begin{pmatrix} 0 \\ u \end{pmatrix}, \quad <\chi>=\begin{pmatrix} \frac{1}{\sqrt{2}}v & 0 \\ 0 & -\frac{1}{\sqrt{2}}v \end{pmatrix}, \quad <h^+>=0 \tag{30}$$

satisfies the equations of minimization for the whole potential. Again we must check that the matrix of second derivatives is positive semidefinite, which is exactly the same as checking that the mass matrices of the scalars are positive semidefinite. For the mass matrix of the neutral scalars there is no problem, it is positive definite if $\lambda_1 > 0$ and $4\lambda_1\lambda_2 - \lambda_3^2 > 0$; however, the mass matrix of the charged scalars is (in the basis $(h^{(+)}, \chi_1^{(+)}, \chi_2^{(+)}, \varphi^{(+)})$)

$$\begin{pmatrix} \mu^2 + \lambda_7 u^2 + \lambda_8 v^2 & \lambda_9 u^2 & 0 & \sqrt{2}\lambda_9 uv \\ \lambda_9^* u^2 & -\lambda_4 u^2 + 2\lambda_5 v^2 & -2\lambda_5 v^2 & -\sqrt{2}\lambda_4 uv \\ 0 & -2\lambda_5 v^2 & \lambda_4 u^2 + 2\lambda_5 v^2 & \sqrt{2}\lambda_4 uv \\ \sqrt{2}\lambda_9^* uv & -\sqrt{2}\lambda_4 uv & \sqrt{2}\lambda_4 uv & 0 \end{pmatrix} \tag{31}$$

Again it contains principal minors that are always negative, so again, it is not the true minimum of the potential and the true minimum of the potential breaks charge conservation. Thus the model as proposed is inconsistent.

[§]In fact, this minimum corresponds to the true minimum of the Gelmini-Roncadelli model; in that case the hypercharge of the triplet is 1 and this vacuum does not break charge conservation.

Obviously to solve the problem we have to add new terms in the potential. We can add lepton number violating terms, in which case there is no reason to use a complex triplet and the natural thing to do is to use a real triplet $\chi = \chi^\dagger$. For a real triplet we have $\chi\chi^\dagger = \chi^2 = \frac{1}{2}Tr\{\chi^2\}1$, and the conditions of eq. (20) are automatically satisfied (or, in other words, the unwanted couplings λ_4 and λ_5 are automatically zero). However, because the lepton number is broken explicitly there is no majoron and the main motivation for the model is lost.[¶] The other solution is to add new fields. The simplest possibility is a scalar singlet. Thus we will try to find the required vacuum by adding a scalar singlet carrying lepton number

$$\sigma \sim (0, 0, -2) \tag{32}$$

The new pieces in the Higgs potential can be written as

$$\begin{aligned} V_3 &= \beta_1(|\sigma|^2 - \omega^2)^2 + \beta_2(|\sigma|^2 - \omega^2)(\varphi^\dagger\varphi - u^2) \\ &+\beta_3(|\sigma|^2 - \omega^2)(Tr\{\chi\chi^\dagger\} - v^2) \\ &+(\beta_4(|\sigma|^2 Tr\{\chi\chi^\dagger\} - \sigma^2 Tr\{\chi^2\}) + \ h.c.) + \beta_5 |h|^2 |\sigma|^2 \end{aligned} \tag{33}$$

where a discrete symmetry $\sigma \to -\sigma$ has been used to simplify the potential by forbidding terms like $\sigma\varphi^\dagger\chi\varphi$ and also for simplicity we will take all the couplings to be real.

