Chapter 7. Inventory Management: Items with Independent Demand

Basic references:

- Heizer, J. & Render, B. (2009): Operations Management. New Jersey: Pearson Prentice Hall

Chapter 7. Inventory Management: Items with Independent Demand

7.1. Functions and types of inventory. Pros and cons of inventory

7.2. Nature of inventories

7.3. Inventory models for independent demand items

Amazon.com

- ☑ Amazon.com started as a ‘virtual’ retailer – no inventory, no warehouses, no overhead; just computers taking orders to be filled by others
- ☑ Growth has forced Amazon.com to become a world leader in warehousing and inventory management

Amazon.com

1. Each order is assigned by computer to the closest distribution center that has the product(s)
2. A ‘flow meister’ at each distribution center assigns work crews
3. Lights indicate products that are to be picked and the light is reset
4. Items are placed in crates on a conveyor. Bar code scanners scan each item 15 times to virtually eliminate errors.

Amazon.com

5. Crates arrive at a central point where items are boxed and labeled with new bar code
6. Gift wrapping is done by hand at 30 packages per hour
7. Completed boxes are packed, taped, weighed and labeled before leaving warehouse in a truck
8. Order arrives at customer within a week

7.1. Functions and types of inventory. Pros and cons of inventory

- **Inventory:** items piled waiting for use (other productive or comercial use)

- **Function of inventory:** To decouple or separate various parts of the production process

  Other functions:

  -To decouple the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers

  -To take advantage of quantity discounts

  -To hedge against inflation

Inventory

- ☑ One of the most expensive assets of many companies and representing as much as 50% of total invested capital
- ☑ Operations managers must balance inventory investment and customer service

Types of inventory

- ☑ Raw material
  - ☑ Purchased but not processed
- ☑ Work-in-process
  - ☑ Undergone some change but not completed
  - ☑ A function of cycle time for a product
- ☑ Maintenance/repair/operating (MRO)
  - ☑ Necessary to keep machinery and processes productive
- ☑ Finished goods
  - ☑ Completed product awaiting shipment

The material flow cycle

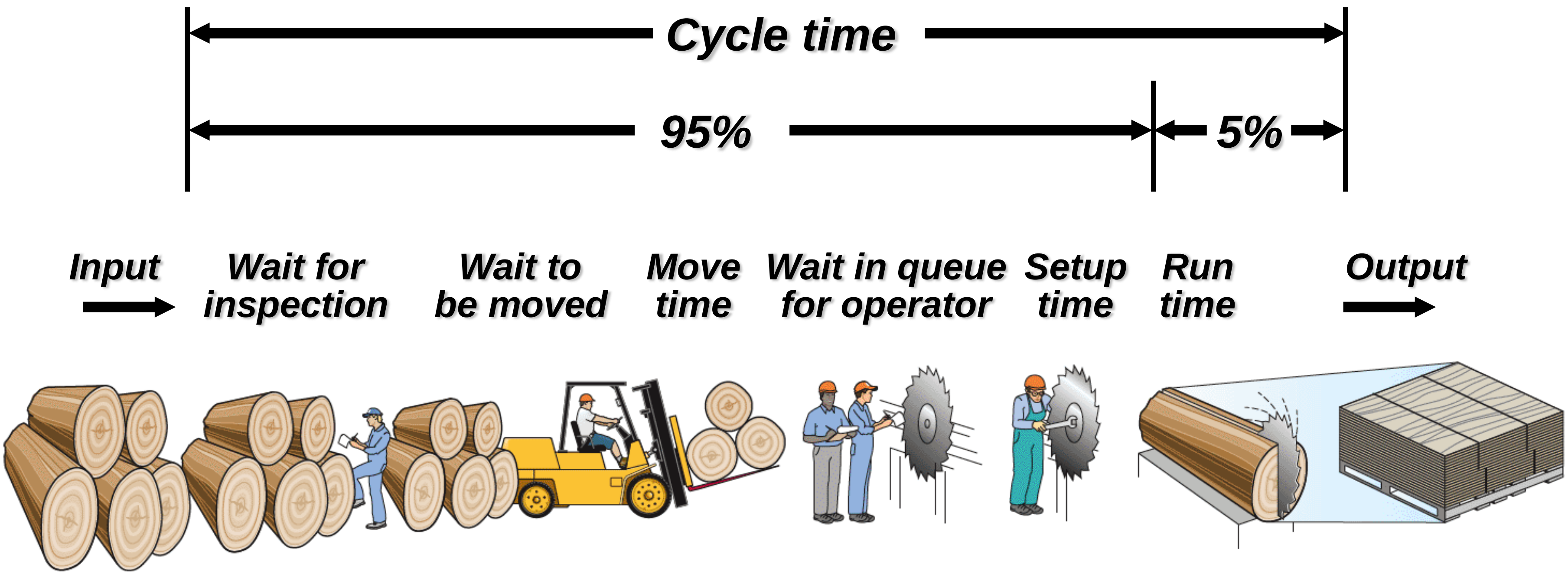


Figure 12.1

Functions of inventory

- ☑ Cycle inventory
- ☑ Safety inventory
- ☑ Seasonal inventories
- ☑ In-transit inventories

Pros and cons of inventory

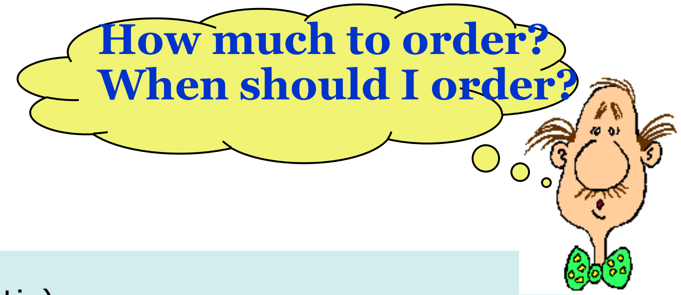
PROS

- Reduction of delivery time
- Increase in flexibility
- Reduction of ordering costs
- Reduction of stock breakage costs
- Reduction of acquisition and production costs.
- Increase in quality

CONS OF A HIGH INVENTORY

- Increase in holding costs
- Increase in financial costs
- Possible obsolescence of inventory
- Hiding inefficiencies
- Associated risks (theft, fire, etc.)

