Broglie and Young, visionaries who shed light in the polar topology that grounds our reality: a hypothesis

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A mathematical observation relating fractal patterns and the convolution operation in the context of digital image processing disrupted an investigation that drives us to hypothesize that the concept of the wave of matter (or duality wave-particle) stands to the dichotomy between the pair weak and a strong topology in the realm of Singular-continuous nowhere-differentiable attractors framework and the Photon-soliton concept by Vigier. Such inference appears to be more evident in the Broglie-Bohm interpretation of quantum mechanics in the local x global features crossover. Follows of this also that the relation of the natural phenomena (scale-free) with the generalized Schröder equation (standing to iteration) is still to be totally explored. The cosmological inflation and ordinary quantum phenomena (by the perspective of after wavefunction collapse) both of them would possibly stand to the solutions of such disregarded generalized Schröder equation that would also trace back to the Singular-continuous nowhere-differentiable attractor framework. Such theoretical insights offer a promising mathematical accommodation that fits recent observations using the PLK+BAO+SN+H datasets (astronomy) suggesting a spontaneous sign change of the cosmological constant that we hypothesize tracks back to a jump discontinuity between 2 Leve-Sets that would have its roots on the concept of topological conjugacy. Such discontinuity would point to the transition between the called radiation phase and ghost dark cosmologies. The most promising route of reasoning devised so far in such a research program seems to occur in the locus that encompasses the connections between fractals and Lebesgue-Cantor measure (Bernoulli convolution). Such concepts seem to accommodate a sort of insulated unforced system which dynamics would be driven by iteration and ruled by initial condition in the realm of perturbation theory (near-integrable). Such locus seems to be preferential for one prospecting about the unification of classical physics, ordinary quantum mechanics and number theory.

Keywords: Interacting Field, total variation filtering, surface of control, Reproducing Kernel, normally hyperbolic invariant manifolds, Level-Set methods, Lebesgue-Cantor measure, convergence of power series, Normal topology, staircase functions, perturbation theory, small-divisors, arithmetic physics

Preamble

As way to hint about our intuition concerning the driven by ordering pattern framework that we will attempt to evidence for the reader, we would like to recover the following quotes:

"..I have to pay a certain sum, which I have collected in my pocket. I take the bills and coins out of my pocket and give them to the creditor in the order I find them until I have reached the total sum. This is the Riemann integral. But I can proceed differently. After I have taken all the money out of my pocket I order the bills and coins according to identical values and then I pay the several heaps one after the other to the creditor. This is my integral"... quote attributed to Heri Lebesgue according to Siegmund-Schultze (2008)

"… mapping from an interval to itself provide the simplest possible examples of smooth dynamical system" (Milnor and Thurston,1998)

"Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity. It is my conviction that purely mathematical construction enables us to discover the concepts and the laws connecting them, which gives us the key to understanding nature… In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed."
Albert Einstein

1-Introduction: a mathematical clue from digital image processing disrupted a search for evidencing iteration (mapping onto itself like) in foundational physics.

In this paper we present the consequences of a mathematical observation relating fractals patterns and the operation of convolution in the context of image processing. Due to the frequent occurrence of fractals in nature (in turbulence for example), such observation inspired and disrupted a series of branched investigations driven by intuition, qualitative reasoning, pairwise comparison organized and named here as routes of reasoning. Such routes touches the mathematics behind concepts like total variation filtering, sliding mode control among others. The joint consideration of these routes seems to reveal more in-depth the invariant topological roots links with the insights by Young and Broglie (1925) and its connection to separable decoupled equations. The pair local diffeomorphism and ordering preserving automorphisms seems to be in the core of such mathematical machinery as the imprint of Lebesgue-Cantor measure.

Such fractality seems to be not be evident in the all cosmological scales possibly due to such decoupling anchored in embedded in the Bellman´s optimality principle (resonating with the Laplace/Legendre transform and or the Laplace-Beltrami operator) causing : 1) the a volumetric inflation already observed using red-shifts that would stands to the convergence between local and global features/statistics 2) occurrence of Lebesgue measure 0 (or cantorian) and different of 0 measure, whose physical imprint would accommodate the dichotomy photon(soliton) x ordinary matter respectively.

Thus, the insights obtained drive us to conclude that the simplest possible method to one obtain dynamics features whose would be grounded on iterations and a single fixed point (or in a saddle equilibrium standing a mathematical group/spinor) would grounds the features that inferred in fundamental physics.

Such theoretical prospective efforts offers a clear mathematical accommodation that fits recent observations using the Plank+BAO+SN+H datasets suggesting a spontaneous sign change of the cosmological constant that we intuit tracks back to a jump discontinuity between 2 level-sets that would have its roots on the concept of topological conjugacy. Such discontinuity would point the transition between the called radiation phase and the ghost dark cosmologies. Moreover such conjectures produce hand-waving arguments that interlock with the long-

standing search for determinism (in the sense of be ruled by initial conditions) in ordinary quantum mechanics.

