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Distributed Clustering Algorithm for Spatial Field Reconstruction in Wireless Sensor Networks

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***Abstract*—In this paper, we consider the problem of distributed spatial estimation for field reconstruction in wireless sensor networks. In order to estimate the field, a geostatistical technique called kriging is used. Centralized spatial estimation algorithms with a large number of sensors lead to significant computational cost and energy wastage. We present a novel distributed clustering algorithm for estimating spatial interference maps, which are essential for operations and management in future wireless networks. In this algorithm, clusters are adaptively formed with a small subset of sensors by minimizing the kriging variance. The semivariogram computation and kriging prediction are locally performed in each cluster in a distributed fashion. The complexity of the clustering algorithm is analyzed and its performance is evaluated by comparing it with centralized and other distributed approaches.**

I. Introduction

Wireless Sensor Networks (WSN) are event-aware systems composed of a large number of sensor nodes with the aim to monitor a spatial physical phenomenon. Typical WSN applications require spatially dense sensor deployments in order to achieve a satisfactory coverage. Due to this high density in the network topology and the recording of a common phenomenon, the sensor network data are spatially correlated. Exploiting such correlation brings potential advantages for the development of efficient communication protocols [1]. In this work, correlations in WSN are exploited for calculating spatial interpolations of the field in spatial locations where no sensors are available.

Spatial field reconstruction in WSN has also gained enormous attention in the research community due to its potential application in 5G heterogeneous networks, where the availability of spatial interference maps could improve the spectrum usage by using more efficient resource allocation algorithms [2][3]. There are two ways to implement spatial interpolation techniques in WSN. In the first approach, all network node measurements are considered for estimation at every possible location. However, global reconstruction using centralized estimation techniques pose several limitations that cannot be neglected. Firstly, full connectivity in WSN is quite difficult to establish and secondly, high energy is required to communicate between all the nodes. Finally, the most important drawback is the computational complexity resulting from solving a large system of equations. An alternative approach to centralized estimation is to perform a cluster based estimation, where only a subset of sensor nodes are utilized for estimation of a given location.

One of the tools that has been employed in the literature for field reconstruction in WSN is the geostatistical tool called Kriging [4]-[6]. In this work, we use kriging since it assumes that the mean of the spatial field is unkown. Geostatistics analysis is a two-stage process: semivariogram modeling and kriging prediction. The semivariogram describes the spatial correlation among samples, while kriging technique uses the semivariogram model for spatial interpolation. In this paper, we will consider ordinary kriging for spatial reconstruction because of its advantages over other kriging variants, i.e., it does not require the mean to be known and shows lower complexity.

The application of distributed kriging algorithms to field reconstruction in WSN is quite limited. In [5], a confident information coverage model based on kriging is proposed for field reconstruction in sensor networks. However, this model uses a fixed number of sensors closest to the unknown location for estimation. In [6], a sparsity promoting ordinary kriging approach via alternating direct method of multipliers is proposed for field reconstruction. Even though this approach penalizes the number of non-zero kriging weights, it still considers a centralized estimation. In [7], the Spatial-Best Linear Unbiased Estimator (S-BLUE) algorithm is developed to solve the problem of spatial field reconstruction in WSN under quantization and imperfect channels. However, node measurements are transmitted to a fusion center for global field reconstruction. In [8], the field reconstruction problem is addressed using shift-invariant spaces by splitting the global reconstruction problem into smaller local problems. Moreover, the cluster head performs local field reconstruction and transmits to the fusion center for global reconstruction.

An algorithm that applies the kriging technique in a distributed way for field estimation is the Distributed Iterative Kriging Algorithm (DIKA) proposed in [4]. Field estimation using DIKA consists of two phases. First, the semivariogram is computed distributively using an iterative algorithm. Secondly, the network nodes distributively apply kriging to estimate the field value. In addition, a clustering algorithm is proposed in which clusters are formed by comparing the kriging variance with a predefined threshold. In our paper, we address the problem of field reconstruction under a clustering approach,

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where we optimize the size of the cluster by successively adding nodes and comparing the resulting kriging variances. Also, we detail the cluster formation and present interpolated maps and various results to evaluate the algorithm performance. Our main objective is to present a distributed clustering algorithm to reconstruct the entire spatial field with the least node measurements by using the kriging variance as a metric to optimize the cluster size.