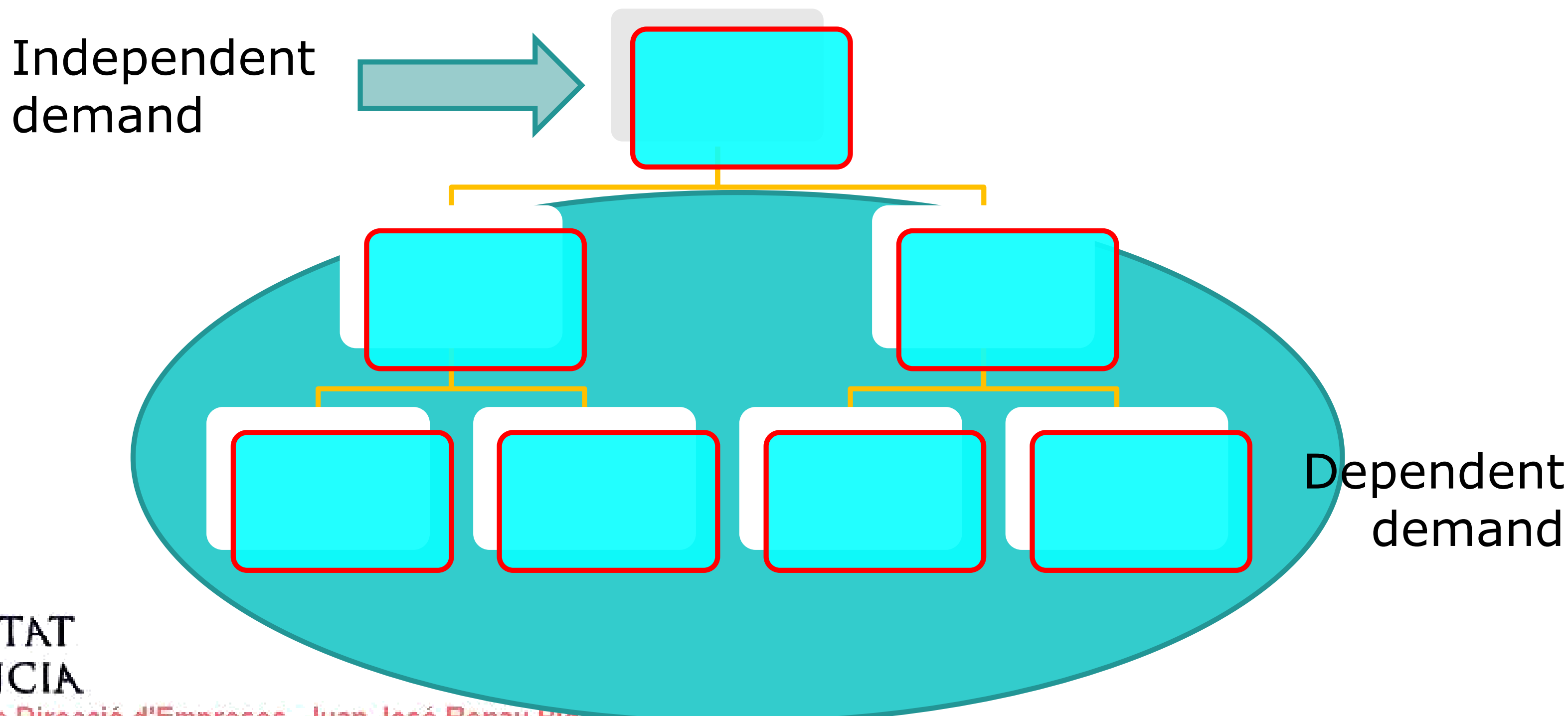
7.2. Nature of inventories



Nature of demand

- Certain (on order)
- Uncertain (probabilistic)

- Independent: does not depend on other items of the firm.
- Dependent: depends on some other ítem demand (derived demand)



Inventory management

- ☑ ***How inventory items can be classified***
- ☑ ***How accurate inventory records can be maintained***

ABC analysis

- ☑ Divides inventory into three classes based on annual dollar volume
  - ☑ Class A - high annual dollar volume
  - ☑ Class B - medium annual dollar volume
  - ☑ Class C - low annual dollar volume
- ☑ Used to establish policies that focus on the few critical parts and not the many trivial ones

