Non-convex Power Allocation Games in MIMO Cognitive Radio Networks

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***Abstract*—We consider a sensing-based spectrum sharing scenario in a MIMO cognitive radio network where the overall objective is to maximize the total throughput of each cognitive radio user by jointly optimizing both the detection operation and the power allocation over all the channels, under a interference constraint bound to primary users. The resulting optimization problems lead to a non-convex game, which presents a new challenge when analyzing the equilibria of this game. In order to deal with the non-convexity of the game, we use a new relaxed equilibria concept, namely, quasi-Nash equilibrium (QNE). We show the sufficient conditions for the existence and the uniqueness of a QNE. A primal-dual interior point optimization method that converges to a QNE is also discussed in this paper. Simulation results show that the proposed game can achieve a considerable performance improvement with respect to a deterministic game.**

I. Introduction

Cognitive radio networks (CRNs) are able to enhance the efficiency in spectrum usage by allowing cognitive radio users (CRs) to access the resources owned by primary users (PUs) in an opportunistic manner [1]. In order to minimize the performance degradation caused to PUs, the CR must first perform spectrum sensing to determine the status of the spectrum [2]. In practice, the reliability of the PU detection at the CR transmitter is limited by several factors. As a consequence, the influence of the sensing accuracy on the throughput of the CR should also be taken into account.

In a multi-user scenario, every CR aims at the transmission strategy that maximizes its own throughput, leading to a non-cooperative game (NCG) [3], where the solution is the well-known concept of Nash equilibrium (NE) [4]. The variational inequality (VI) method [5] has been used in [6]–[8] to analyze the solution for the NCG. However, no sensing is performed by CRs in these works. Recently, the sensing information is addressed in [9] as a part of the game, and the analysis of the equilibria of this game is based on a new concept called quasi-Nash equilibrium (QNE) [10].

The incorporation of MIMO techniques into CRNs can improve the channel capacity by sending independent data streams simultaneously over different antennas. There are some works that attempt to protect PUs in a MIMO CRN while maximizing the CRN's throughput [11]–[13]. However, due to the challenges associated with power and spectrum optimization, all the existing works on MIMO CRNs do not consider the joint optimization over the sensing information.

In this paper, we move a step ahead from current approaches, and consider a sensing-based spectrum sharing scenario in a MIMO CRN where the overall objective is to maximize the total throughput of each CR by jointly optimizing both the detection operation and the power allocation over all channels, under the interference constraint bound to PUs. The optimization problem is analyzed as a strategic NCG, where the transmit covariance matrix, sensing time, and detection threshold are considered as variables to be optimized. The resulting game is non-convex, hence, we use the new relaxed equilibria concept QNE introduced in [10]. We give the sufficient condition of the existence and uniqueness of the QNE for the proposed game, by making use of the VI method. Furthermore, a primal-dual interior point optimization (IP) method that converges to a QNE is discussed in the paper. Simulations show that the proposed game can achieve a considerable performance improvement with respect to the deterministic game in [14].

The rest of the paper is organized as follows. Section II presents our system model. The NCG is discussed in Section III. The concept and the existence of a QNE and the outline of the IP method is shown in Section IV. The simulation results are presented in Section V. Section VI states the conclusions.

Notation: Matrices and Vectors are indicated in boldface. We use $(.)^H$ to denote the Hermitian matrix transpose, Tr(.) for the trace, det(.) for the determinant, $\nabla_{\mathbf{x}} f(\mathbf{x})$ for the gradient of function $f(\mathbf{x})$ at point $\mathbf{x}$, $\nabla^2_{\mathbf{x}} f(\mathbf{x})$ for the second order of function $f(\mathbf{x})$ at point $\mathbf{x}$.