The organization of this paper is as follows: In section II, we will define the problem considered and the goal pursued in this work. A brief overview of the kriging backround and modeling is presented in section III. Section IV describes the distributed clustering algorithm and complexity analysis. Simulation results and discussion considering different scenarios are presented in section V. Finally, conclusions are drawn in section VI.

II. Problem Statement

We will model the WSN as a connectivity graph $\mathbf{G}(\mathbf{V}, \mathbf{E})$, composed by a set of n nodes $\mathbf{V} = \{1, 2, ..., n\}$, so that node i is located at position $\mathbf{x}_i$, where $\mathbf{x}_i \in \mathbb{R}^2 \ \forall i$, and a set of links $\mathbf{E}$. Each node of the network is located at a physical position denoted by (x, y). Because of the limited capabilities of the nodes, we assume the range of transmission for each node is limited to a distance R. As a consequence, the link E_{ij} is established only if the euclidean distance between nodes i and j, denoted by d_{ij}, is lesser than R. Each network node i measures the field at location $\mathbf{x}_i$, denoted by $V(\mathbf{x}_i)$, where $i = \{1, 2, ..., n\}$ is the set of all network samples. We assume that nodes are equipped with GPS that allows them to calculate the distance with respect to the nodes inside their transmission range. Otherwise, we assume that some underlying algorithm is used to estimate relative inter-node distances.

The objective of this paper is to estimate the value of the field $\hat{V}(\mathbf{x}_0)$ at unknown locations $\mathbf{x}_0$ using the available network samples $V(\mathbf{x}_i)$ and position of the sensor nodes. The nodes cooperatively work in order to estimate the field. In the case of clustered estimation, nodes are successively included in the cluster from closest to the farthest based on the distance to the unknown location. Assuming that we are sensing spatial power spectrum maps, node measurements $V(\mathbf{x}_i)$ can be modeled as:

$$V(\mathbf{x}_i) = P(\mathbf{x}_i) + S(\mathbf{x}_i), \tag{1}$$

where $P(\mathbf{x}_i)$ is the average received power depending on the path loss model and $S(\mathbf{x}_i)$ is the shadow fading obeying a lognormal distribution [9]. The received power at location $\mathbf{x}_i$ from N_t single antenna transmitters is calculated by the simple path-loss model:

$$P(\mathbf{x}_i) = K_{dB} + 10\gamma \log_{10} d_0 + 10 \log_{10}(\sum_{t=1}^{N_t} d_{it}^{-\gamma}), \tag{2}$$

where K is the constant path-loss factor, γ is the path loss exponent, d_0 is a reference distance for antenna far field and d_{it} is the distance between node location $\mathbf{x}_i$ and transmitter location $\mathbf{x}_t$.

The location of the network nodes is an important issue that must be considered to reach an optimal sampling. Node placement, number of nodes and the quality of node measurements determine the field reconstruction quality. In this work, a realistic spatial deployment is considered and a random distribution of the network nodes is applied (see figure 1).

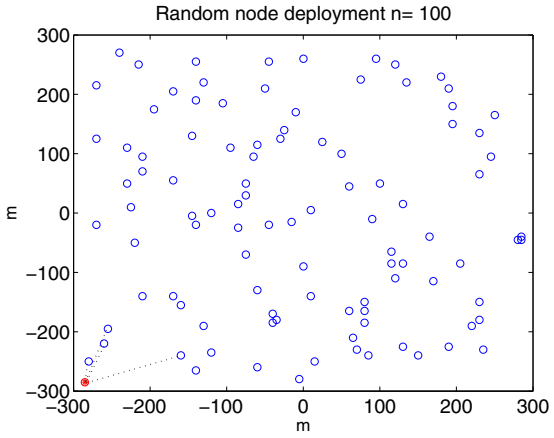


Fig. 1: Random node placement of size n=100

III. Kriging Basics

Geostatistics describes the spatial variability of a phenomenon and estimates the values of this phenomenon at an unknown location [10]. When using ordinary kriging, it consists of a semivariogram analysis and kriging prediction.