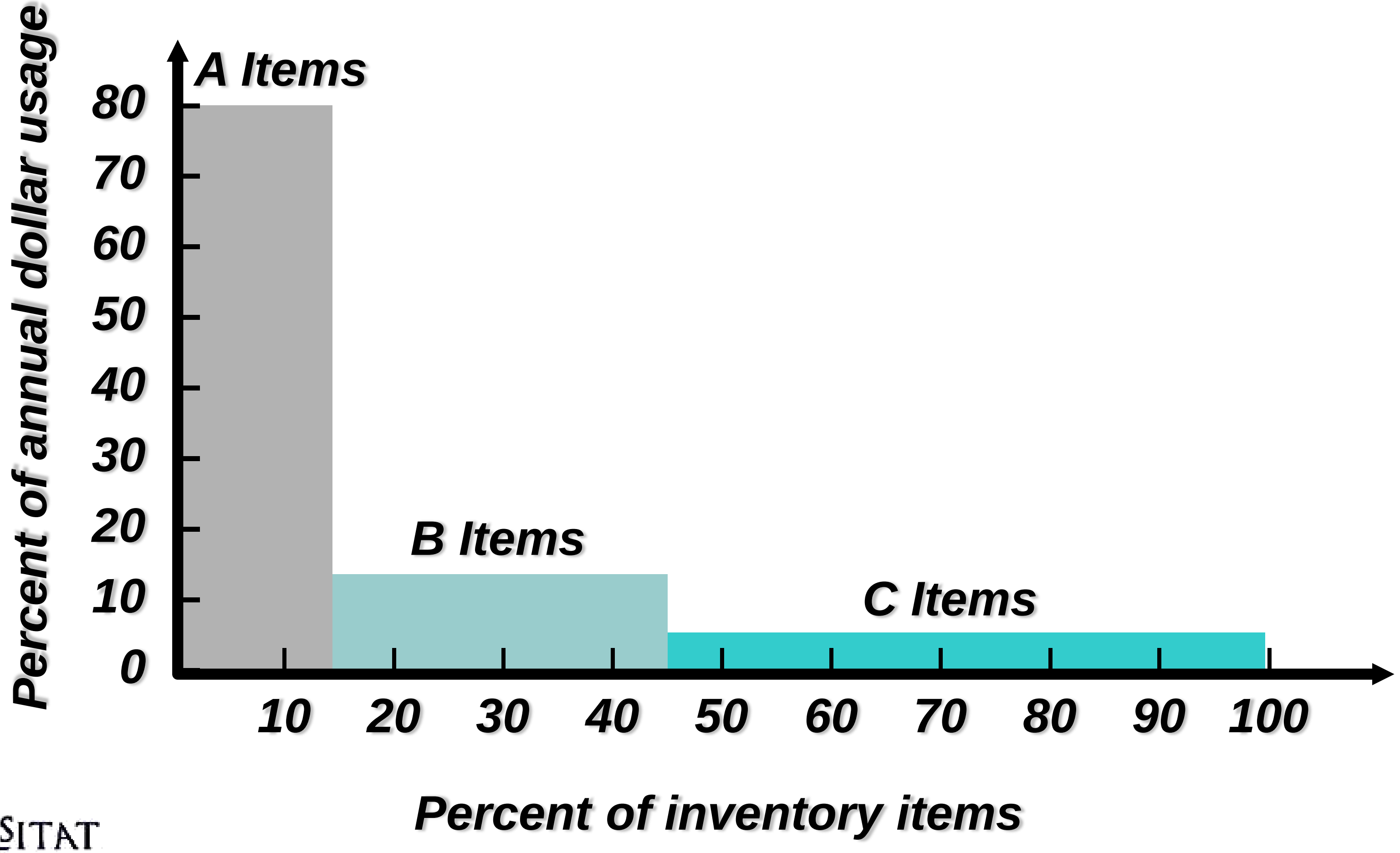
ABC analysis

Item stock number	Percent of number of items stocked	Annual volume (units)	x	Unit cost	=	Annual dollar volume	Percent of annual dollar volume		Class
#10286	*20%*	*1,000*		*$90.00*		*$90,000*	*38.8%*	72%	A
#11526		*500*		*154.00*		*77,000*	*33.2%*		A
#12760		*1,550*		*17.00*		*26,350*	*11.3%*		B
#10867	*30%*	*350*		*42.86*		*15,001*	*6.4%*	23%	B
#10500		*1,000*		*12.50*		*12,500*	*5.4%*		B

ABC analysis

Item stock number	Percent of number of items stocked	Annual volume (units)	x	Unit cost	=	Annual dollar volume	Percent of annual dollar volume		Class
#12572		*600*		*$14.17*		*$8,502*	*3.7%*		C
#14075		*2,000*		*.60*		*1,200*	*.5%*		C
#01036	*50%*	*100*		*8.50*		*850*	*.4%*	**5%**	C
#01307		*1,200*		*.42*		*504*	*.2%*		C
#10572		*250*		*.60*		*150*	*.1%*		C
		8,550				*$232,057*	*100.0%*		

ABC analysis



ABC analysis

- ☑ Other criteria than annual dollar volume may be used
  - ☑ Anticipated engineering changes
  - ☑ Delivery problems
  - ☑ Quality problems
  - ☑ High unit cost

ABC analysis

- ☑ Policies employed may include
  - ☑ More emphasis on supplier development for A items
  - ☑ Tighter physical inventory control for A items
  - ☑ More care in forecasting A items

Record accuracy

- ☑ Accurate records are a critical ingredient in production and inventory systems
- ☑ Allows organisation to focus on what is needed
- ☑ Necessary to make precise decisions about ordering, scheduling, and shipping
- ☑ Incoming and outgoing record keeping must be accurate
- ☑ Stockrooms should be secure

Control of service inventories

- ☑ Can be a critical component of profitability
- ☑ Losses may come from shrinkage or pilferage
- ☑ Applicable techniques include
  1. Good personnel selection, training, and discipline
  2. Tight control on incoming shipments
  3. Effective control on all goods leaving facility