Boils down from such prospection too that Einstein's equations and Schrödinger equations have some of its realistic roots interlocked with the generalized Schröder equation and iteration. The cosmological inflation the and ordinary quantum phenomena (after the wave function collapse by the Schrödinger perspective) both of them would relates to the Iteration Problem and to solutions of the translation equation that traces back the to analytical mechanics.

Such paper does not have the pretension of claim definitive answers about such fundamental questions but contrastingly only to perhaps suggest some new avenues for research as for example one gain insights on Big Bang using small scales phenomena generated by vortex ring head collision of fluid dynamics as a toy model among others.

2- The most promising route of reasoning devised so far: the Fractal-Bernoulli convolution interlock in the realm of Takagi function

Some evidences collected by us suggests that this work by Disconzi et al (2015) potentially could be considered on a search for advances in the program called arithmetic Physics (see Varadarajan (2002) and there in references). Our investigation concerning an in-deep understanding physical meaning of the singularity highlighted after the evolution towards a ghost dark cosmology model in this paper by Disconzi et al. (the called Big Rip) drive us to resemblances with the Takagi function and the big picture delineated by Singular-continuous nowhere-differentiable attractors (Tsuda et. al 1998 and therein references) that would connects photon with information processing using Cantor Set and that the we will identify in such notes as “Cantorization”.

Our proposition has some of its grounds/ roots in the conceptual connection of solitons with the photon proposed by Vigier (1991) and such analogy is key because it would endows the reality that we perceive in daily basis with a discrete flavor that at a glimpse seems to be not intuitive. We claim that the photon in such

prospection possibly could be understood in the framework of “Cantorization” and in the realm singularity theory and near integrable systems. Photons would stands to a dense set of resonances (possible mapping to a Level-set approach alternatively) in charge of small divisors as touched elsewhere (Broer et al. 1990, Hanßmann 2007, Broer and Hanßmann 2008, Broer and Sevryuk 2008).

Grounded in such delineated framework and using the bridge offered by the very possible interlock of cosmological inflation with bifurcations, we evoked here the results by Sinai (1970). Our prospective claim is that the ghost dark cosmologies stands to the following unstable system as a toy model: the called scattering billiards balls (Hamiltonian 3D) as stated by Sinai (1970) and the Big Rip would occur when the energy density (volumetric) would imply reflections anymore in such “billiard table” delimiting perimeter and this would then threshold its vanishing as an analog of space-time fabric ripping apart in the called Big rip. The math machinery of such prospective setup would relies on a Lebesgue-Cantor measure (in the realm of Bernoulli convolution) and convergence radii of formal powers series as an analog of such thresholding.

It hints renormalization (in physics jargon) as a kind “house keeping” to trap of dynamics exclusively on the framework features related to the normal topology i.e normal trajectories (phase space) regarding crossing control surfaces delimiting an invariant manifold (and the dynamics) which origins relies in an unique fixed point and multiplicative features as a type of order driven propagation due to accommodate some tolerance regarding small perturbation.

Moreover , is the perturbation theory and its “Cantorization” concepts that would offers accommodation to such accelerated cosmological inflation (ghost dark cosmology) as one of element of a pair in the realm of a Whitney smooth conjugation (standing to crossing a jump discontinuity between two Level-sets approaches or “cross the phantom divide”) inserted in the framework standing to a nearly integrable system. The other conjugated element of such pair would stand to the cosmos dynamic dominated by the gravitational (Newtonian) attraction. Such conjugate system would stand to that which shows up in Sinai´s work as related to the intersection that originates the set of singular points of a boundary. Thus, probably such set also would stand to a dense set of resonances and would be the mathematical incarnation of the photons.

By the lens of the called mathematical resources theory the photons importance would possibly stand to the class of process that is implemented with no cost (Coecke et al 2014) whose we claim that one could read it as generating zero thermodynamic entropy and presenting Lebesgue measure equals 0. This indeed traces back, by the Bernoulli convolution lens, to a mandatory transition for an infinite slope closely as math analog of Big Rip and related to that observed for the Takagi function (see Lagarias 2012). Interlocking all of this with number theory .

The background of this discussion is the long standing core is discussion of how initial conditions rules the sequent dynamics of systems. In our perspective such ideas do not implies two Level-set dominating equally the dynamics simultaneously but, contrastingly, would stand to a cross-over between both of them. This would rule out static universe model as proposed elsewhere (Rossler 2011). Such cross-over that we conjecture seems to be supported by evidences due astronomical instrumentation regarding spontaneous signal change of the cosmological constant (Akarsu 2020).

Recently the blow up of solutions of Müller-Israel-Stewart equations of relativistic viscous fluids as related to “localized perturbations of constant states” (Disconzi et al 2020) and such results appear to be interlocked with the concepts exposed above and such “constant states” seems to resonate with the concept of kernel (mask or convolution matrix) that shows up in convolution usage in image processing.

An alternative lens is to understand such cosmological dynamics would be by the lens of a limited capability of novelty generation due to a given causal set as discussed elsewhere (Krioukov 2012, Cortes and Smolin 2014, Cohen et al 2019 and therein references). Such limitation in capability of novelty generation seems to have its grounds on a the features of dynamic related to an unique fixed point generating an analog of principle holographic i.e. the emergence from a basis on R^2 dimensions of a R^3 setup in the spirit of pairing and/or Euclid method to generate primitives which will be presented more in detail in sequent section of this paper.