II. System Model

We consider a multi-user environment of M CR transmitter-receiver (Tx-Rx) pairs and N PUs, where each PU uses a different channel (PU k uses channel k, $k = 1, ..., N$). The spectrum to be allocated is comprised of N OFDM channels, and each node is equipped with L antennas, as shown in Fig.1. Each CR can simultaneously communicate over multiple channels, thus, muti-user interference (MUI) is considered in this work. Spectrum sensing in MIMO CRN exploits the available spatial domain observations and has been proposed

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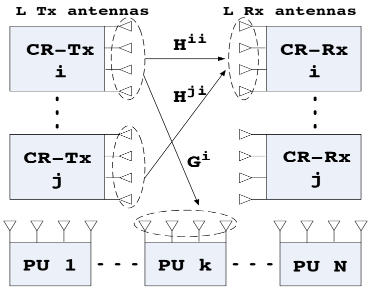

Fig. 1. The system model.

in [15]–[17]. In [15], a blind energy detector based on SNR maximization has been proposed and its performance has been evaluated in difference cases. [16] proposes a Generalized Likelihood Ratio detector, which is a blind and invariant detector with a low computational complexity. Finally, a summary of spectrum sensing is given in [17].

In this paper, we investigate the spectrum sensing problem by using multiple antennas when the PU signal can be well modeled as a complex Gaussian random signal in the presence of an Additive White Gaussian Noise. We assume that simultaneous spectrum sensing of each spacial channel is performed by multiple antennas at each CR-Tx using an energy detection scheme. The detection problem on each channel is modeled as a hypothesis test, where hypothesis $H_{0,k}$ represents the absence of a PU in channel k, and the alternative hypothesis $H_{1,k}$ represents the presence of a PU in channel k. For channel k, at the discrete sample l, the received signal $\mathbf{y}_k^i(l) \in \mathbb{C}^{L\times 1}$ at the CR-Rx i, $i = 1, 2, ..., M$, is given by:

$$H_{0,k}: \quad \mathbf{y}_k^i(l) = \mathbf{n}_k(l) \tag{1}$$

$$H_{1,k}: \quad \mathbf{y}_k^i(l) = \mathbf{S}_k^i(l) + \mathbf{n}_k(l) \tag{2}$$

where $\mathbf{n}_k(l) \in \mathbb{C}^{L\times 1} \sim \mathcal{N}(0, (\sigma_{k,n}^i)^2\mathbf{I})$ denotes additive background noise on the k-th channel. $\mathbf{S}_k^i(l) = \mathbf{G}_k^i \mathbf{s}_k(l)$ stands for the PU transmit signal in channel k, where $\mathbf{s}_k(l) \in \mathbb{C}^{L\times 1} \sim (0, (\gamma_k)^2\mathbf{I})$ is a column vector of L information symbols, and $\mathbf{G}_k^i \in \mathbb{C}^{L\times L}$ is the channel matrix on channel k from PU to CR-Rx i. Then, the covariance matrix is $\mathbf{Y}_k^i = \gamma_k \mathbf{G}_k^i (\mathbf{G}_k^i)^H + (\sigma_{k,n}^i)^2\mathbf{I}$. Let $L_s = tf_s$ denote the number of samples, where t is the sensing time and f_s represents the sampling frequency. Under an energy detection scheme the decision is based on:

$$\sum_{l=1}^{L_s} \mathrm{Tr}(\mathbf{Y}_k^i) \gtrless_{H_{0,k}}^{H_{1,k}} \tau_k^i, \quad k = 1, 2, \ldots, N. \tag{3}$$

where τ_k^i denote the decision thresholds. According to the Central Limit Theorem, for large L_s, $\mathbf{Y}_k$ are approximately normally distributed: $\mathbf{Y}_k^i \sim \mathcal{N}(\mu_{k,0}^i, (\sigma_{k,0}^i)^2)$ for $H_{0,k}$, and $\mathbf{Y}_k^i \sim \mathcal{N}(\mu_{k,1}^i, (\sigma_{k,1}^i)^2)$ for $H_{1,k}$, where:

$$\begin{cases} \mu_{k,0}^i = L_s L (\sigma_{k,n}^i)^2 \\ (\sigma_{k,0}^i)^2 = L_s L^2 (\sigma_{k,n}^i)^4 \end{cases} \tag{4}$$

$$\begin{cases} \mu_{k,1}^i = L_s (L(\sigma_{k,n}^i)^2 + \gamma_k \mathrm{Tr}(\mathbf{G}_k^i (\mathbf{G}_k^i)^H)) \\ (\sigma_{k,1}^i)^2 = L_s (L(\sigma_{k,n}^i)^2 + \gamma_k \mathrm{Tr}(\mathbf{G}_k^i (\mathbf{G}_k^i)^H))^2 \end{cases} \tag{5}$$