A. Semivariogram analysis

The semivariogram is a structural and descriptive tool through which second order properties of a spatial process can be studied. It is a measure of the degree of spatial dependence between the samples. The empirical semivariogram (EV), denoted by $\hat{\gamma}(\mathbf{h})$, is defined as half the average squared difference between samples separated by a lag distance distance $\mathbf{h}$:

$$\hat{\gamma}(\mathbf{h}) \equiv \frac{1}{2|\mathcal{N}(\mathbf{h})|} \sum_{\mathcal{N}(\mathbf{h})} (V(\mathbf{x}_i) - V(\mathbf{x}_j))^2, \tag{3}$$

where $V(\mathbf{x}_i)$ and $V(\mathbf{x}_j)$ are data values at spatial locations $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. $\mathcal{N}(\mathbf{h})$ is set of all location pairs $(\mathbf{x}_i, \mathbf{x}_j)$ such that $\mathbf{x}_i - \mathbf{x}_j = \mathbf{h}$ and $|\mathcal{N}(\mathbf{h})|$ denotes the number of distinct pairs in $\mathcal{N}(\mathbf{h})$.

Spatial correlation in a phenomenon can be observed by computing the EV value for several lags. However, in the case of kriging techniques, the EV has to be replaced with an acceptable semivariogram model in order to satisfy certain numerical properties. Hence, a phenomenon's spatial correlation model is established by fitting a curve onto the computed EV values. Semivariogram modeling involves choosing a theoretical model and fitting process. In this paper, the spherical model is used as a theoretical model, since it models the data well for all sort of distances, whereas weighted least-squares are used to fit the model. The spherical model is given by:

$$\overline{\gamma}(\mathbf{h}) = \left\{ \begin{array}{l} c_1 + c_2 \left\{ \frac{3}{2}(\frac{\mathbf{h}}{c_3}) - \frac{1}{2}(\frac{\mathbf{h}}{c_3})^3 \right\}, \end{array} \right.$$

where c_1, c_2 and c_3 are nugget, sill and range, respectively.

B. Kriging prediction

Once a semivariogram model is established for a given phenomenon, it can then be used for spatial interpolation using kriging. Kriging can be viewed as a weighted average method where the estimation of a phenomenon at a given location is a linear combination of the neighbouring values. Let $\mathbf{V} = (V(\mathbf{x}_1), V(\mathbf{x}_2), ..., V(\mathbf{x}_n))$ be the data observed at spatial locations $(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$, respectively. The kriging interpolator at an unknown location $\mathbf{x}_0$ is given by:

$$\hat{V}(\mathbf{x}_0)|_n = \sum_{i=1}^{n} w_{i|n}(\mathbf{x}_0) V(\mathbf{x}_i), \tag{4}$$

where n is the number of nodes, $w_{i|n}$ is the weight assigned for node i from an estimation performed using n nodes and $\hat{V}(\mathbf{x}_0)|_n$ is the estimated value. These weights can be obtained by solving the following system of linear equations:

$$\sum_{i=1}^{n} w_{i|n}(\mathbf{x}_0)\mathbf{A}(\mathbf{x}_i, \mathbf{x}_j) + \mathcal{L}(\mathbf{x}_0) = \mathbf{b}(\mathbf{x}_i, \mathbf{x}_0), \quad j = 1, 2, \ldots, n \tag{5}$$

where $\mathbf{A}(\mathbf{x}_i, \mathbf{x}_j) = \overline{\gamma}(\mathbf{x}_i, \mathbf{x}_j)$ is the spatial correlation matrix between samples from locations $\mathbf{x}_i$ and $\mathbf{x}_j$, $\mathbf{b}(\mathbf{x}_i, \mathbf{x}_0) = \overline{\gamma}(\mathbf{x}_i, \mathbf{x}_0)$ is the spatial correlation vector between samples from locations $\mathbf{x}_i$ and $\mathbf{x}_0$ and $\mathcal{L}(\mathbf{x}_0)$ is the Lagrange multiplier which guarantees the kriging universality condition:

$$\sum_{i=1}^{n} w_{i|n}(\mathbf{x}_0) = 1. \tag{6}$$

The kriging variance can be calculated as:

$$\sigma^2(\mathbf{x}_0)|_n = \sum_{i=1}^{n} w_{i|n}(\mathbf{x}_0)\overline{\gamma}(\mathbf{x}_i, \mathbf{x}_0) + \mathcal{L}(\mathbf{x}_0). \tag{7}$$

IV. Distributed Clustering Algorithm

In the case of a centralized estimation technique, equations (4) and (5) are solved considering the field data observed by all the sensors in the network. In this paper, our aim is to build clusters of nodes in such a way that the entire spatial field can be reconstructed with the least number of node measurements.

A. Algorithm description

The following steps describe the algorithm operation:

1) Create clusters of nodes based on kriging variance.
2) Distributed semivariogram computation in each cluster.
3) Distributed kriging prediction in each cluster.

The distributed clustering algorithm employs only a subset of nodes to estimate the value of the field. The objective is to improve the field estimation by local semivariogram computation and kriging prediction with the least number of nodes. This is achieved by minimizing the kriging variance to optimize the cluster size. Note that node sharing is allowed in this work in order to create a complete map. We refer the reader to [4] for details on the distributed semivariogram and kriging computation.

Algorithm 1 Distributed Clustering Algorithm

// $\mathbf{t}$: Initial number of nodes in the cluster // $\mathbf{k}$: Total number of nodes in the cluster after updating process // $\hat{V}(\mathbf{x}_0)|_k$: Final field estimated value obtained through clustering // n: Number of nodes in the network.

1: **for** all $\mathbf{x}_0$ **do**
2: **for** $k = t$ **to** $n - 1$ **do**
3: Distributed kriging is performed to estimate fields value $\hat{V}(\mathbf{x}_0)|_k$ and $\hat{V}(\mathbf{x}_0)|_{k+1}$ using k and $k+1$ node sets, respectively.
4: Kriging variances $\sigma^2(\mathbf{x}_0)|_k$ and $\sigma^2(\mathbf{x}_0)|_{k+1}$ are computed by using equation (7).
5: **if** $\sigma^2(\mathbf{x}_0)|_k \leq \sigma^2(\mathbf{x}_0)|_{k+1}$, **then**
6: Final Kriging variance and estimated values are obtained.
$\hat{V}(\mathbf{x}_0) = \hat{V}(\mathbf{x}_0)|_k$
$\sigma^2(\mathbf{x}_0) = \sigma^2(\mathbf{x}_0)|_k$
7: Updating process is terminated and the cluster is formed by the node set k
8: **else**
9: A new node is incorporated to the cluster and process is restarted from line 3
$k = k + 1$
10: **end if**
11: **end for**
12: **end for**