All of this concepts ate interlocked with previous efforts described elsewhere (Prado 2019 and therein references) and also appears to have close resemblance with a least action driven approach to thermodynamics (Garcia-Morales et al 2008).

Some resemblance with Michael Atyiah : on the realm of the contraction mapping theorem (invoke by this work as essential to approach cosmologic inflation) and variance identities both invoked evaluate the blow up of the non linear Schrödinger equation (Cazenave 2012 and therein references) seems to show up a clue for the Atyiah insights reported on “The fine structure constant” that relates fine structure with sequence of iterated exponentials linked hyperfinite type II-1 factor A by Von Neumman and or in multiplicative sequences by Hirzebruch. The Lie flow considered in the work seems to have resemblance with “the stable homotopy group π_{1+k} (SO(3 + k)) = 0, for k > 1 and not congruent to 3 mod 4” evoked by Atiyah who also touched the theme of flow with jumps and topological invariance resonating with the role topological conjugacy role discussed in this paper. The transition from SO(3) to SO(3+k) indeed seems to track back to the Kaluza-Klein accommodation of gravitational force as touched previously here.

Joint considered, boils down for us that perhaps the fine-structure constant is the limit that flags to the transition between an analogue of interacting field (driven by bulk density for example) to a free field (possibly standing to the chaos related to thermalization as hinted by Fermi-Pasta-Ulam-Tsingou experiment. Such transition seems, as touched earlier, to stands to the Big Rip of the universe (Disconzi et al 2015).

Such deep connections of Bernoulli convolution with multiplicative structures beyond appear to have some partial resemblance with Atyiah´s prospections about cosmological constant (Azcarraga 2018) also suggests an work in progress that has been done by us that is to extend the theorems used elsewhere (Li and York (1975) to identify the period concept (in realm of chaos studies) with Euclidian dimensions. If such extension indeed holds possibly it would grounds the conclusion that beyond period 3 implies chaos , 3 Euclidian dimension (R^3) also would do it. Another future discussion we will save by now is the very probable interlock of we discussed above with the small-network model (Watts,

and Strogatz 1998), specifically parametrized with p=0.5 bringing to the table a stochastic viewpoint (that would due Feynman–Kac formula to trace back to partial differential equations) named the martingale named Polya´s urn that we guess stand to Hamiltonian approach contrasting with a Lagrangian one.We conjecture that Watts and Strogatz approach to that by causal networks by Krioukov (2012) in the spirit of the Legendre transform.

2.1-The background of such a prospection: a tentative presentation of the interconnections explored so far

2.2– Revisiting the Planck law

The Planck law models the called blackbody radiation and was proposed by Max Planck in 1901 in a seminal paper (Planck,1901) entitled “Über das Gesetz der Energieverteilung im Normalspektrum” (On the Law of Distribution of Energy in the Normal Spectrum) in the Annalen der Physik journal. The quantization of energy evoked by Planck allowed the avoidance of the called ultraviolet catastrophe and helped to pave the way for the further development of quantum mechanics.

Such a mathematical model proposed by Planck has a pivotal role ins cosmology studies because of its fit of the called cosmic background radiation (CMB) that might be related to inflationary dynamics (universe expansion) that has been occurring since the Big Bang accordingly to mainstream cosmology.

The in-depth physical interpretation of the energy quantization evoked for the Planck law statement is far from been related to some scientific consensus despite its huge importance. In a previous short note by this author (Prado 2019 and therein references) it was conjectured that the Planck constant would be the imprint of some specific invariant linked to the topology.

2.3- Does convolution has been an under-explored tool to one gain insight about Planck law and the foundation of physics?

An occasional byproduct of a doctorate thesis work (Prado PF 2018c) spotted a relation between fractal patterns and the mathematical operation of convolution hinted by the aptitude of the called “spot noise” method (Wijk, 1991). Such a method has the convolution operation as its main core and is recommendable for the synthesis of fractals texture of digital images.

An insight emerged immediately: meanwhile, fractals are very common in nature´s phenomenological observation, its possible in deep relation with the convolution operation suggests many other opportunities of to one explore connection mathematics and physics and perhaps could be an important bridge to interlock these 2 scientific branches.

Whereas a fixed fractal dimension suggests an information (pattern) preservation another noteworthy outcrop of the convolution usage seems to be the de-convolution operation in the realm of the retrieval of the unit hydrograph. Such method is a consolidated procedure in hydrological sciences and its usage also seems to interlock fractal and convolution for been endowed with a flavor of feature encoding/preservation too. This unit hydrograph concept could be interpreted as a kind of a specific Kernel (or a “fingerprint”) whose iteration can be used as the core of modeling the runoff of water of a given hydrological basis and it seems to resonate with more complex concepts like reproducing kernel Hilbert space (Theodoridis,2015).