The probabilities of detection $\mathcal{P}_{k,d}^i$ and false alarm $\mathcal{P}_{k,fa}^i$ for the kth channel for CR-Tx i, $i = 1, 2, ..., M$, are expressed in closed forms as:

$$\mathcal{P}_{k,fa}^i(\tau_k^i, t) = \mathcal{Q}\left((\tau_k^i - \mu_{k,0}^i)/\sigma_{k,0}^i\right) \tag{6}$$

$$\mathcal{P}_{k,d}^i(\tau_k^i, t) = \mathcal{Q}\left((\tau_k^i - \mu_{k,1}^i)/\sigma_{k,1}^i\right) \tag{7}$$

Formally, for channel k , let $\mathbf{q}_k^i \in \mathbb{C}^{L\times 1}$ be a column vector of L information symbols, sent from CR-Tx i to its destination node CR-Rx i. Each element of $\mathbf{q}_k^i$ belongs to one data stream. Specifically, for channel k, the received signal $\mathbf{z}_k^i \in \mathbb{C}^{L\times 1}$ at the CR-Rx i, $i = 1, 2, ..., M$, is given by:

$$\mathbf{z}_k^i = \mathbf{H}_k^{ii}\mathbf{q}_k^i + \sum_{j=1, j\neq i}^{M} \mathbf{H}_k^{ji}\mathbf{q}_k^j + \mathbf{n}_k \tag{8}$$

where $\mathbf{H}_k^{ii} \in \mathbb{C}^{L\times L}$ is the channel matrix on channel k from CR-Tx i to the intended CR-Rx i, and $\mathbf{H}_k^{ji} \in \mathbb{C}^{L\times L}$ is the cross-channel matrix on channel k from CR-Tx j to CR-Rx i. The elements in $\mathbf{H}_k^{ii}$ and $\mathbf{H}_k^{ji}$ are complex Gaussian variables with zero mean and unit variance. The first term on the right-hand side is the desired signal sent from CR-Tx i, the second term represents the MUI from other CR-Tx that share the channel k. For the sake of simplicity, we consider here only the case where the channel matrices $\mathbf{H}_k^{ii}$ and $\mathbf{H}_k^{ji}$ are square nonsingular. The opportunistic achievable throughput of the CR i, denoted as $U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i)$, can be formulated as the following:

$$U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i) = (1 - t^i/T) R^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i) \tag{9}$$

where $R^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i)$ is given by:

$$\sum_{k=1}^{N} (1 - \mathcal{P}_{k,fa}^i(\tau_k^i, t^i)) \log\det\left(\mathbf{I} + (\mathbf{C}_k^i)^{-1}\mathbf{H}_k^{ii}\mathbf{Q}_k^i(\mathbf{H}_k^{ii})^H\right)$$

$\mathbf{Q}^i = (\mathbf{Q}_k^i)_{k=1}^N$, $\boldsymbol{\tau}^i = (\tau_k^i)_{k=1}^N$, $\mathbf{Q}_k^i$ denotes the covariance matrix of the symbols transmitted by CR-Tx i on channel k. $\mathbf{C}_k^i$ is the noise-plus-interference covariance matrix at CR-Rx i over channel k, given by $\mathbf{C}_k^i = \mathbf{I} + \sum_{j=1, j\neq i}^{M} \mathbf{H}_k^{ji}\mathbf{Q}_k^j(\mathbf{H}_k^{ji})^H$. For CR-Tx i, the total transmit power over all channels should not exceed its maximum allowed power $P_{\max}$. Consequently, the power budget constraint can be formulated as:

$$\sum_{k=1}^{N} \mathrm{Tr}(\mathbf{Q}_k^i) \leq P_{\max} \tag{10}$$

Furthermore, in order to effectively protect the PU from harmful performance degradation in case of missed detection, we consider an interference constraint, denoted as:

$$(1 - \mathcal{P}_{k,d}^i(\tau_k^i, t^i)) \mathrm{Tr}(\mathbf{G}_k^i \mathbf{Q}_k^i (\mathbf{G}_k^i)^H) \leq P_{mask} \tag{11}$$