It is easy to see that the equations of minimization for the potential, $V_1 + V_2 + V_3$, are satisfied by the field configuration of eq. (30) together with $<\sigma>=\omega$. Developing around this point we can obtain the mass matrices of the scalars. For the real parts of the neutral components we obtain that the mass matrix is positive definite if

$$\lambda_1 > 0, \quad 4\lambda_1\lambda_2 - \lambda_3^2 > 0, \quad 4\lambda_1\lambda_2\beta_1 + \lambda_3\beta_2\beta_3 - \lambda_2\beta_2^2 - \lambda_1\beta_3^2 - \beta_1\lambda_3^2 > 0 \tag{34}$$

For the imaginary parts of the neutral components we find that $\mathrm{Im}(\varphi^{(0)})$ does not get any mass because it corresponds to the would-be-Goldstone boson eaten by the Z^0 gauge

[¶]Nevertheless it can be a nice alternative to the Zee model. It contains one less degree of freedom and, more importantly, the equivalent of the arbitrary trilinear coupling in the Zee model is fixed to be here of the order of the electroweak scale.

boson. For the other two components $(\sqrt{2}\,\mathrm{Im}(\chi^{(0)}), \sqrt{2}\,\mathrm{Im}(\sigma))$ we obtain the following mass matrix

$$4\beta_4 \begin{pmatrix} \omega^2 & v\omega \\ v\omega & v^2 \end{pmatrix} \tag{35}$$

It has a zero eigenvalue corresponding to the majoron field

$$J = \sqrt{2}\left(\frac{v}{\sqrt{v^2+\omega^2}}\,\mathrm{Im}(\chi^{(0)}) - \frac{\omega}{\sqrt{v^2+\omega^2}}\,\mathrm{Im}(\sigma^{(0)})\right) \tag{36}$$

while the orthogonal combination gets a mass squared $4\beta_4(v^2+\omega^2)$, positive if $\beta_4 > 0$. However for the mass matrix of the charged scalars we obtain exactly the same structure as eq. (31) by changing only $\lambda_8 v^2 \to \lambda_8 v^2 + \beta_5\omega^2$ and $2\lambda_5 v^2 \to 2\lambda_5 v^2 + 2\beta_4\omega^2$. Thus, again it is not positive semidefinite, and the model will break charge conservation. We can understand this result in the following way; the model, without the neutral singlet, breaks charge conservation because the minimum of the potential is found for an angle between the doublet and the triplet incompatible with charge conservation. The problem cannot be solved by adding a term like $\sigma^2 Tr\{\chi^2\}$, because this term is independent of that angle, and therefore cannot modify the pattern of symmetry breaking. The same argument can be applied for a model with a singlet $\sigma' \sim (0,0,-4)$ with the lepton number conserving coupling $\sigma Tr\{\chi^2\}$. We have checked this explicitly. Following this argument it seems that the only possible solution of the problem is to add couplings that depend on the angle formed between the doublet and the triplet. The only term we can use maintaining lepton number conservation is

$$V_4 = \frac{1}{\sqrt{2}}\alpha\sigma\varphi^\dagger\chi\varphi \;+\; h.c. \tag{37}$$

At first look it seems that adding such a term cannot modify the mass submatrix of the charged scalars $(\chi_1^{(+)}, \chi_2^{(+)})$, which is not positive definite. This is because it is linear in χ. However by adding the term of eq. (37) the vacuum is shifted inducing additional terms in this mass matrix. The minimum of the potential will now be

$$<\varphi>= \begin{pmatrix} 0 \\ \bar{u}^2 \end{pmatrix} \qquad <\chi>= \begin{pmatrix} \frac{1}{\sqrt{2}}\bar{v}^2 & 0 \\ 0 & -\frac{1}{\sqrt{2}}\bar{v}^2 \end{pmatrix} \qquad <\sigma>=\hat{\omega} \qquad <h^{(+)}>=0 \tag{38}$$

where the new $\tilde{u}$, $\tilde{v}$ and $\tilde{\omega}$ must satisfy the following equations

$$\begin{aligned} &2\bar{u}[2\lambda_1(\bar{u}^2-u^2)+\lambda_3(\bar{v}^2-v^2)+\beta_2(\bar{\omega}^2-\omega^2)-\alpha\bar{\omega}\bar{v}]=0 \\ &2\bar{v}[2\lambda_2(\bar{v}^2-v^2)+\lambda_3(\bar{u}^2-u^2)+\beta_3(\bar{\omega}^2-\omega^2)]-\alpha\bar{\omega}\bar{u}^2=0 \\ &2\bar{\omega}[2\beta_1(\bar{\omega}^2-\omega^2)+\beta_2(\bar{u}^2-u^2)+\beta_3(\bar{v}^2-v^2)]-\alpha\bar{v}\bar{u}^2=0 \end{aligned} \tag{39}$$