Guided by this image processing and hydrological pieces of evidence, we then expand this argumentation to one explore nature by the lens of the mathematical framework related to convolution like the shift operator, iterative maps and Lie flows, kneading theories, positive/negative definiteness, linear time-invariant systems, the Dirichlet problem among others.

Among such kneading theories, we spot the Milnor–Thurston kneading theory due to the concept of the topological conjugacy with emphasis in the the unimodal case because of its connection with monotone piecewise maps that suggests a sense of order i.e. an arrow of time. It also touches the energy quantization (concerning its piecewise feature) standing to Planck Law and consequently to cosmic background radiation fairly fitted by such law.

3- Development: memory of the routes of reasoning that has been explored in this prospection

Inspired by mathematical-physics explorations reported elsewhere (Prado PF 2017; Prado PF 2018a,b,c; Prado PF 2019a,b,c,d,e,f) this work aims to present some new insights and the historical record of our collection modeling of mounting evidence, here identified as routes of reasoning, that disrupted such analogies and insights. The interlocked consideration of such routes prospectively hints, accordingly to our mathematical gut intuition, a framework for disrupt further discussions and perhaps research programs on the consequences the fractal-convolution identification that potentially could interlock pure mathematics, ordinary quantum mechanics and classical physics by means perhaps of the most simple and fundamental pattern: ordering.

3.1- Route 1: on the potentially disregarded identification of fractal and convolution pair with concepts like boundary, unitarity and its features

Marolf (2009) presented a piece of argumentation denominated boundary unitarity. Marolf states that the AdS boundary preserves the information of time t_1 in a time t_2 in the context of the collapse of asymptotically anti-de Sitter (AdS) black holes. Such argument was used to support the generalization of the Black holes information paradox to the named boundary paradox information as discussed elsewhere (Jacobson and Nguyen 2019) who highlighted the diffeomorphism as the key theme concerning an in-depth understanding of this variant and generalized paradox.

Marolf details that:

> "*In the AdS context, much weaker assumptions imply that similar superselection effects must occur. Specifically, whether or not the set of boundary observables is complete, boundary unitarity follows directly from the assumption that, in the non-perturbative theory, the algebra of boundary observables again contains a self-adjoint Hamiltonian. While complete information may never be present at the boundary, any information present*

there at one time t1 is also contained in boundary observables at any other time t2."

The evoked superselection by Marolf seems to stands to some very specific type of fundamental embedding characterized by iteration like the circle mapping onto itself (Essl,2006) as an analog of the harmonic oscillatory setup that supported the Planck law proposition. Indeed, such circle mapping onto itself can present chaotic behavior that seems linked to the Markov partitions, hyperbolic dynamics (and deterministic chaos), and local structural stability concepts. Perhaps a rough translation of the dimension reduction stands to the specificity of the tangent bundles linked to hyperbolic dynamics i.e. a locus to accommodate a cross-over (and thus convolution and commutation) between 2 natural topologies linked to Axiom A (omega stable) diffeomorphism as follows: stable manifold stands to the strong topology whereas the unstable one would stand to the weak topology. Both of them interlocked to the supremum norm as a fingerprint of infinitesimal changes (the same that probably relates to the concept of the sliding window used for convolution in image processing for example) and that would be the core feature of what is called unitarity. Noteworthy that the geometrically driven crossing of functions convolution seems to interlocks with the geometrical grounds of Einstein equations seems and it is very specific features related to the hyperbolic dynamics.

Such concepts seem to be fairly accommodated by Bellman's equation and the principle of optimality. This principle would trackback to hyperbolics' structural stability and this very specific feature would have a fingerprint: a coarse-grained that we, in a hand-waving conjecture identify with the concept of quanta of energy as the palpable feature of a "superselection". This principle plays a major role in the far-flung of dynamic programming studies resonating also with known as viscosity solution regarding some discrete problems that seem to touch the pair: bulk viscosity consideration and Big Rip cosmological concept (Disconzi et al 2015). Remarkably all of this seems to resonate with an integral transformation ruling off a temporal variable.

In short what we are trying to grasp is that we intuit that Marolf, Jacobson, and Nguyen arguments about the boundary feature very possibly could be interpreted by a cross-over (or commuting/ convolution like) that drives to very fine-tuned

and equilibrated convergence of microstates of both manifolds: stable and unstable to a slow manifold (i.e interlocked with the concept of the boundary) implying in dimension reduction due ordering features intrinsic to iteration encoded Kernels (i.e in infinitesimal orderd changes that seem to read as adiabatic in physics) in the same spirit that grounds Lebesgue integration for example.

3-2 Route 2: following the track of Bellman´s equation

In such physical-mathematics prospection, the invocation of Bellman´s equations seems to be an adequate step because it is a bridge to the named eikonal equation through the Hamilton–Jacobi–Bellman equation. Indeed The eikonal equation shows up in modeling the origins of the black holes as trapped surfaces (An and Luk 2014). In this work also shows up an L_2-L_{inf} Sobolev embedding and supremum norm that potentially adds up consistency and fine detailing regarding the invariant slow manifold evoked earlier by us.

