

Stability of the VEV Hierarchy and Higgs Boson Invisibility in Majoron Models

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Abstract

We study the stability of the lepton number breaking VEV under radiative corrections in the doublet and triplet majoron models, including the effects of a heavy top quark. We find that it is possible to maintain the hierarchy between the VEV's, at the one loop level, by fine tuning the ratio of two coupling constants in the Higgs potential. For a top quark heavier than $50 - 60 \text{ GeV}$ the required relation depends strongly on the top mass. We use this relation to show that the decay mode of the neutral Higgs boson to majorons may be the dominant one for a wide range of the Higgs mass, thus making its detection, in this class of models, more problematic.

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The spontaneous breaking of the lepton number represents, among the different explanations of the origin of small neutrino masses, a very interesting and attractive possibility. Some recent work [?,?] has been indeed devoted to the possibility that a solution of the apparent solar neutrino deficit [?] through matter enhanced neutrino oscillations (Mikheyev-Smirnov-Wolfenstein mechanism [?]), may find a natural implementation in models that exhibit the spontaneous breaking of the lepton number. In particular, the present authors have proposed a simple extension of the standard electroweak theory in which lepton number is spontaneously broken by a Higgs *doublet* and neutrino masses are generated radiatively, so that they naturally fall in the correct regime for the MSW mechanism [?]. The analysis of various phenomenological implications of the model shows that the range of variation of the parameters is extremely restricted and allows for a number of characteristic predictions which will be testable in the near future [?,?]. This is mainly a consequence of the presence of the *majoron*, the Nambu-Goldstone boson associated with the spontaneous breakdown of lepton number [?,?], and the related astrophysical bound on the lepton number breaking VEV [?,?]. Analogous features are present in the supersymmetric majoron model of ref. [?]. It is appealing that this class of models not only provide a rationale for the smallness of the neutrino mass and a scenario for a possible solution of the solar neutrino problem, but also exhibit a predictivity which goes beyond neutrino phenomena (for instance, a decay rate for $\mu \rightarrow e\gamma$ likely to be within two orders of magnitude from the present experimental limit in the “doublet” majoron model of ref. [?] or the correction to the Michel parameter in τ and μ decays in the supersymmetric majoron model of ref. [?]).

As already mentioned, a key element in the aforementioned analysis is the strong astrophysical bound on the lepton number breaking VEV v ($v \lesssim 10 \text{ KeV}$) [?], coming from majoron emission in the cores of red giant stars, which generally holds in non-singlet majoron models [?,?,?,?]. This fact introduces therefore a hierarchy problem in this class of

models: the lepton number breaking VEV must be at least seven orders of magnitude smaller than the standard electroweak scale. Although this may be conveniently implemented at the tree level, one should generally expect that the hierarchy may be spoiled by higher order corrections. Indeed, in general, to construct a gauge hierarchy of order $\alpha^{-n/2}$ it is necessary to compute the effective potential up to order α^n , find the minimum and then adjust, if possible, the parameters of the Higgs potential to avoid the mixings between the large and small VEV's [?].

This hierarchy problem, in the framework of the triplet model of Gelmini and Roncadelli [?], was studied by Grzadkowski and Pich in ref. [?]. They conclude that, at the one loop level, it is possible to maintain the VEV's hierarchy by adjusting the coupling constants of the Higgs potential in a way compatible with all the phenomenological constraints of the model. Here we extend their analysis to include the important effects of a heavy top quark and discuss the implications for the recently introduced doublet majoron model. We will show that the relation among scalar coupling constants and top mass, required to maintain the VEV hierarchy at the one loop level, may lead to a relevant contribution to the standard Higgs width from the decays into two majorons or a pair of light neutral scalars, $\rho_L\rho_L$. Depending on the top and the standard Higgs mass, this may justify the experimental difficulty for present and future experiments of detecting the standard neutral scalar [?], thus invalidating, in the hypothesis of the existence of the majoron, some of the bounds on the Higgs mass presently claimed.