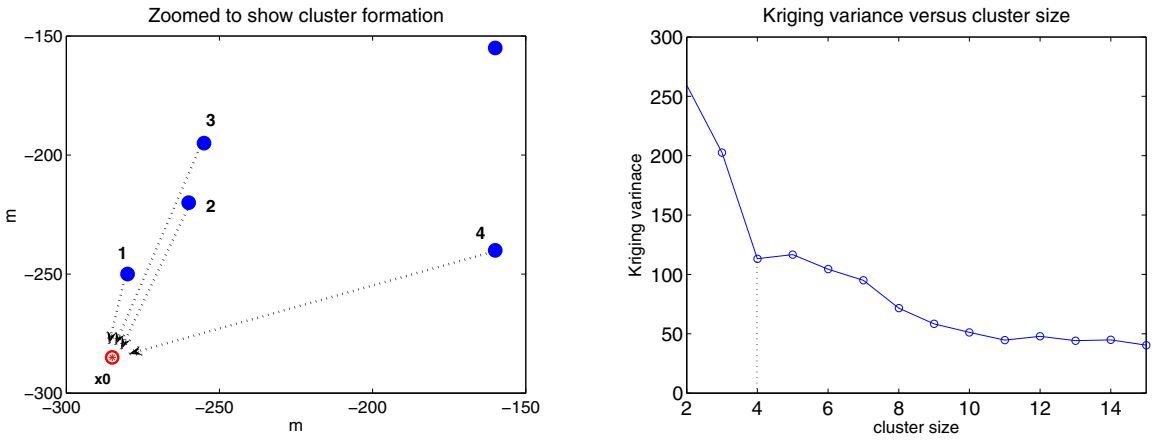
The distributed semivariogram computation uses an iterative algorithm to calculate the semivariogram by expressing equation (3) in a distributed way. Thus, a local semivariogram is computed between all the nodes in the cluster. The distributed kriging algorithm performs the modified Gauß-Jordan elimination method in an iterative way. As a result, the kriging system of equations is solved to obtain the weights and Lagrange multiplier. Finally, the kriging estimate $\hat{V}(\mathbf{x}_0)|_n$ is obtained by multiplying the weights with the corresponding field value in a distributed way.

The clustering procedure is explained in Algorithm 1. An initial set of t nodes, closest to the unknown location $\mathbf{x}_0$, begins the estimation procedure by performing kriging to obtain the estimate $\hat{V}(\mathbf{x}_0)|_t$ and computing the kriging variance $\sigma^2(\mathbf{x}_0)|_t$. The kriging variance of this estimation is calculated using equation (7). Then, a new node close to the unknown location is incorporated to the cluster and kriging estimation $\hat{V}(\mathbf{x}_0)|_{t+1}$ is performed to obtain a new variance $\sigma^2(\mathbf{x}_0)|_{t+1}$. The following equations are used to find the field estimate and weights for a newly added node [11]:

$$\hat{V}(\mathbf{x}_0)|_{t+1} = \hat{V}(\mathbf{x}_0)|_t - w_{t+1|t+1}(\mathbf{x}_0)[\hat{V}(\mathbf{x}_{t+1})|_t - V(\mathbf{x}_{t+1})], \tag{8}$$

$$w_{i|t+1}(\mathbf{x}_0) = w_{i|t}(\mathbf{x}_0) - w_{t+1|t+1}(\mathbf{x}_0) w_{i|t}(\mathbf{x}_{t+1}), \quad i = 1, 2, \ldots, t \tag{9}$$

where $\hat{V}(\mathbf{x}_0)|_{t+1}$ is the estimate at $\mathbf{x}_0$ using $t + 1$ nodes, $w_{t+1|t+1}$ is the weight assigned to the node $t + 1$ when predicting $V(\mathbf{x}_0)|_{t+1}$, $\hat{V}(\mathbf{x}_{t+1})|_t$ is the kriging estimate at



(a) (b)

Fig. 2: (a) Figure 1 zoomed to show cluster formation (b) Kriging variance versus cluster size for location $\mathbf{x_0}$=[-285,-285]

$\mathbf{x}_{t+1}$ from nodes located at $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_t$ and $V(\mathbf{x}_{t+1})$ is the measurement of node $t+1$ at location $\mathbf{x}_{t+1}$.

The variances of $\sigma^2(\mathbf{x}_0)|_t$ and $\sigma^2(\mathbf{x}_0)|_{t+1}$ are compared. If the variance with $t+1$ nodes is higher, then an update process is carried by adding a new node. This process is iteratively repeated until the incorporation of a new node no longer improves the estimation quality. As a result, a cluster is formed with an optimal number of sensors. If m nodes are finally added to the cluster, we can find the weights and estimate the field value using the following update model equations:

$$\hat{V}(\mathbf{x}_0)|_{t+m} = \hat{V}(\mathbf{x}_0)|_t - \sum_{j=1}^{m} w_{t+j|t+m}(\mathbf{x}_0)[\hat{V}(\mathbf{x}_{t+j})|_t - V(\mathbf{x}_{t+j})], \quad (10)$$

$$w_{i|t+m}(\mathbf{x}_0) = w_{i|t}(\mathbf{x}_0) - \sum_{j=1}^{m} w_{t+j|t+m}(\mathbf{x}_0) w_{i|t}(\mathbf{x}_{t+j}). \quad (11)$$

The advantage of this model is that the estimation can be updated once without performing kriging for every increase in the cluster size. Hence, the update model has lower complexity than solutions requiring to solve the system of equations each time.