Independent versus dependent demand

- ☑ Independent demand - the demand for an item is independent of the demand for any other item in inventory
- ☑ Dependent demand - the demand for item is dependent on the demand for some other item in the inventory

Holding, ordering, and setup costs

- ☑ Holding costs - the costs of holding or ‘carrying’ inventory over time
- ☑ Ordering costs - the costs of placing an order and receiving goods
- ☑ Setup costs - cost to prepare a machine or process for manufacturing an order

Holding costs

Category	Cost (and range) as a percent of inventory value
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	*6% (3 - 10%)*
Material handling costs (equipment lease or depreciation, power, operating cost)	*3% (1 - 3.5%)*
Labour cost	*3% (3 - 5%)*
Investment costs (borrowing costs, taxes, and insurance on inventory)	*11% (6 - 24%)*
Pilferage, space, and obsolescence	*3% (2 - 5%)*
Overall carrying cost	*26%*

Holding Costs

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech items have holding costs greater than 50%.

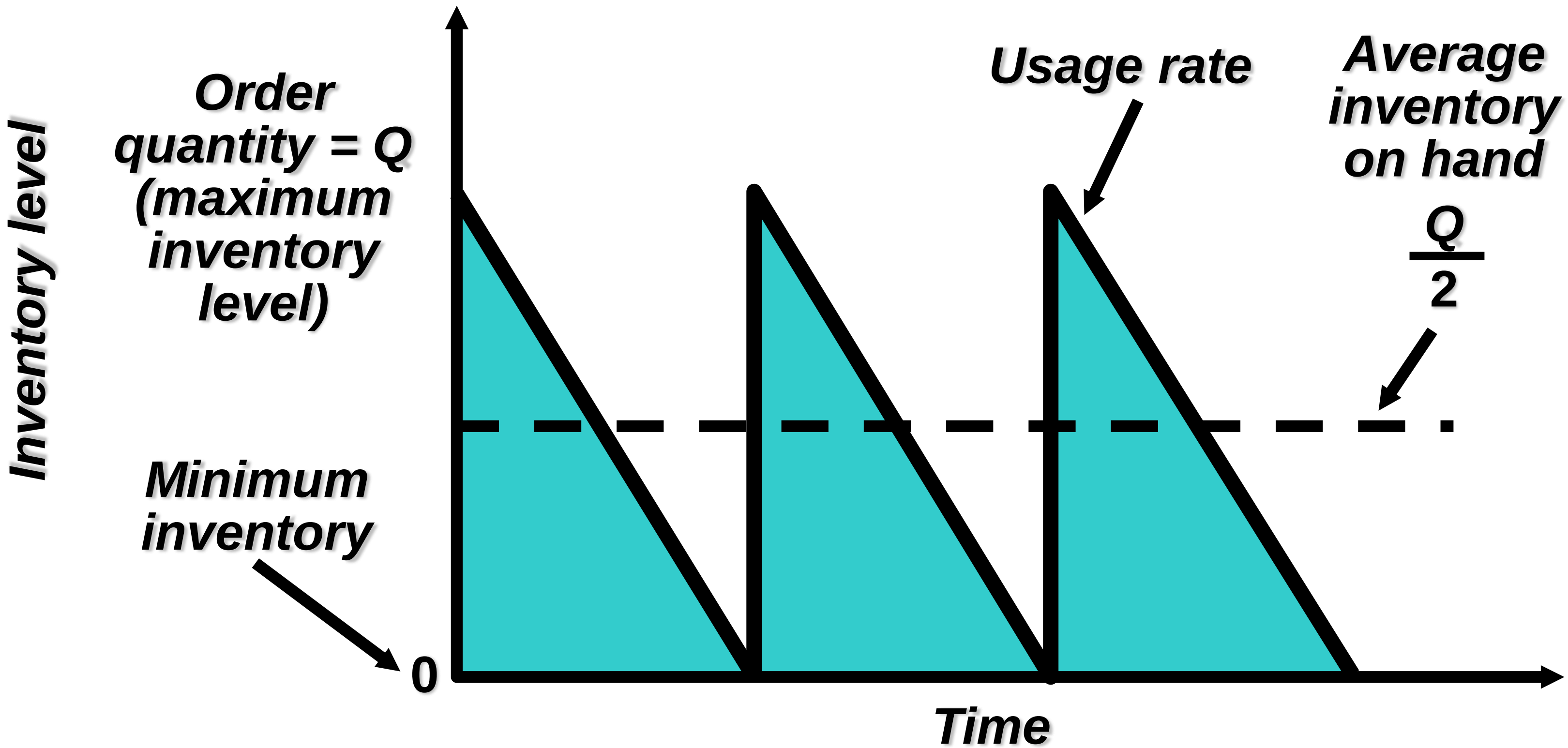
	Cost (and [illegible])	
[illegible]		
La[illegible]	*3%*	*(3 - 5%)*
Investment costs (borrowing costs, taxes, and insurance on inventory)	*11%*	*(6 - 24%)*
Pilferage, space, and obsolescence	*3%*	*(2 - 5%)*
Overall carrying cost	*26%*	