Additionally, the well-known relation of Hamilton–Jacobi–Bellman equation with stochastic process tracks back to iteration, recursion, and the Markov partition already touched. Moreover, Hamilton–Jacobi–Bellman methods seem to have a resemblance to the archetypal of 2 stage dimension reduction method: 1) first by been trapped in limits (with an adjunction flavor) and 2) followed by achieving the optimal control strategy standing to the settle up of optimizers of the Hamiltonian.that suggests again an embedding indeed.

Along these lines of thought, most surely those concepts linked to Bellman´s equations has also deep connections with those concepts approached in the paper "Algorithms for computing normally hyperbolic invariant manifolds" (Broer et al 1997). In this paper, the called graph transform is proposed to allow one to reduce dimensions by contracting the space of embedding with a correspondence between a fixed point end a specific invariant manifold that we conjecture could be the slow one as touched previously. The emergence of the w-lambert function on the deviation of Wien displacement law starting by the

Planck law seems to be signaling about the consistency of our reasoning so far because it would possibly interlock with the show up of implicit function related to an invariant manifold as touched by (Broer et al 1997).

3-3 Route 3: probable fingerprints of diffeomorphism in spheres harmonics expansion

´T Hooft (2016) claims that a unitary evolution operator relates to spherical partial waves expansion of matter in going to blackholes regarding studying Hawking radiation. Another claim stressed in this work, is about the existence of an antipodal identification of the regions I and II of Penrose diagram in a called in the called CPT x PT relation with a linkage to a Mobius Strip. Considering that Einstein's equations general relativity contemplate blackholes existence, we suggest a generalization grounded on the following:

We intuit such identification is just the same already spotted in the dichotomy pair strong x weak topology already touched in another route. Moreover, it could also seem to emerge in the Planck Law, specifically, in its final formulation after the half Planck addition (Mehra, J., Rechenberg 1999 and there in references, that roughly reads eq 1).

$u(f,T)= u1(f,T) + u2(f)$ where f-frequency and T-temperature eq-1

This eq-1 stands to the Planck Law and represents the radiation of electromagnetic energy per unit volume (of an ideal blackbody) as the sum of two functions that relies on different variables. In this model, the parcel u2 stands to the called vacuum or zero Kelvin energy.

Therefore, tied in the fact that one can derive the Wien displacement Law from Planck Law, we speculated to be plausible to one identify the u2(f) parcel with CPT and diffeomorphism´s strong topology with the notion of a group (mathematics) induced by the cavity feature (spherical for example). By

exclusion, the PT/ weak topology features would be linked to the u1(f, T) that would emerge as the locus of an arrow of time and or the convergence of the free energy for a minimum (thermodynamics) that would underlie cosmic inflation. In such a scenario, free energy arises as an index about the dissimilarity between these 2 natural topologies.

The notion of an order that possible is intrinsic to the cosmological inflation would have resemblance with an induced hierarchy offered by spherical harmonics (assuming a spherical holorum as the reference). Indeed this optical setup seems to encode deep insights because it allowed the deviation of the Wien displacement law based on thermodynamics arguments in a way redundant to that whose start from Planck law.

Such hierarchy related to cosmic inflation seem to have resemblance with the interplay volume x probability of collision concerning the concept of temperature and seems to trace back to studying far from thermodynamics equilibrium system through the Fermi-Pasta-Ulam- Tsingou problem and its connection to the setup lattice plus 2 competing frequencies (as studied by Born and von Karman) problem that grounds ionic-cristal studies (Carati et al 2019, Galgani 2012) and there in reference.

The adiabatic expansion of the holorum that bases Wien Law deduction seems to have analogies with the averaging of values related to numeric measures a turbulent flow occurrence been analyzed what links to the called chaotic hypothesis. These 2 last ideas will be saved for further discussion.

Such discussion on fractal /hierarchy theme seems to fit the cosmological approach by Susskind (2012) named Fractal flow that evokes source and sink concepts that seem to be fairly accommodated by the framework delineated above.

3-4 Route 4: Is there a total variation digital images denoising driven avenue for cosmological research far beyond only imaging black holes?

The Rudin, Osher, and Fatemi (ROF) model, which relies on total variations (Rudin et al 1992), recently was highlighted due to its usage in the pipeline that

generated the first digital image of a blackhole albeit as pointed elsewhere (Prado,2019b). we speculate that its utility goes very far beyond help imaging such structures. Indeed possibly they (blackholes and the ROF model) share conceptual mathematical grounds that interlocks both and thus the mathematics of ROF is a preferential bridge for one understand mechanistically the natural phenomena in general (Prado,2019b).

Through the theme of diffeomorphism encoded in a kind of ordered “flow of shapes” captured by the Level-Set method and linked to an L_2 norm. The interplay about avoiding the initial condition the in the ROF models stands to filter the noise of digital images (interpreted as an analog to the weak topology contrasting with the original image has been the strong topology). The connection of ROF with iterations (concerning a fixed point) suggests consistency of souch route of reasoning about our investigation on scale-free outcrops of weak x strong topology.