where P_{mask} is the interference bound, and $\mathbf{G}_k^i \in \mathbb{C}^{L\times L}$ is the channel matrix for CR-Tx i on channel k. Since sensing accuracy is typically required in a real system, in this work, without loss of generality, we restrict the target detection probability to $\mathcal{P}_{k,d}^i \geq \frac{1}{2}$ and false alarm to $\mathcal{P}_{k,fa}^i \leq \frac{1}{2}$, respectively. Hence, these constraints are equivalent to:

$$\tau_{k,min}^i \leq \tau_k^i \leq \tau_{k,max}^i \tag{12}$$

where $\tau_{k,min}^i = \mu_{k,0}^i$, $\tau_{k,max}^i = \mu_{k,1}^i$. We aim at maximizing the total opportunistic throughput of CR i over all channels. The optimization problem can be formulated as:

$$\begin{aligned}
\max_{\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i} \quad & U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i) \\
\text{subject to} \quad & \sum\nolimits_{k=1}^{N} \operatorname{Tr}(\mathbf{Q}_k^i) \leq P_{max} \\
& (1 - \mathcal{P}_{k,d}^i(\tau_k^i, t^i)) \operatorname{Tr}(\mathbf{G}_k^i \mathbf{Q}_k^i (\mathbf{G}_k^i)^H) \leq P_{mask} \\
& \tau_{k,min}^i \leq \tau_k^i \leq \tau_{k,max}^i \\
& 0 \leq t^i \leq T \quad \forall k = 1, 2, \ldots N
\end{aligned} \tag{13}$$

III. Non-convex game and QNE

In our scenario, all the CRs are selfish and strive to maximize their own total throughput under several constraints, which leads to a NCG. The resulting game is non-convex due to the presence of the sensing information, thus, the traditional tools are not applicable to show the existence of a NE. In this section, we use a relaxed equilibrium concept, namely, the quasi-Nash equilibrium (QNE) from [10]. The QNE is by definition a tuple that satisfies the Karush-Kuhn-Tucker (KKT) conditions of all the players' optimization problems; the prefix quasi is intended to signify that a NE must be a QNE under certain constraint qualifications (CQs) [10].

In order to simplify the game, the sensing time t is not considered as a variable, and finally we optimize t by exhaustive search. Assume that there are M players, corresponding to the M CR-Txs, each one controlling the variables $\mathbf{x}^i = (\mathbf{Q}^i, \boldsymbol{\tau}^i)$, $i = 1, ..., M$, and the utility function for each player is the total opportunistic throughput. Since the gradients of the constraints (10), (11), and (12) are linearly independent at $\mathbf{x}^i$, the Linear Independent Constraint Qualification (LICQ) holds at $\mathbf{x}^i$ [18], [19], and we can conclude that the KKT conditions are necessary conditions for the proposed game.

Instead of explicitly accounting all the multipliers as variables of the KKT conditions, we introduce multipliers only for the non-convex constraints (11), denoted as $h_{\mathbb{C}}^i(\mathbf{x}^i) \leq 0$, while the convex constraints are embedded in the defining set $\boldsymbol{\mathcal{X}}_{\mathbb{C}}^i$. Denote by α_k^i the multipliers associated with $h_{\mathbb{C}}^i(\mathbf{x}^i)$. For CR i, the KKT conditions are given by (14).

We reformulate the KKT conditions to an equivalent variational inequality (VI) problem, denoted as $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, where $\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}}$ are given in (14), involving the following variables: $\mathbf{x}^i$ and multipliers $\boldsymbol{\alpha}^i = (\alpha_k^i)_{k=1}^N$ of $h_{\mathbb{C}}^i(\mathbf{x}^i) \leq 0$. The convex constraints (10), (12) are embedded in the defining set $\mathbf{F}_{\mathbb{C}}$. The $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ is to find a vector $\mathbf{x}^\star \in \mathbf{F}_{\mathbb{C}}$ and multipliers $\boldsymbol{\alpha}^\star$, such that $(\mathbf{x} - \mathbf{x}^\star)\boldsymbol{\Theta}_{\mathbb{C}}(\mathbf{x}^\star, \boldsymbol{\alpha}^\star) \geq 0$ (the basic concepts of VI problem are shown in [5], the details are omitted here).

Definition 1: A quasi-Nash equilibria (QNE) is defined and formed by the solution tuple $(\mathbf{x}^\star, \boldsymbol{\alpha}^\star)$ of the equivalent $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$, which is obtained under the first-order optimality conditions of each player's problems, while retaining the convex constraints in the defined set $\mathbf{F}_{\mathbb{C}}$.