B. Complexity analysis

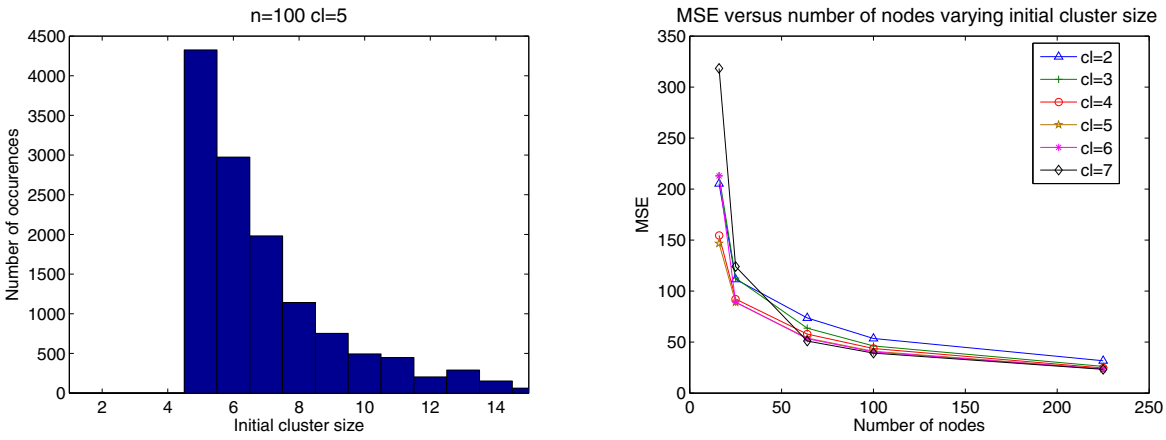
According to our algorithm, a cluster is formed with $k = t + m$ number of nodes, where t is the initial number of nodes and m is the number of nodes added to the initial cluster. Initial cluster estimation with t nodes requires one operation to estimate while adding nodes requires m number of operations. Therefore, the clustering operation has a complexity of $m+1$ times the local semivariogram and kriging estimation. Kriging variance calculation has complexity of $cost_{variance} \in \mathcal{O}(N)$. Distributed semivariogram computation involves exchanging packets between the nodes, distance calculation and semivariogram modeling. Therefore, the cost to estimate the semivariogram is given by $cost_{SV} \in \mathcal{O}(N^2)$. Kriging prediction procedure consists of solving a system of linear equations by the Gaussian elimination method to obtain weights and estimating an unknown field value from equation (4). Gaussian elimination complexity is well known and is given by $cost_{gauss} \in \mathcal{O}(N^3)$. The kriging estimation $\hat{V}(\mathbf{x}_0)|_n$ requires only local multiplication and addition. Therefore, the complexity for distributed kriging prediction is $cost_{kriging} \in \mathcal{O}(N^3)$. The complexity of the clustering approach is given by $cost_{cluster} = |m+1| \times (cost_{SV} + cost_{kriging} + cost_{variance}) \in \mathcal{O}(N^3)$.

V. Simulations

Simulations were performed considering the scenario depicted in Figure 1, where $n = 100$ sensor nodes are randomly deployed in a square area of $570m \times 570m$. In order to test our algorithm performance, we create a field that follows a normal distribution based on the spatial statistical model from equations (1) and (2) with path loss exponent γ=6, reference distance d_0=10m, number of transmitters N_t=3, sampling spacing ls=5, loss at 10m AA=20dB, correlation length $lcorr$=60m, location variablity sl=10dB and maximum radius of simulated field $maxradius$=300m.