Basic EOQ model

Important assumptions

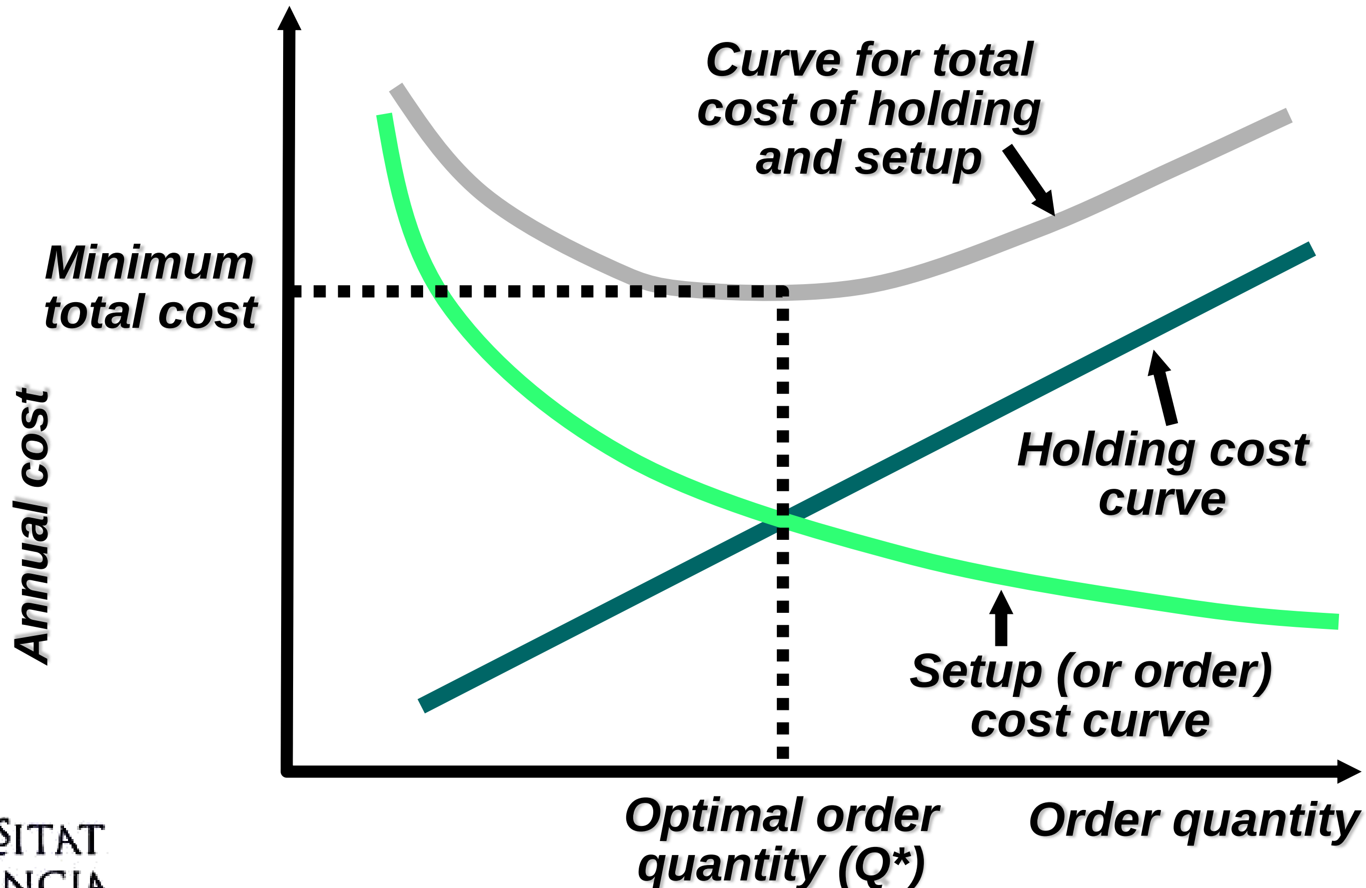
1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup and holding
6. Stockouts can be completely avoided

Inventory usage over time



Minimising costs

Objective is to minimise total costs



The EOQ model

$$\textit{Annual setup cost} = \frac{D}{Q}S$$

Q = *Number of pieces per order*
Q* = *Optimal number of pieces per order (EOQ)*
D = *Annual demand in units for the inventory item*
S = *Setup or ordering cost for each order*
H = *Holding or carrying cost per unit per year*

Annual setup cost = (*Number of orders placed per year*) x (*Setup or order cost per order*)

$$= \left(\frac{\textit{Annual demand}}{\textit{Number of units in each order}}\right)\left(\begin{matrix}\textit{Setup or order}\\ \textit{cost per order}\end{matrix}\right)$$

$$= \left(\frac{D}{Q}\right)(S)$$

The EOQ model

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\text{Annual holding cost} = \frac{Q}{2}H$$

Q = *Number of pieces per order*
*Q** = *Optimal number of pieces per order (EOQ)*
D = *Annual demand in units for the inventory item*
S = *Setup or ordering cost for each order*
H = *Holding or carrying cost per unit per year*

$$\text{Annual holding cost} = (\text{Average inventory level}) \times (\text{Holding cost per unit per year})$$

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H)$$

The EOQ model

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\text{Annual holding cost} = \frac{Q}{2}H$$

Q **= *Number of pieces per order***
Q* **= *Optimal number of pieces per order (EOQ)***
D **= *Annual demand in units for the inventory item***
S **= *Setup or ordering cost for each order***
H **= *Holding or carrying cost per unit per year***

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

Solving for Q*

$$2DS = Q^2H$$

$$Q^2 = 2DS/H$$

$$Q^* = \sqrt{2DS/H}$$

An EOQ example

Determine optimal number of needles to order
D* = 1,000 *units
S* = $10 *per order
H* = $.50 *per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ example

Determine optimal number of needles to order

D* = 1,000 *units $\quad$ ***Q**** **= 200 *units***

S* = $10 *per order

H* = $.50 *per unit per year

$$\text{Expected number of orders} = N = \frac{\text{demand}}{\text{order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = 5 \text{ orders per year}$$