The doublet majoron model is obtained by adding to the standard model a singly charged scalar which couples to the standard leptons and carries two units of lepton number, together with a new Higgs doublet with an analogous lepton charge and therefore no couplings to fermions. The lepton number, driven from the fermion sector to the scalar sector by the

charged singlet, is spontaneously broken by the VEV v of the neutral component of the new doublet. As a consequence, the neutrinos obtain a radiative mass proportional to v . Given the scalar fields of the model, $h^+ \sim (0, 1, 2)$, $\varphi \sim (\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi \sim (\frac{1}{2}, \frac{1}{2}, 2)$ (the numbers between brackets refer to the transformation properties of the multiplets under $SU(2)_L$, $U(1)_Y$ of hypercharge and $U(1)_l$ of lepton number), the most general, renormalizable, gauge invariant and lepton number conserving Higgs potential can be written as [?]

$$\begin{aligned}
V^{(0)} &= \lambda_1(\varphi^\dagger\varphi - u^2)^2 + \lambda_2(\phi^\dagger\phi - v^2)^2 + \lambda_3(\varphi^\dagger\varphi - u^2)(\phi^\dagger\phi - v^2) \\
&+ \lambda_4((\varphi^\dagger\varphi)(\phi^\dagger\phi) - (\varphi^\dagger\phi)(\phi^\dagger\varphi)) \\
&+ \lambda_5 | \mu h^+ - \varphi^T i\tau_2 \phi |^2 + \lambda_6 | \mu h^+ + \varphi^T i\tau_2 \phi |^2 \\
&+ \lambda_7 | h^+ |^4 + \lambda_8 | h^+ |^2 | \phi |^2 + \lambda_9 | h^+ |^2 | \varphi |^2
\end{aligned} \tag{1}$$

We remark that terms like $(\varphi^\dagger\varphi)(\varphi^\dagger\phi)$ or $(\phi^\dagger\phi)(\varphi^\dagger\phi)$ are forbidden because of lepton number conservation. Hermiticity requires all the λ_i 's to be real and the dimensional coupling μ may be taken, without loss of generality, real by absorbing its phase in the definition of the singlet h^+ . For $\lambda_i > 0$, $i \neq 3$, and $|\lambda_3| < 2\sqrt{\lambda_1\lambda_2}$ the potential is semi-positive definite ($V \geq 0$) and assumes its minimum for $\langle \varphi \rangle = (0, u)$, $\langle \phi \rangle = (0, v)$ and $\langle h^+ \rangle = 0$, where the VEV's u and v can be chosen, without loss of generality, real and u ($\simeq 174 \text{ GeV}$) identifies the standard electroweak scale.

For $v \neq 0$ the global symmetry associated with lepton number is broken spontaneously and a massless Goldstone boson, the majoron, arises which is mainly the imaginary part of the new doublet. The real part gives origin to a scalar ρ_L with a mass proportional to the lepton number breaking VEV, and thus very light. This pattern of light scalars is a general characteristic of non-singlet majoron models and is therefore present in the triplet majoron model as well.

In terms of u and v , the masses of the gauge bosons in the doublet majoron model are

given by $m_W^2 = \frac{1}{2}g^2(u^2 + v^2)$ and $m_Z^2 = \frac{1}{2}(g^2/\cos^2\theta_W)(u^2 + v^2)$ respectively. As in any model with only Higgs doublets (or singlets), the standard relation $m_W = m_Z \cos\theta_W$ is preserved, unlike in the triplet majoron model where the dependence on the triplet VEV of the W and the Z^0 gauge bosons exhibits an extra factor 2 and 4 respectively.

Following the work of Coleman and E. Weinberg [?], we write the one-loop corrected effective potential as,

$$V = V^{(0)} + V_G + V_F + V_S + V_C \quad (2)$$

where $V^{(0)}$ is the tree level potential, V_G , V_F , V_S are the contributions from gauge bosons, fermions and scalar loops and V_C includes the counterterm contributions.

We can neglect the scalar contribution if the scalar coupling constants are small enough ($\lambda_i \lesssim g^4$). However the fermion part contains the contribution of the top quark which, as we are going to show, can be important if the top mass is above 50 – 60 GeV, as the experimental results on the $B_d^0 - \bar{B}_d^0$ mixing suggest.