In the distributed clustering algorithm, initial clusters close to the location target are selected to begin the estimation. Nodes in the cluster distributively compute the semivariogram and solve the kriging equations by using only the readings from the nodes. As a result, local semivariogram and weights are obtained for each node in the cluster. The advantage of this approach is that nodes with high spatial correlation to the target location are given more weights while the non-cluster nodes are neglected. Because of the adaptive nature of the algorithm, the initial cluster size can be increased by adding new nodes. However, increasing the size of the cluster does not guarantee a better estimation quality. Hence, our clustering algorithm employs the kriging variance to choose an optimal cluster size.

We demonstrate the cluster formation procedure with an example. The size of the cluster at each unknown location



(a) (b)

Fig. 3: (a) Histogram (b) MSE as a function of initial cluster size

$\mathbf{x}_0$ changes adaptively depending on the kriging variance. In Figure 2(a), an unknown location $\mathbf{x}_0 = [-285, -285]$ is considered, where an initial set $t = 3$ composed by $\{1, 2, 3\}$ performs kriging. Figure 2(b) shows the kriging variance as a new nodes are added in the cluster. In this case, the addition of a new node $\{4\}$ improves the estimation and the cluster will be formed by the set of 4 nodes. From Figure 2(b), one can see that including additional nodes in the cluster may further decrease the kriging variance. However, the computational complexity increases significantly, without having a clear impact on the estimation quality.

An important factor that must be taken into account is the initial cluster size. The only parameter influencing the formation of initial clusters is the parameter R and battery restrictions or communication constraints which are out of the scope of this paper. Therefore, we present simulations obtained with several initial cluster sizes. In order to measure the quality of reconstruction, we consider the Mean Squared Error (MSE) between the estimated field value and actual field value. The histogram plot 3(a) illustrates the number of occurrences of initial cluster sizes for reconstructing the entire area. By observing the MSE as a function of the initial cluster size in Figure 3(b) and comparing with the histogram plot, we chose a realistic initial cluster size (cl) of 5 nodes.

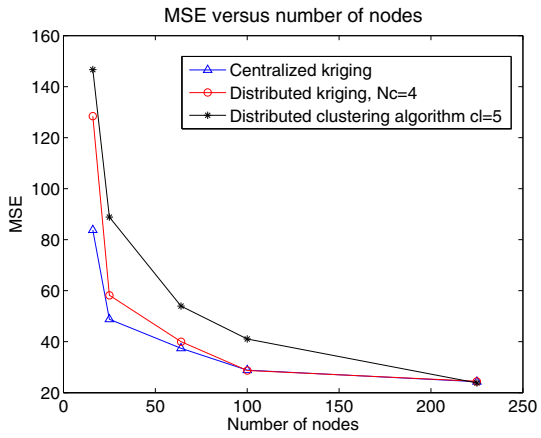


Fig. 4: MSE versus number of nodes

To evaluate the performance of our algorithm, we compare it with two proposals. The first proposal is a centralized kriging algorithm in which all nodes send their information to the fusion center for global field reconstruction. The second proposal is implemented by dividing the area into four clusters Nc=4 and applying the distributed kriging algorithm in each cluster. Figure 4 shows the performance comparison plot and Figure 5 are examples of the obtained interpolated maps. The distributed clustering algorithm uses an initial cluster size of 5 nodes. As shown in Figure 4, the centralized scheme has better performance than distributed kriging with Nc=4 and distributed clustering algorithm. This is due to the fact that it uses all the node measurements for estimation. However, our clustering algorithm uses an average of 5 nodes (see Figure 3(a)) to estimate an unknown location $\mathbf{x}_0$. In the centralized scheme, failure of fusion center leads to the complete failure of reconstruction. Due to battery life constraints, communication range R of sensors is limited. Hence, a multi hop routing protocol may be required to implement the centralized scheme, resulting in huge delays. However, the clustering scheme reduces the communication requirements due to local computations and would be beneficial in realistic scenarios for designing an energy efficient WSN. Lastly, the field reconstruction due to faulty node measurements affects a small subregion in the clustered scheme, where as in the centralized scheme, the entire region is affected.