An EOQ example

Determine optimal number of needles to order

D* = 1,000 *units | ***Q**** = 200 *units*

S* = $10 *per order | ***N*** = 5 *orders per year*

H* = $.50 *per unit per year

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{N}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ example

Determine optimal number of needles to order

D* = 1,000 *units	***Q**** **= 200 *units***
S* = $10 *per order	***N*** **= 5 *orders per year***
H* = $.50 *per unit per year	***T*** **= 50 *days***

Total annual cost = setup cost + holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$TC = (5)(\$10) + (100)(\$.50) = \$50 + \$50 = \$100$$

Robust model

- ☑ The EOQ model is robust
- ☑ It works even if all parameters and assumptions are not met
- ☑ The total cost curve is relatively flat in the area of the EOQ

An EOQ example

Management underestimated demand by 50%

D = ~~1,000 *units*~~ 1,500 *units* *Q** = 200 *units*

S = $10 *per order* *N* = 5 *orders per year*

H = $.50 *per unit per year* *T* = 50 *days*

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50) = \$75 + \$50 = \$125$$

Total annual cost increases by only 25%

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An EOQ example

Actual EOQ for new demand is 244.9 units

D* = ~~1,000 *units*~~ 1,500 *units ***Q**** **= 244.9 *units***

S* = $10 *per order ***N*** **= 5 *orders per year***

H* = $.50 *per unit per year ***T*** **= 50 *days***

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$TC = \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$TC = \$61.24 + \$61.24 = \$122.48$$

Only 2% less than the total cost of $125 when the order quantity was 200

Reorder points

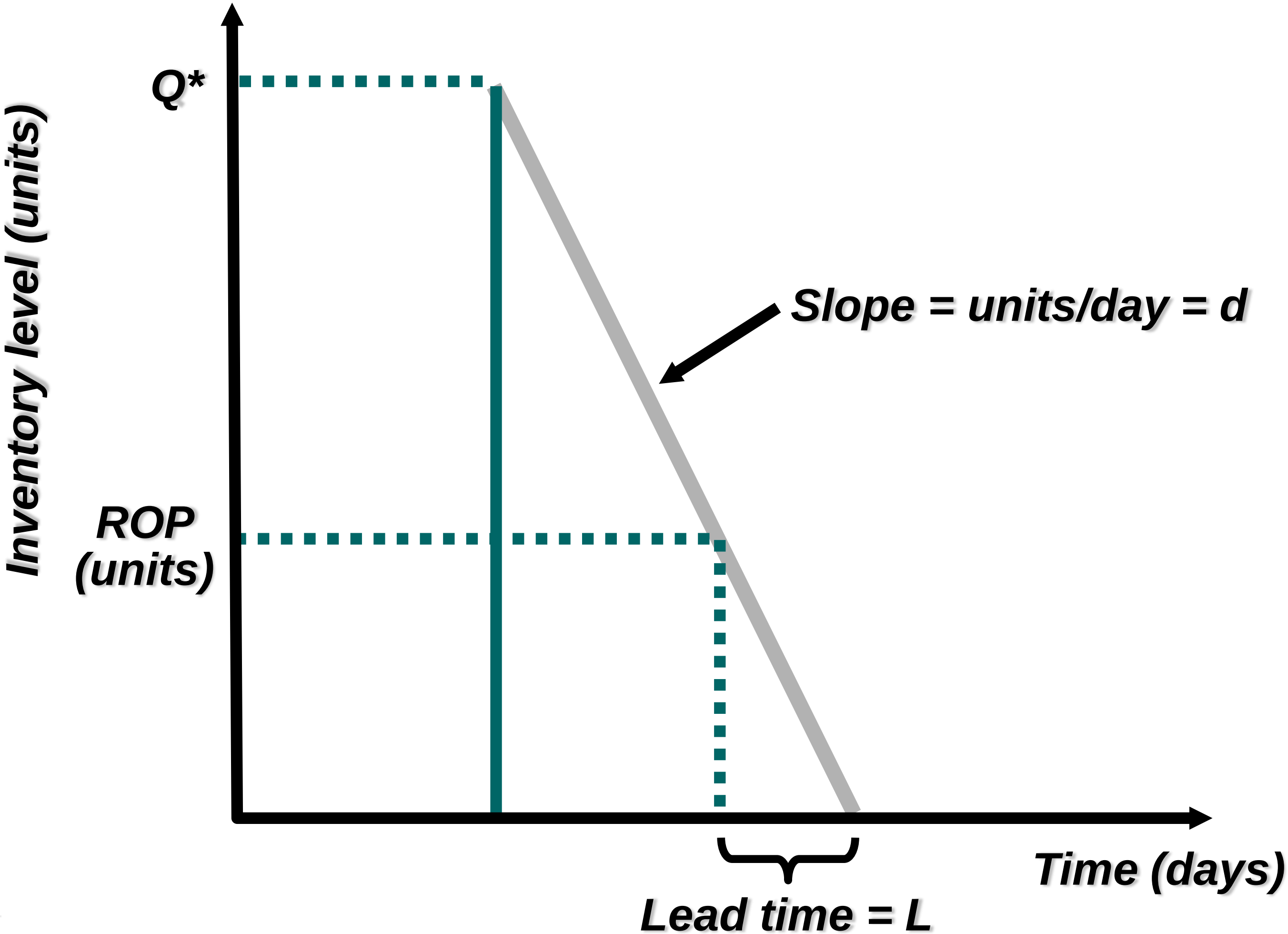
- ☑ ***EOQ answers the 'how much' question***
- ☑ ***The reorder point (ROP) tells when to order***

$$ROP = \left(\begin{array}{c}\text{Demand}\\ \text{per day}\end{array}\right)\left(\begin{array}{c}\text{Lead time for a}\\ \text{new order in days}\end{array}\right)$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder point curve