From the whole potential, making the shift in the fields and using eq. (39) we can write the mass matrices of all the particles in terms of the new VEV's. Thus for the imaginary part of the fields $(\sqrt{2}\,\mathrm{Im}(\chi^{(0)}), \sqrt{2}\,\mathrm{Im}(\sigma))$ we get the following mass matrix

$$\left(4\beta_4 + \frac{1}{2}\alpha\frac{\bar{u}^2}{\bar{v}\bar{\omega}}\right)\begin{pmatrix} \bar{\omega}^2 & \bar{v}\bar{\omega} \\ \bar{v}\bar{\omega} & \bar{v}^2 \end{pmatrix} \tag{40}$$

It has exactly the same form, apart from the global factor, as eq. (35), therefore, in this case the majoron is also given by eq. (36) changing only $u \to \bar{u}$, $v \to \bar{v}$ and $\omega \to \bar{\omega}$. For the mass matrix of charged scalars we obtain

$$\begin{pmatrix} \bar{a} & \bar{b} & 0 & \bar{b}\epsilon \\ \bar{b} & -\bar{c}+\bar{d}+\bar{e} & -\bar{d} & (-\bar{c}+\bar{e})\epsilon \\ 0 & -\bar{d} & \bar{c}+\bar{d}+\bar{e} & (\bar{c}+\bar{e})\epsilon \\ \bar{b}\epsilon & (-\bar{c}+\bar{e})\epsilon & (\bar{c}+\bar{e})\epsilon & 2\bar{e}\epsilon^2 \end{pmatrix} \tag{41}$$

where we have defined

$$\begin{aligned} &\bar{a} \equiv \mu^2+\lambda_7\bar{u}^2+\lambda_8\bar{v}^2+\beta_5\bar{\omega}^2 \qquad \bar{b}\equiv\lambda_9\bar{u}^2, \qquad \bar{c}\equiv\lambda_4\bar{u}^2 \\ &\bar{d}\equiv 2\lambda_5\bar{v}^2+2\beta_4\bar{\omega}^2 \qquad \bar{e}\equiv\frac{1}{2}\alpha\frac{\bar{\omega}\bar{u}^2}{\bar{v}} \qquad \epsilon\equiv\sqrt{2}\frac{\bar{v}}{\bar{u}} \end{aligned} \tag{42}$$

Only the term $\bar{e}$ contains the dependence on the new coupling. From this expression it is clear that in the limit $\bar{e} \to 0$ the mass matrix of charged scalars reduces to the form of eq. (31) and so it is not positive semidefinite. However, the pieces with $\bar{e}$ destroy the previous structure of eq. (31). On the other hand, it is easy to check that this matrix contains a zero mode, as it should, corresponding to the would-be-Goldstone boson eaten by the W gauge boson. It is given by the linear combination

$$\phi_W^{(+)} = \frac{1}{\sqrt{1+2\epsilon^2}}(\epsilon\chi_1^{(+)}+\epsilon\chi_2^{(+)}-\varphi^{(+)}) \tag{43}$$

Performing a rotation to get rid of this spurious degree of freedom we obtain

$$\begin{pmatrix} \bar{a} & \bar{b}/\sqrt{2} & \bar{b}z/\sqrt{2} \\ \bar{b}/\sqrt{2} & \bar{e}+2\bar{d} & -\bar{c}z \\ \bar{b}z/\sqrt{2} & -\bar{c}z & \bar{e}z^2 \end{pmatrix} \tag{44}$$

where $z \equiv \sqrt{1+2\epsilon^2}$. This mass matrix is positive definite if

$$\begin{aligned} &\bar{e} > 0 \\ &\bar{e}(\bar{e}+2\bar{d}) - \bar{c}^2 > 0 \\ &\bar{a}(\bar{e}(\bar{e}+2\bar{d}) - \bar{c}^2) - \bar{b}^2(\bar{e}+\bar{c}+\bar{d}) > 0 \end{aligned} \tag{45}$$

These conditions can be easily satisfied if the new coupling we have introduced in eq. (37) is different from zero. In particular they can be fulfilled for a singlet VEV $\bar{\omega}$ of the same order of magnitude as the triplet VEV. If that is the case, the resulting majoron is a roughly equal combination of triplet and singlet and all the interesting phenomenology related to the nonsinglet nature of the majoron can be maintained. In particular, all the aspects discussed in section 3 are maintained.