VI. Conclusions

In this paper, a distributed clustering algorithm for spatial field reconstruction in wireless sensor networks has been proposed. The algorithm builds clusters of nodes which perform local semivariogram and kriging computations. The algorithm is performed in a distributed way, is scalable to network size, has lower complexity and is power efficient. Our future work aims to study the problem of field reconstruction for both spatially and temporally correlated fields.

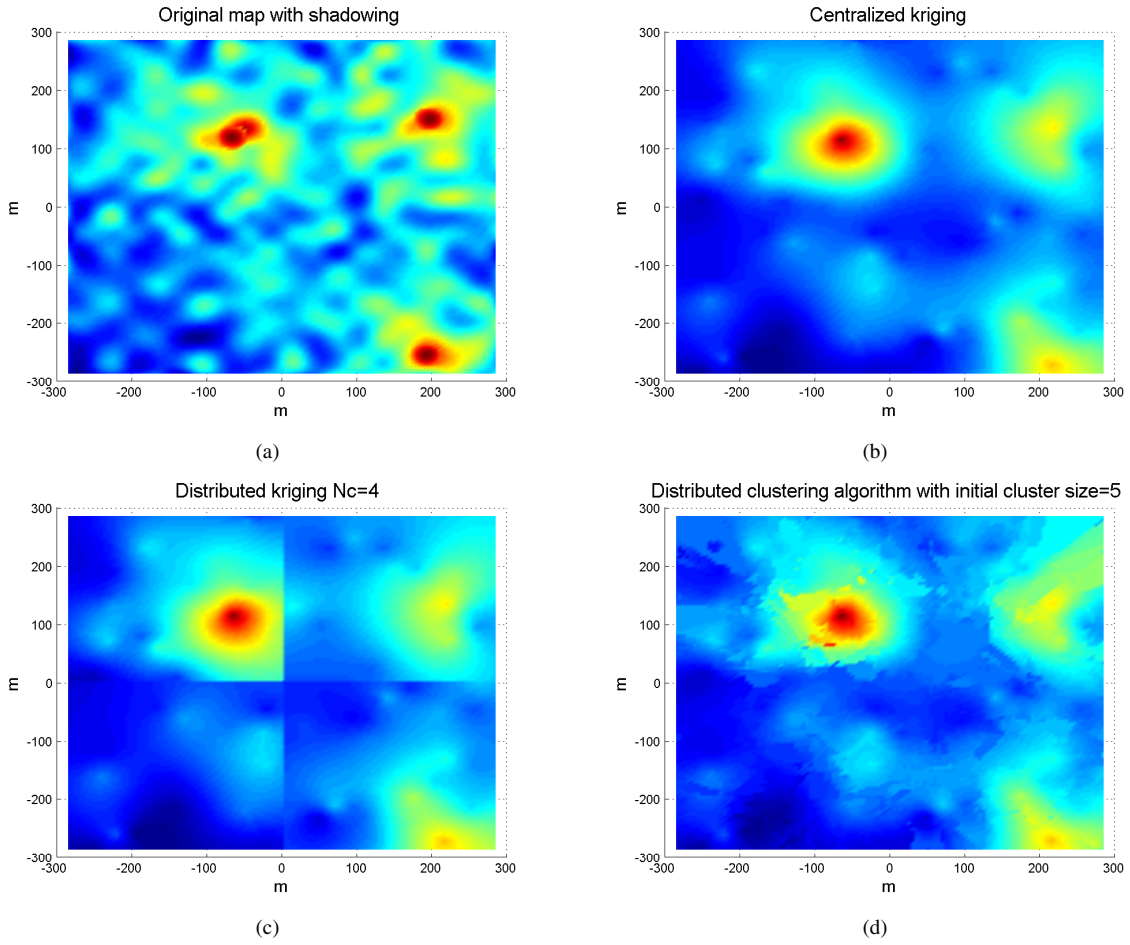


Fig. 5: Interpolated maps with n=100 (a) Original map (b) Centralized kriging (c) Distributed kriging with Nc=4 (d) Distributed clustering algorithm with cl=5

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