Reorder point example

Demand* = 8,000 *iPods per year
250 *working day year*
Lead time for orders is* 3 *working days

$$d = \frac{D}{\text{Number of working days in a year}}$$

$$= 8{,}000/250 = 32 \text{ units}$$

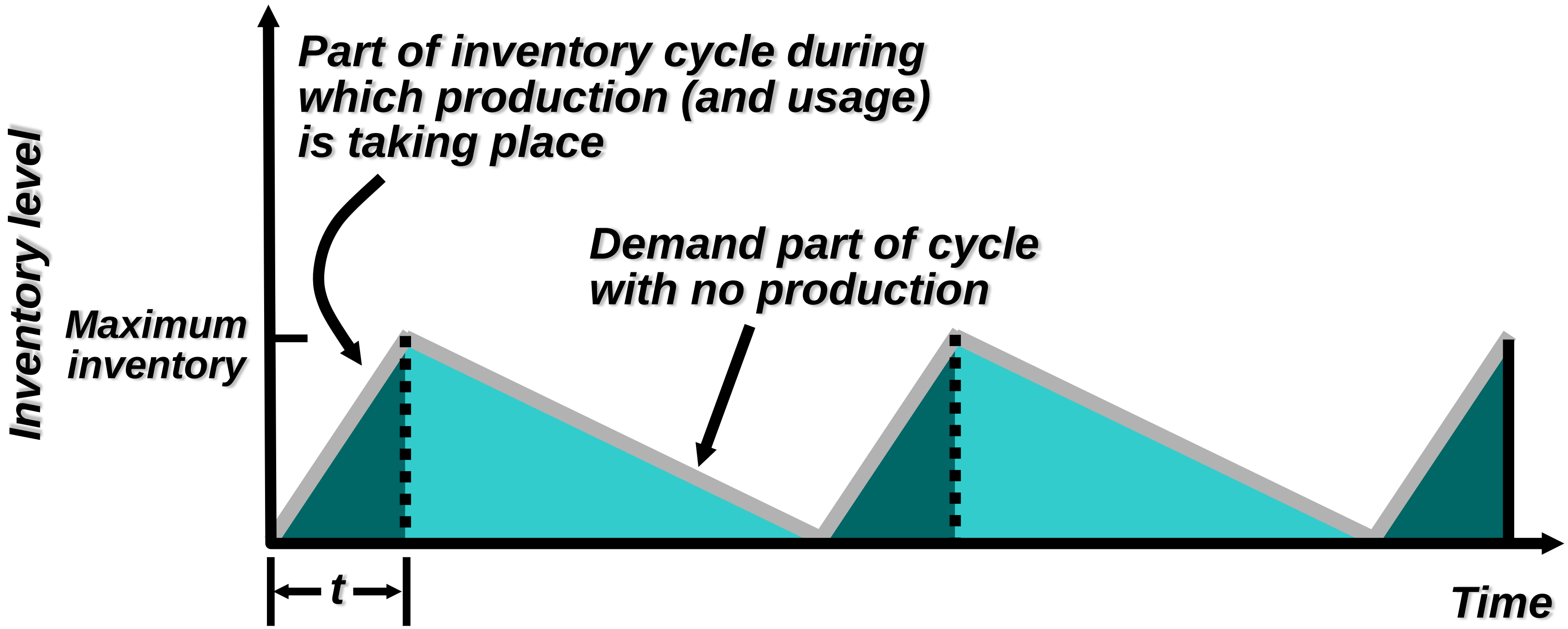
$$ROP = d \times L$$

$$= 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

Production order quantity model

- ☑ Used when inventory builds up over a period of time after an order is placed
- ☑ Used when units are produced and sold simultaneously

Production order quantity model



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Production order quantity model

Q **=** ***Number of pieces per order***
H **=** ***Holding cost per unit per year***
t **=** ***Length of the production run in days***
p **=** ***Daily production rate***
d **=** ***Daily demand/usage rate***

$$\left(\begin{array}{c}\textit{Annual inventory}\\ \textit{holding cost}\end{array}\right) = (\textit{Average inventory level}) \times \left(\begin{array}{c}\textit{Holding cost}\\ \textit{per unit per year}\end{array}\right)$$

$$\left(\begin{array}{c}\textit{Annual inventory}\\ \textit{level}\end{array}\right) = (\textit{Maximum inventory level})/2$$

$$\left(\begin{array}{c}\textit{Maximum}\\ \textit{inventory level}\end{array}\right) = \left(\begin{array}{c}\textit{Total produced during}\\ \textit{the production run}\end{array}\right) - \left(\begin{array}{c}\textit{Total used during}\\ \textit{the production run}\end{array}\right)$$

$$= pt - dt$$

Production order quantity model

Q = *Number of pieces per order*
p = *Daily production rate*
H = *Holding cost per unit per year*
d = *Daily demand/usage rate*
t = *Length of the production run in days*

$$\left(\begin{matrix}\text{Maximum}\\ \text{inventory level}\end{matrix}\right) = \left(\begin{matrix}\text{Total produced during}\\ \text{the production run}\end{matrix}\right) - \left(\begin{matrix}\text{Total used during}\\ \text{the production run}\end{matrix}\right)$$

$$= pt - dt$$

However, Q = total produced = pt ; thus t = Q/p

$$\left(\begin{matrix}\text{Maximum}\\ \text{inventory level}\end{matrix}\right) = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Production order quantity model