The sufficient conditions for the existence and the uniqueness of the solution for $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ are given in [9], [10], [20], which are related to the positivity properties of the Hessian matrix of $U^i(\mathbf{x}^i)$ and $h_{\mathbb{C}}^i(\mathbf{x}^i)$. For the proposed game, the $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ has a unique optimal solution, if the following sufficient condition is satisfied:

$$\frac{\max\limits_{k=1,\ldots,N} \max\limits_{i=1,\ldots,M} \operatorname{Tr}(\mathbf{G}_k^i (\mathbf{G}_k^i)^H)}{\sqrt{2\pi} \min(\min\limits_{i=1,\ldots,M} \nabla^2_{\mathbf{Q}^i} U^i, (\nabla^2_{\boldsymbol{\tau}^i} U^i - \nabla^2_{\boldsymbol{\tau}^i} h_{\mathbb{C}}^i))} \leq 1 \tag{15}$$

Based on condition (15), from the definition of QNE, we can state that our game admits a unique QNE. The proof is omitted here due to lack of space, while the proof for a SISO system is given in [20].

Equi-sensing time for all the CR user: The proposed decision model so far is based on the assumption that only the PUs' signals are involved in the detection process, the interference from other CR-Txs in the same channel is ignored. Since the energy detector is not able to discriminate between different received energy contributions, the interference generated by the transmitting CR users in the same frequency channel would affect the result. To overcome this issue, we can force the same sensing time for all the CR users. The original games is modified to the following problem:

$$\begin{aligned}
\max_{\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i} \quad & U^i(\mathbf{Q}^i, \boldsymbol{\tau}^i, t^i) - c(t^i - \frac{1}{M} \sum\nolimits_{j=1}^{M} t^j)^2 \\
\text{s. t.} \quad & h_{\mathbb{C}}^i(\mathbf{x}^i) < 0, \mathbf{x}^i \in \boldsymbol{\mathcal{X}}_{\mathbb{C}}^i
\end{aligned} \tag{16}$$

In the new objective function of each player, there is an additional term that works like a penalization in using different sensing times for the players. Because of this penalization, we would expect that, for sufficiently large c, the equilibrium of the game tends to have equal sensing times.

Primal-dual interior point optimization method: We reformulate the $VI(\mathbf{F}_{\mathbb{C}}, \boldsymbol{\Theta}_{\mathbb{C}})$ to the equivalent constrained equations (CEs) and solve them by the primal-dual interior point (IP) method. The IP method combines a line search step that computes iterative steps by factoring the primal-dual equations, and a trust region step that uses a conjugate gradient iteration. The IP method can run at each node in parallel, since it requires only the local information of each CR user (e.g. its own transmit power and the channel gain), and hence, it can be regarded as a distributed solution. We first compute the steps using line search whenever the conditions of these steps can be guaranteed, and turn to the trust region step otherwise. The resulting method is ensured to have global convergence, thus achieving a QNE of the game. We outline the IP method in **Algorithm 1**, for more details, we refer to [20]–[22].

IV. Simulation results

We consider a CRN with $M = 6$ CR Tx-Rx pairs and $N = 3$ PU channels. The antenna array size is $L = 4$. PUs and CRs are randomly placed in a 50 meter $\times$ 50 meter square, and the radio environment map is shown in Fig. 2, where the colorbar shows the received power from PUs in Watt. We use the channel model from the 3GPP Indoor scenario for LTE [23]. Denote by $d = d_{ji}/d_{ii}$ (m) the relative distance between CR-Tx j and CR-Rx i, where d_{ii} and d_{ji} are the distance between CR-Tx i and CR-Rx i, CR-Tx j and CR-Rx i, respectively. We