The decision on how to strip out the local features (noise to be removed) in parallel to preserving the global theme (image to be restored) drive us directly again to the discussed on the route 1 regarding Bellman's equation and principle. Total variation problems filtering are usually related to mathematical concepts like regularization, maximum a posterior theory, and functions with bounded variation.

This Total variation filtering methods of image processing includes the resolution of an optimization problem related to Lagrange multipliers (close related to regularization) that ties with iteration driven methods (based on an single fixed point) relying on Theorem of Picard-Lindelöf (and Banach–Caccioppoli fixed-point theorem / the contraction mapping theorem) that very possibly are very useful on the scale-free generalization to gain insights about our reality (marked by the convergence of free energy to a minimum by the lens of thermodynamics with a possible incarnation in convergence of Cauchy sequences) that we have been trying to grasp and evidence here. Such mathematical approach appears to emerge also in the next the concepts of the sliding mode control and in the theme surfaces of control.

3-5 Route 5 - Boundary, unitarity, diffeomorphism by the lens of a sliding mode control

By this route of reasoning, we compare/interpret the called big bang as kind or perturbation that would be handled by the emergence of a flow (related to with cosmological Inflation) that we claim to be analog on how a sliding mode controller (Young et al 1999) tracks a set of intersecting control surfaces, constrained by a tolerance delimited by the concept normality of trajectories.

The invariant manifold related to such obligatory normality of trajectories that possibly could be interpreted as a slow manifold related also to Anosov diffeomorphism. The same pattern repeats: the constraints imply reduced dimensions that ground an asymptotic convergence to stability again that we interpret as an analog to the maximization of thermodynamic entropy. The volume of space-time would be accommodated as inversely proportional to the magnitude of perturbation not suppressed yet regarding a standard (default) mode of the operation of a system that seems to trace back to vacuum (or a strong topology). The related mathematical framework includes the Picard–Lindelöf theorem and Banach fixed point theorem also possibly acmodating constants, perhaps the Planck´s one, in the concept of best Lipschitz constant and Holder exponents. The coarse-grained feature intrinsic to the concept of embedding and the slow manifold suit the context of bounded functions (that shows up in total variation filtering) related to the required normality of trajectories would act as a "control" signal with a resemblance to Planck constant perhaps. Mathematically the action of the controller possible stands to the convergence of Cauchy sequences of functions with bounded variation.

In this route, such a "controlling" flow would stands to a Lie flow plus small perturbation, and the solution of the translational equation just like the regular stable (pre-perturbation) would stand specifically to the group related to this potential flow. Indeed, alternatively, probably we here are touching here are the solutions the translation equation in formal power series rings that has its roots linked to analytical mechanics (Birkhoff. 1922).

Such roots seem to be reasonable and consistent for dealing also with surfaces and using the called “Iteration Problem” and the with embedding showing up appearing explicitly. Such fact interlocks with the linkage with the Sobolev Embedding (L_2-L_{inf}) evoked concerning another route of reasoning. Formal power series C seems to be an asymptotic limit: a convergence linked to the concept of locally analytic diffeomorphisms of C (resonating also with the called powerset construction of computational studies).

In such interplay between minimally handled perturbation (or maximal dissimilarity between weak and strong topology) and maximum handled perturbation (or minimum dissimilarity between weak and strong topology) one can devise a duality between these two extremes conditions schematized as follows :

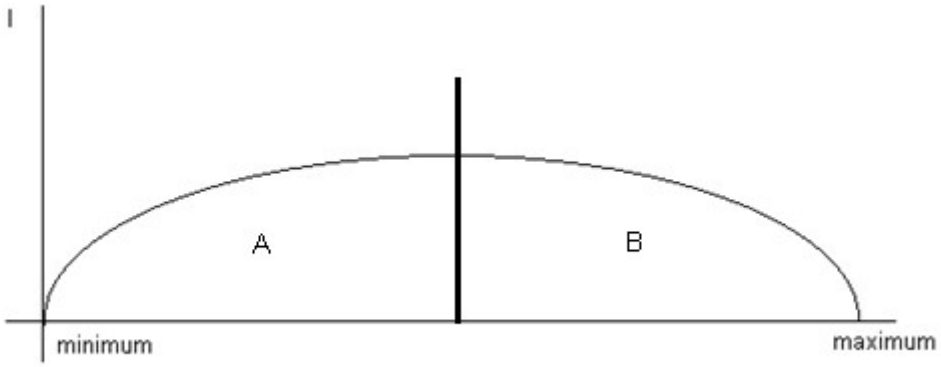

Fig 1-Schematics of a possible duality concerning perturbation handled

Where I axis would stand to an index that points out the dissimilarity regarding the extremes of this "toy model" controller action. The junction A and B supported by route 4 seems to be related to the meet of two-level sets. This scenario would interlock cosmic inflation with the concept of the staircase like function, endowed with a winding number that stands to a single saddle point equilibrium (and an unique fixed point parametrization of the dynamics). Such a staircase would have its roots related to Cantor parts of mathematical functions.

Such argumentation resonates with the efficiency intrinsic to generating dynamics based in mapping onto itself and with the concept of the Emergent universe see Kohli and Haslam (2016) and therein references.