Q = *Number of pieces per order*
H = *Holding cost per unit per year*
D = *Annual demand*
p = *Daily production rate*
d = *Daily demand/usage rate*

$$\text{Setup cost} = (D/Q)S$$

$$\text{Holding cost} = \frac{1}{2}HQ[1 - (d/p)]$$

$$(D/Q)S = \frac{1}{2}HQ[1 - (d/p)]$$

$$Q^2 = \frac{2DS}{H[1 - (d/p)]}$$

$$Q_p^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

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Production order quantity example

D = 1,000 *units*
S = $10
H = $0.50 *per unit per year*

p = 8 *units per day*
d = 4 *units per day*

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50[1 - (4/8)]}} = \sqrt{80{,}000}$$

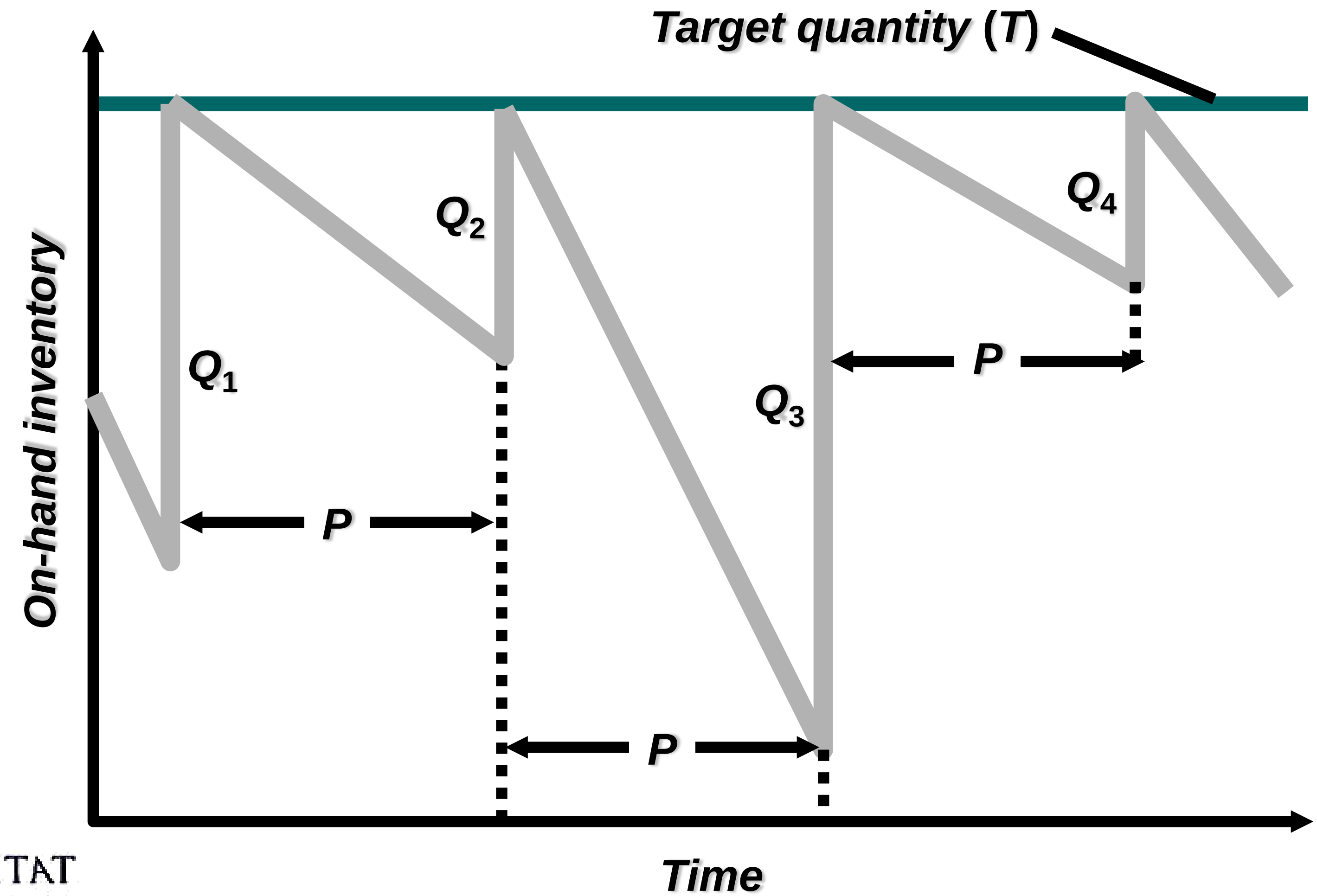
= 282.8 *or* 283 *hubcaps*

Fixed-period (P) systems

- ☑ Orders placed at the end of a fixed period
- ☑ Inventory counted only at end of period
- ☑ Order brings inventory up to target level

  - ☑ Only relevant costs are ordering and holding
  - ☑ Lead times are known and constant
  - ☑ Items are independent from one another

Fixed-period (P) systems



Fixed-period (P) example

3 jackets are back ordered
It is time to place an order

No jackets are in stock
Target value = 50

Order amount (Q) = Target (T) - On-hand inventory - Earlier orders not yet received + back orders

Q = 50 - 0 - 0 + 3 = 53 jackets

Fixed-period systems

- ☑ Inventory is only counted at each review period
- ☑ Can be scheduled at convenient times
- ☑ Appropriate in routine situations
- ☑ May result in stockouts between periods
- ☑ May require increased safety stock