$$\begin{pmatrix} \mathbf{Q}-\mathbf{Q}^{\star} \\ \boldsymbol{\tau}-\boldsymbol{\tau}^{\star} \\ \alpha_k-\alpha_k^{\star} \end{pmatrix}^T \underbrace{\begin{pmatrix} -\nabla_{\mathbf{Q}^i} U^i(\mathbf{Q}^i,\boldsymbol{\tau}^i)+\alpha_k^i(1-\mathcal{P}_{k,d}^i(\tau_k^i))\,\mathrm{Tr}(\mathbf{G}_k^i(\mathbf{G}_k^i)^H) \\ -\nabla_{\boldsymbol{\tau}^i} U^i(\mathbf{Q}^i,\boldsymbol{\tau}^i)-\alpha_k^i \nabla_{\tau_k^i}\mathcal{P}_{k,d}^i(\tau_k^i)\,\mathrm{Tr}(\mathbf{G}_k^i\mathbf{Q}_k^i(\mathbf{G}_k^i)^H) \\ P_{mask,k}-(1-\mathcal{P}_{k,d}^i(\tau_k^i))\,\mathrm{Tr}(\mathbf{G}_k^i\mathbf{Q}_k^i(\mathbf{G}_k^i)^H) \end{pmatrix}_{i=1}^{M}}_{\boldsymbol{\Theta}_{\mathbb{C}}(\mathbf{x}^\star,\boldsymbol{\alpha}^\star)} \geq 0, \quad \forall (\mathbf{x}^i,\alpha_k^i) \in \underbrace{\prod_{i=1}^{M} \boldsymbol{\mathcal{X}}_{\mathbb{C}}^i \times \mathbb{R}_+^r}_{\mathbf{F}_{\mathbb{C}}} \qquad (14)$$

Algorithm 1 Primal-Dual Interior Point Optimization

Initialize $\mathbf{x}^i, \boldsymbol{\alpha}^i, \beta^i, \boldsymbol{\mu}_0^i, \boldsymbol{\mu}_1^i$. Trust-region radius $R_t^i > 0$. Set the maximum number of search steps N_b, $LS = 0$.
For $t^i = 1 : T$
  Repeat
    For $i = 1 : M$
     Compute the search direction $\mathbf{d}$ by Newton's method.
      Repeat 1. Determine the step length s.
           2. Line search and update.
       If $U^i(j+1) \leq U^i(j)$, set $LS = 1$.
       Else $j = j + 1$. **End If**
      Until $j \geq N_b$ or $LS == 1$.
      If $LS == 0$
      1. Compute the trust-region radius R_t^i
      2. Trust region method and update R
      End If
    End For
  Until $\mathbf{x}^i$ and U^i satisfy the stopping tests.
End For

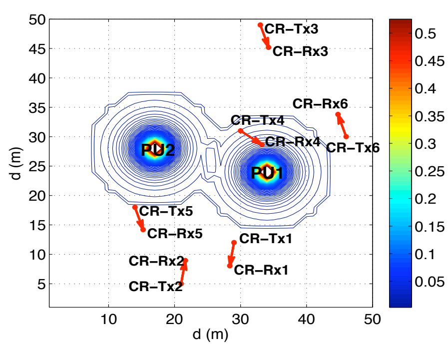

Fig. 2. Radio environment map for 2 PUs and 6 CR pairs

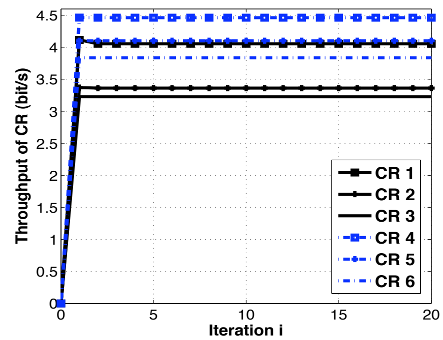

Fig. 3. Throughput vs. iteration value for different CRs

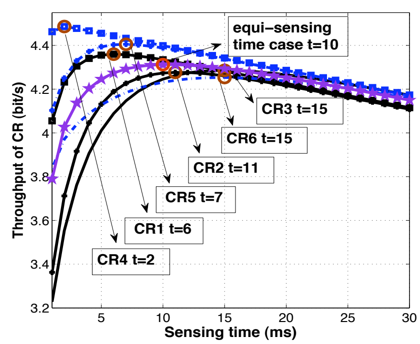

Fig. 4. Throughput vs. optimal sensing time for different CRs

consider the shadowing as a lognormal variable with variance $10dB$, $T = 100ms$, $f_s = 2MHz$, $(\sigma_{k,n}^i)^2 = 1$, according to [24]. The received minimum SNR from PU at the CR-Rx is equal to $\mathrm{SNR}_{pu} = \gamma_k^2/(\sigma_{k,n}^i)^2 d_{max} = -20dB$, where P_{pu} is the transmission power of PU, d_{max} is the longest relative distance between CR-Rx and PU. The maximum power at each CR-Tx is equal to $P_{max}/(\sigma_{k,n}^i)^2 d_{ii} = 5dB$, and $P_{mask} = 10^{-4}$ on all channels. We assume that the sensing environment is stable in the optimization process, and the channel state information is known by both the CRs and the PUs. The simulation results are base on game (13).