This analogy (fig1) could potentially be accommodated in its 1st derivative the spontaneous sign switch of the cosmological constant verified in observational data elsewhere (Akarsu et al 2020). Such change suggests a topological conjugacy (driven by the kneading theory analogy) that would accommodate the transition between a dynamic dominated by Newtonian gravity to another one dominated by what is named ghost and dark energy cosmology standing to an accelerated cosmological expansion.

These idea would also be interlocked with the solution of generalized Schröder equation and such dichotomy (Newtonian x dark) would stand to the same of semi canonical forms concerning conjugation by the lens of mathematics. Such mathematics regarding the solution of generalized Schröder equation concepts are detailed elsewhere (see Reich1982 and therein references).

Indeed, driven by mathematical instinct, appear to us that this dichotomy has interlocks with the switch between positive-definite to negative-definite concerning complex Hermitian matrices (and/or a Hessian Matrix) suggesting that possible it could be spotted in some specific parametrization of Einsteins´ field equations.

Its seems not to be mere coincidence that positive-definiteness and negative-definiteness also play a major role in the Hammersley–Clifford theorem used statistics mechanics. Such observation suggests that the flow that would grounds with a dynamical system would stand with the Markov properties and the interplay Gibbs x Markov random fields by the lens of a probabilistic approach. Such discussion tracks back to the Route 1 discussion on interlocking ordering, convolution and preservation of information as touched in the Marolf (2009) arguments.

Our intuition suggests a possible deep interlocking of topological conjugacy with the theme of inhomogeneous dilation and the diagonalizable matrices that perhaps has its grounds on the geometric features intrinsic to the matrices concept and it will be saved for future discussions.

3.6 - Route 6 - The Lie and Jordan Algebras interplay

Baez (2020) explores in depth, using the mathematics, the called Noether theorem in the branch of physics studies touching the theme of convexity regarding states of a physical system.

By doing this he sheds light on the emergence of a complex number in the context of ordinary quantum mechanics, specifically in the realm of the mapping observables and generators that would be captured by the interplay of unital JB algebra (encompassed by Jordan Algebras) with the Lie algebras interlocked with the Noether´s theorem accordingly to him. This author states also that the Noether´s theorem relates 'self-conservation principle' and to the occurrence Lagragian also.

Considering that the Langrangian shows up also in the math of total variation denoising (route 4) and that the Lie algebras possibly related to iteration mediated by an unique fixed points. If we extended the our topological reasoning for those ideas, weak topologies would stand for observables and the unital JB-algebra while generator would stand to Lie algebras.

Again the convergence of weak to strong topology appears to be interlocked with the supremum (and the mandatory sense of order) norm that also shows up in the unital JB norm. Such duality weak x strong topology that seems to fairly accommodate wave x particle duality (especially by the Bohm interpretation lens) in the spirit of cross over between local x global features.

3.6.1 Route 6 -number theory, Euclid method , pairing, and polar topology

As an intuitive attempt to turn this approach of route 6 more palpable/realistic we go a step further and present here an insight: that an asymptotic convergence (driven by quadratic irrationals) could accommodate the features of the Anosov diffeomorphism as discussed earlier. For such objective, it would induces some kind of "synthetic" or auxiliary dimension (that would tracks back to iteration also)

induced by the concept of self-consistency or persistency in the spirit of perturbation theory. Such machinery could acomodate the weak topology convergence to the strong one as approached as an analog of a flow originated by an iterative mapping onto itself.

Such ideas drive us to claim for identification of dimensions with primitives (and the math concept of pairing in the framework of polar topology).These steps could be done by relying on the very old idea by Euclid who proposed a method to generate primitive triplets of integers based on a single pair of integers (m,n) and m>n to generated such auxiliary dimension as follows and also described elsewhere (Satija 2018). $A^2 + B^2 = C^2$ eq-1. All of this seems to touch the Holographic principle indeed.

$A = m^2 - n^2$, $B = 2mn$ and $C = m^2 + n^2$ for satisfy eq-1

Grounded by such reasoning, follows the that relation unital–JB algebras (as observables) approximation driven by a perturbation related flow and our perception of geometry *per si,* we conjecture, would stand to such algebra.

In brief, such flow would be triggered by the approximation of a quadratic irrational standing to a saddle point equilibrium and the Lie algebra endowed with the role of a generator. These analogies would be related the with Pythagorean trees and Diophantine equation (specifically equation of Pell´s type) driven by the settle up of an invariant path (rooted in a periodic string of h-matrices) in such tree (see Satija 2018 for detail) that would produce equations of this type. And these ideas suggest a locus for unifying classical physics, ordinary quantum mechanics and number theory.

3.7 Route 6 – A way to grasp the concept of antimatter?

We conjecture that the concept of matter and antimatter could be spotted on two separatrices that together with a saddle focus would compose the unstable manifold in a framework delineated by the called Shilnikov bifurcation. Indeed our gut instinct suggests that Satija (2018) also touched the concept of slow manifolds possibly related to such a saddle focus when this author identified the connection of right triangles (Pythagorean triplet) with the integer null vectors in

the light cone of a real 3-dimensional Minkowski space. This role of Pythagorean triples in its turn seems to be interlocked with to conversion of pathological functions one in “nice” ones in the framework of Lebesgue integration or the concept of non-defective matrices i.e. very unique ordering driven conditions resonating with the exceptional condition tracing back again to emergent universe as already touched in this paper previously (see Kohli and Haslam (2016) and therein references).