Assuming that the renormalized vacuum preserves the electric charge, it will be enough to consider, for the purpose of minimization of the potential, only the neutral components of the scalar fields. The effective Higgs potential, along the neutral direction, may then be written as [?]

$$\begin{aligned} V = & \lambda_1(\varphi_0^2 - u^2)^2 + \lambda_2(\phi_0^2 - v^2)^2 + \lambda_3(\varphi_0^2 - u^2)(\phi_0^2 - v^2) \\ & + \frac{3}{64\pi^2} \left[2\bar{m}_W^4(\varphi_0, \phi_0) \ln \frac{\bar{m}_W^2(\varphi_0, \phi_0)}{\bar{m}_W^2(\mu_1, \mu_2)} + \bar{m}_Z^4(\varphi_0, \phi_0) \ln \frac{\bar{m}_Z^2(\varphi_0, \phi_0)}{\bar{m}_Z^2(\mu_1, \mu_2)} \right. \\ & \left. - 4\bar{m}_t^4(\varphi_0) \ln \frac{\bar{m}_t^2(\varphi_0)}{\bar{m}_t^2(\mu_1)} \right] + \dots \quad (3) \end{aligned}$$

where a colour factor 3 has been taken into account for the top quark contribution. The ellipses represent the contribution from the quadratic and quartic counterterms relative to

the neutral scalars. The functions \bar{m}_W , \bar{m}_Z and \bar{m}_t are given, in terms of φ_0 and ϕ_0 , by

$$\bar{m}_W^2 = \frac{1}{2}g^2(\varphi_0^2 + \phi_0^2), \quad \bar{m}_Z^2 = \frac{1}{2}\frac{g^2}{\cos^2\theta_W}(\varphi_0^2 + \phi_0^2), \quad \bar{m}_t^2 = h_t^2\varphi_0^2 \quad (4)$$

where h_t is the top Yukawa coupling. The same functional dependence applies to the arbitrary scales μ_1 and μ_2 , at which the renormalized couplings of the neutral sector are defined. It is worthwhile at this point to make a few remarks on the role of the counterterms in eq. (??). Indeed, by a convenient choice of the renormalization conditions the quadratic counterterms may be taken equal to zero. Then, it is not difficult to show that the effect of the remaining quartic counterterms leads, in eq. (??), to the replacement

$$\ln \frac{\bar{m}_i^2(\varphi_0, \phi_0)}{\bar{m}_i^2(\mu_1, \mu_2)} \rightarrow \ln \frac{\bar{m}_i^2(\varphi_0, \phi_0)}{\bar{m}_i^2(\mu_1, \mu_2)} + C \quad (5)$$

where $i = W, Z, t$. It is important to remark that the constant C is exactly the same for the three terms in eq. (??). In this way, the effect of the quartic counterterms can be reabsorbed in a global redefinition of the two arbitrary renormalization scales, namely $e^{-C}(\mu_1, \mu_2) \rightarrow (\mu_1, \mu_2)$. This shows that we may neglect the effect of the counterterms in the minimization of the effective potential in eq. (??). In addition, if we choose the renormalization scales, μ_1 and μ_2 , equal respectively to the two doublet VEV's

$$\mu_1^2 = \langle \varphi_0 \rangle^2 \quad \mu_2^2 = \langle \phi_0 \rangle^2, \quad (6)$$

the minimization of the potential simplifies considerably, since the logarithmic terms cancel at the minimum.

The solution of the extremizing equations leads to

$$\langle \varphi_0 \rangle^2 \simeq u^2 \left[1 - \frac{3g^4}{64\pi^2\eta} \frac{2\lambda_2(1-\eta t) - \lambda_3}{4\lambda_1\lambda_2 - \lambda_3^2} \right], \quad (7)$$

$$\langle \phi_0 \rangle^2 \simeq v^2 \left[1 - \frac{3g^4}{64\pi^2\eta} \left(\frac{2\lambda_1 - \lambda_3}{4\lambda_1\lambda_2 - \lambda_3^2} + \frac{2\lambda_1 - \lambda_3(1-\eta t)u^2}{4\lambda_1\lambda_2 - \lambda_3^2} \frac{u^2}{v^2} \right) \right] \quad (8)$$