5. Conclusions

We have studied the conditions needed to avoid the tree level coupling of the majoron to electrons, and hence, the astrophysical bound on the lepton number breaking VEV. It turns out that this is only possible if the majoron multiplet does not carry weak hypercharge. To avoid fractional charges and high SU(2) multiplets the only possibilities are singlets or triplets. Triplets are in principle more interesting because their phenomenology is richer and because the lepton number breaking scale is bounded, at least, by the electroweak scale. We tried to build a hyperchargeless triplet majoron model by adding to the standard model a charged singlet carrying two units of lepton number and a triplet without weak hypercharge carrying also lepton number 2. In the minimal version the model is inconsistent because the minimum of the potential breaks charge conservation.

However the problem can be solved by adding a neutral singlet σ carrying two units of lepton number and adding the lepton number conserving coupling $\sigma\varphi^\dagger\chi\varphi$. The resulting majoron can be a roughly equal combination of triplet and singlet and most of the triplet model phenomenology can be maintained. In particular there is no tree level coupling of the majoron to electrons. There is a bound on the triplet vacuum expectation value coming from the correction to the $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$ parameter ($\langle\chi\rangle < 10\ GeV$). There are no additional decay modes of the Z^0 into invisible particles. The neutrino mass is generated at the one loop level and can be in the range $1 - 100\ eV$ for a mass of the charged singlet between $100 - 1000\ GeV$. For masses of the charged scalar singlet in this range the radiative decay $\mu \to e\ \gamma$ can proceed with branching ratios at the verge of the present experimental limit. The masses of the charged triplet scalars are bounded from above because they contribute, through radiative corrections, to the ρ parameter. And finally, because the majoron and the charged scalars feel the weak interactions they contribute, if they are light enough, to the decay width of the gauge bosons.

After completion of most of this work, we became aware that models containing a complex scalar triplet without weak hypercharge have also been considered by the authors of ref. [22]. However they did not study the Higgs potential.

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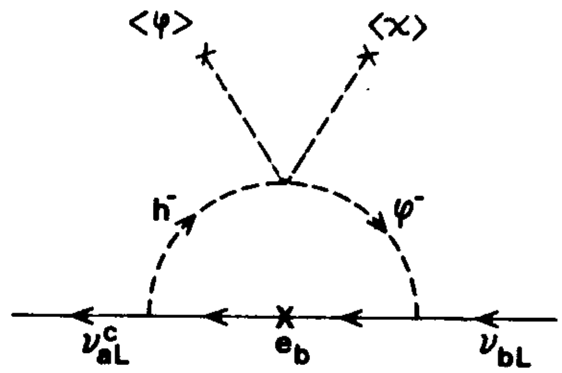
Figure Captions

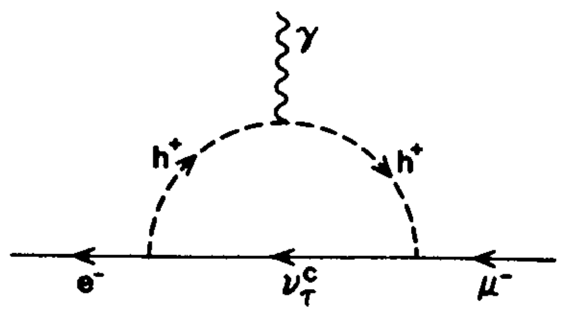
Fig. 1 The one-loop diagram that generates the Majorana entries of the neutrino mass matrix.

Fig. 2 The diagram for $\mu \to e\,\gamma$ in the hyperchargeless majoron model.



$\langle\varphi\rangle$
$\langle\chi\rangle$
h^-
φ^-
ν_{aL}^c
e_b
ν_{bL}

Fig. 1



γ
h^+
h^+
e^-
ν_τ^c
μ^-

Fig. 2