This reasoning leads to the analogy between the concept called Static Universe of Einstein with the called Pythagorean spinor (Satija 2018) related to the pair (m,n). The static feature stands to absence of infinitesimal rotation information. Such observation extends automatically the settle up of Euclid´s like a method to generate a triplet based on this pair (m,n) very probably someone that relates with a Pell´s equation (Satija 2018) that reads like this (eq -2) and whose emergence would stand the disruption of a Lie flow due to conjunction of a group and an initial condition appear i.e generating the solution of the translation equation standing as the mathematical incarnation/analog of the what is named Big Bang.

$x^2 – sy^2 = 1$ *eq-2:*

This equation has fundamental flavor because d=1 stands for the occurrence of invariants (that tracks back to an earlier discussion on dimension reduction) along all branches of the related Pythagorean tree. The (m,n) pair and show up of a spinor (tracking back to a group) also seems to resonate with the antipodal identification and its relation to the Mobius Strip regarding blackholes (´T Hooft 2016). If this spinor connection is omnipresent as the mounting evidence presented here suggests scenario whose fractality in the big picture would be the connection to a recursion linked to a specific spinor (group, probably, standing to formal power series ring).

Interlocking and reinforcing the fractal-convolution identification that disrupted such studies, Akhiezer and Peletminskii (2002) reports that convolution for higher ranks spinors resulting in low ranking spinors (resonating with dimension reduction) would be linked with an upper and lower index of the same type.

These ideas perhaps trackback to the module of the cosmological constant as standing to the result of a convolution i.e as being a kind of scalar representative a functional related to the cross-relation between the unstable and stable manifold related to the saddle equilibrium point.

Conclusions and final remarks

We attempt to present hand-waving mounting evidences and to invite for the studies of our reality (scale-free) by the lens of Singular-continuous nowhere-differentiable attractors framework and the photon-soliton concept by Vigier.

Possibly the most friendly way to sense such theme of discussion is through identifying iterations with the adiabatic constraint invoked regarding the Wien displacement law as the source of slow manifold. Such setup would then arises from R^2 by originating the “synthetic” dimension completing the R^3 with perceive.

The very low expansion of a holorum probably maximized a notion of order and continuity with an analog in allowing a kind of "memoryless" stochastic process. Such constraint would have a mathematical counterpart in the Lebesgue integration and its deep relation with volume and area measure in our daily reality or perhaps the notion of order of infinitesimal rotation related to spinors. The driven by iteration and simplest possible way of obtaining a notion of order seems to be the conceptual grounds of the 2nd law of thermodynamics.

Baby universes, Many worlds, before Planck era, before wave collapse: all of this concept seems to be interlocked only with the mathematical concept of a group (power series ring for example) but not endowed with any consideration about infinitesimal rotation i.e. without the totality of the information necessary to disrupt a Lie Flow (interpreted as th machinery of renormalization).

Such ideas driven by spinors Such seems to accommodate a closed universe as touched elsewhere (Di Valentino et al. 202) and suggests that could be reasonable the concept of the multiverse. Also this group, with possible links to the quaternion concept, seems to relate to the Kaluza-Klein method to

accommodate gravitational force by adding initial data to the group information and that would reveal a deep insight about String theory potentialities.

An expected singularity would arises in this transition of been related to group contrasting with been related to the group plus an initial condition that points out the consistency of our reasoning concerning the framework of Einstein´s equations i.e. our reasoning doesn't lead to singularity free scenario.

Due to the theme of origins blackholes linked trapped surfaces and the role viscosity in the discussed earlier, we have the intuition that gains insights about the Big bang by comparing it with the induced origins of small by head-on vortex head on ring collisions (McKeown 2018) that would encode the tolerance concept linked to perturbation theory i.e. the emergence of a non zero Lebesgue measure related to such collision standing to the emergence of the wave-particle duality too.

The mounting evidence suggests that the ultimate simplicity intrinsic to the generation of dynamics based on a saddle point equilibrium (and an unique fixed point) would be determinant to the emergence of order-preserving automorphism regarding, possibly, the ring of formal power series that would interlock and ground the unification of ordinary quantum mechanics, classical physics and a specific branch of mathematics.

Whereas we are exploring foundational themes is expected that it outcrops in a huge variety of branches of the scientific investigation and such seems to occur. The same type of local diffeomorphism spotted here seems to emerge also in: the rare features of short-time Fourier Transform, memristor modelling, Van de Pol oscillator all of these are potential sources of a new perspective and will be saved for discussion of future works.

Our intuition that we are only starting to grasp all the consequences of an in-depth interpretation of the duality by such remarkable scientists that Louis de Broglie and Thomas Young were.

Competing interests

We don’t have any competing interests to declare.

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