where $\eta \equiv 4 \cos^4 \theta_W / (1 + 2 \cos^4 \theta_W) \simeq 1.08$ and $t \equiv m_t^4 / m_W^4$. It is clear, from eq. (??), that the lepton number breaking VEV $\langle \phi_0 \rangle^2$ gets, in general, a correction proportional to the large VEV u^2 . In order to maintain the hierarchy a fine tuning of the parameters in the potential is needed. If we demand that the tree level formulation of the theory can be sensibly used for predictive calculations (or, in other words, $v \simeq \langle \phi_0 \rangle_{ren}$), then we must make shure that the one-loop corrections in eq. (??) are globally small. This amounts, as follows from direct inspection of eq. (??), to requesting that the term proportional to u^2 is conveniently suppressed. For the doublet majoron model the required relation is, up to terms $O(v^2/u^2)$,

$$(1 - \eta t)\lambda_3 = 2\lambda_1 \quad (9)$$

The minus sign in front of the top contribution is a direct consequence of the minus sign appering for the fermion contributions in the one loop effective potential in eq. (??). In the case of the triplet majoron model, with minimal modifications related to the different dependence of the masses of the gauge bosons on v , we obtain the relation

$$(1 - \eta t)\lambda_3^{(T)} = 2(4 - \eta)\lambda_1^{(T)} \quad (10)$$

In the limit of a small top quark mass, eq. (??) coincides with the result of ref. [?]. It is interesting to notice that even though the majoron does not couple directly to quarks in any of the models considered, the renormalization of the lepton number breaking VEV turns out to be strongly dependent on the top quark mass. This should not surprise since the minimization of the potential involves a system of two equations strongly coupled and any modification on the equation for the large VEV u may indirectly affect the other. In this fact rests, after all, the origin of the hierarchy problem.

The conditions of eq. (??) or eq. (??) are compatible with the tree-level requirement of the positivity of the potential, namely $4\lambda_1\lambda_2 - \lambda_3^2 > 0$. In fact, after using eq. (??), the

positivity condition reads

$$\lambda_1 < (1 - \eta t)^2 \lambda_2 \quad (11)$$

A similar relation can be obtained for the triplet majoron model by using eq. (??).

A first remark that follows from eq. (??) is that if the mass of the top quark happens to be very close to the W mass, maintaining the hierarchy would require a very small λ_1 and, in turn, would imply a very small mass for the standard Higgs boson (we recall that $m_H^2 = 4\lambda_1 u^2$). In majoron models, as we shall see, the decay properties of the standard Higgs may change radically, because of the decay to majorons and ρ_L , thus making more difficult its detection. This may spoil the bounds on the Higgs mass which are related to specific decay modes. However it seems quite difficult to avoid the bounds coming from the absence of Higgs mediated “long-range” forces in neutron-electron [?] and neutron-nucleus [?] scattering, which are independent of the nature of the Higgs decays. In this way one obtains a lower bound on the Higgs mass typically of $O(1 - 10 \text{ MeV})$. Thus, we must conclude that if the top quark mass is contrively “chosen” in such a manner that the right-hand side of eq. (??), or eq. (??), turns out to be extremely small, it is quite problematic to maintain the hierarchy between the two VEV’s in the non-singlet majoron models here considered.

It is remarkable that the relation between λ_1 and λ_3 which we obtain on the grounds of the VEV stability, is just what is needed, in the models here considered, in order to discuss a number of relevant phenomenological issues. A first example was given in ref. [?] in the analysis of the decay $K^+ \rightarrow \pi^+ + \text{nothing}$. Two other instances are presented in the following. Indeed, the ratio between the coupling constants λ_1 and λ_3 fixes the mixing between the real parts of the neutral component of the two Higgs doublets (or the standard doublet and the triplet in the GR model). In other words, it allows for a determination of

the coupling of the light scalar ρ_L , always present in nonsinglet majoron models, to charged fermions. In the doublet majoron model the coupling of the ρ_L to electrons can be written as [?]

$$\mathcal{L}_{e\rho_L} = \frac{m_e}{\sqrt{2}u} \left(\frac{\lambda_3}{2\lambda_1} \right) \frac{v}{u} \bar{e} e \rho_L = \frac{m_e}{\sqrt{2}u} \frac{1}{(1-\eta t)} \frac{v}{u} \bar{e} e \rho_L \quad (12)$$

This may have an effect on the bounds on the lepton number breaking VEV coming from the recent supernova event [?]. Indeed, the mass of the light majoron neutral partner ρ_L is bounded to be of the same order or smaller than the lepton number breaking VEV v and, although it is not clear if it can be produced in red giant stars, it may be emitted in the supernova together with the majoron*. If this is the case, eq. (??) would imply a slight improvement over the bound on the lepton number breaking VEV v given in ref. [?] (see also ref. [?]), depending on the model considered and the top mass.