Fig. 3 shows that all the CRs are able to achieve the QNE within a few iterations. Specifically, the nearby CRs, which are closer to the PU, are able to achieve higher throughputs compared with the distant CRs, which are far from the PU. Because all the CRs are bounded by the power budget constraint, the CRs can exhaust all the power budget without causing harmful interference to the PU. Hence, the distant CRs have to increase the sensing time and decrease the detection threshold to satisfy the target $\mathcal{P}_{k,d}$ and $\mathcal{P}_{k,fa}$, yielding a decrease in the data transmission time and the throughput.

Fig. 4 shows the optimal sensing time versus the achievable throughput for each CR. We highlighted the optimal t^i in the figure. According to the result, there exists an optimal t^i for each CR at which its throughput is maximized. Moreover, as expected, the optimal t^i for the nearby CRs are smaller than the distant ones due to the target sensing accuracy. We also show the result for the equi-sensing time case, where the optimal equi-sensing time for all the CRs is which the sum-throughput (average) is maximized.

In Fig. 5, we plot the throughput achieved at QNE by one CR versus the optimal sensing time for different values of the interference constraint bound. It can be observed that the optimal

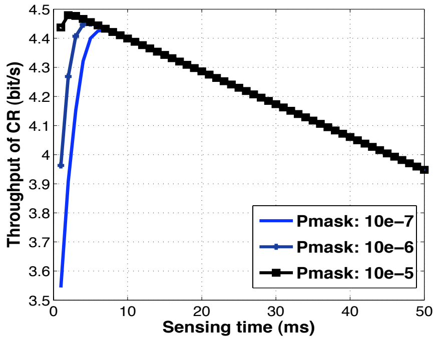

Fig. 5. Throughput vs. sensing time for different values of P_{mask}

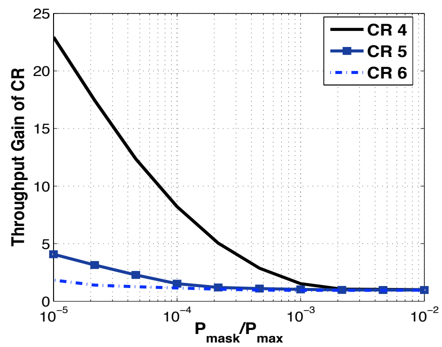

Fig. 6. Throughput gain U/U_d vs. normalized interference P_{mask}/P_{max}

sensing time increases as the interference constraints become more stringent. More specifically, more stringent interference constraints impose lower missed detection probabilities as well as false alarm rates, which require more accurate detection information by increasing the sensing time, leading to the degradation in the throughput.

Fig. 6, compares the performance achieved by the proposed sensing based game with those achieved by the deterministic game in [14]. For the deterministic game, all the frame length is used for the transmission, the sensing information is not considered as a part of optimization. To quantify the throughput gain achievable at the QNE of the proposed game, we plot the ratio U/U_d versus the normalized interference constraint bound P_{mask}/P_{max}, where U_d is the throughput achievable at the NE of the deterministic game. It is clear in the figure that the proposed joint optimization of the sensing information and transmission power yields a considerable performance improvement with respect to the disjoint case, especially when the normalized interference constraint bound is stringent. Moreover, the performance improvement becomes more significant with the nearby CRs. This is because a more stringent normalized interference constraint bound impose lower transmission power for the nearby CRs in the deterministic game, while higher transmission power is allowed due to the accurate sensing information in the proposed game.

V. Conclusions

In this paper, we proposed a sensing-based non-convex NCG for a multi-user MIMO CRN. To deal with the non-convexity of the game, we used a new relaxed equilibria concept, namely, QNE. In particular, we the sufficient condition of the existence and the uniqueness of a QNE for the proposed game.

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