More important, however, are the consequences of eq. (??) or eq. (??) in the decay properties of the “standard” Higgs boson (henceforth denoted by H). The coupling of the Higgs boson to two majorons or two ρ_L is proportional to λ_3 [?]

$$\mathcal{L}_H \simeq -\frac{1}{\sqrt{2}} \lambda_3 u (J^2 + \rho_L^2) H \quad (13)$$

Thus, the decay rate to majorons and ρ_L 's is

$$\Gamma(H \rightarrow J J) = \Gamma(H \rightarrow \rho_L \rho_L) = \frac{1}{16\pi} \left(\frac{\lambda_3}{2\lambda_1} \right)^2 \frac{G_F}{\sqrt{2}} m_H^3 = \frac{1}{16\pi} \frac{1}{(1-\eta t)^2} \frac{G_F}{\sqrt{2}} m_H^3 \quad (14)$$

where we have used the relation $m_H^2 = 4\lambda_1 u^2$ and in the last step we have eliminated λ_3 in favour of λ_1 by using eq. (??). For the triplet majoron the last term in eq. (??) is a factor $(4-\eta)^2$ larger, as a consequence of eq. (??). Because of the dependence on m_H^3 of the rate, these invisible modes may dominate over the other standard channels for a very wide range

*Barring the possibility that the temperature of the supernova is enough to restore the lepton conserving vacuum [?].

of the Higgs mass, making somewhat difficult its detection (for a fairly complete discussion of the various contributions to the Higgs boson width in the standard model see refs. [?,?]).

In fig. 1 we show the branching ratio due to the decay to majorons and ρ_L , for the doublet majoron model, as a function of the Higgs mass, for different values of the top quark mass ($m_t = 60 \text{ GeV}$, $m_t = 100 \text{ GeV}$ and $m_t = 120 \text{ GeV}$). From the figure we see that, for a relatively light top quark, the invisible decay to scalars is dominant for most of the Higgs mass range considered. Only for top masses above 100 GeV the decay rate of the Higgs boson to the light scalars falls below 50%, in the considered range of the Higgs mass.[†] In the case of the triplet majoron model the rate for the “invisible” decays is roughly a factor 10 larger than in the doublet model and the curves in Fig. 1 must be scaled accordingly, thus making the Higgs boson even more elusive.

In the previous discussion, we have purposely avoided values of the top mass in the close neighbourhood of the W boson mass because, as we have already commented, this would require a Higgs mass at the limit imposed by nuclear scattering experiments. From eq. (??) we see that such values would also lead to a pathologically large decay rate to majorons.

In conclusion, we have seen that in non-singlet majoron models the problem of the stability of the hierarchy between the lepton number breaking VEV and the standard VEV, required by astrophysical arguments, may be avoided, at the one-loop level, by adjusting in a convenient (and consistent) way the ratio of two coupling constant in the Higgs potential. We have shown that the required relation depends critically on the top quark mass, if larger than $50\text{--}60 \text{ GeV}$. This fine tuning, however, may be problematic if the mass of the top quark is very close to the mass of the charged gauge boson. The relation needed to maintain the

[†]For a Higgs mass substantially heavier than $2m_Z$, the W^+W^- and Z^0Z^0 decay channels may become competitive with the majoron modes. Such large values of the Higgs boson mass fall however outside the range of applicability of the present analysis.

VEV hierarchy, may have various relevant phenomenological consequences. In particular, it allows for an evaluation of the coupling of the standard Higgs boson to majorons and ρ_L in terms of the top quark mass. As a consequence, we showed that the decay mode to “invisible” scalars may become the dominant one for a wide range of the Higgs mass, thus contributing to the experimental elusiveness of the neutral Higgs boson.

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Figure Captions

Fig. 1 The branching ratio of the neutral Higgs decay to majorons and ρ_L is shown as a function of the Higgs mass, for different values of the top quark mass. The solid line corresponds to $m_t = 60 \text{ GeV}$, the dashed line to $m_t = 100 \text{ GeV}$ and the dotted line to $m_t = 120 \text{ GeV}$.

