Article

Circular Economy and Value Creation: Sustainable Finance with a Real Options Approach

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Abstract: This paper presents a methodological proposal that integrates the circular economy concept and financial valuation through real options analysis. The Value Hill model of a circular economy provides a representation of the course followed by the value of an asset. Specifically, after the primary use, the life of an asset may be extended by going through four phases: the 4R phases (Reuse, Refurbish, Remanufacture and Recycle). Financial valuation allows us to quantify value creation from firms' asset circularity under uncertainty, modelled by binomial trees. Furthermore, the 4R phases are valued as real options by applying no-arbitrage opportunity arguments. The major contribution of this paper is to provide a quantitative approach to the value of circularity in a general context that is adaptable to firms' specific situations. This approach is also useful for translating relevant information for stakeholders and policy makers into something with economic and financial value.

Keywords: circular economy; sustainable development goals; investment strategies; asset valuation models; real options; binomial trees

1. Introduction

In recent decades, sustainability has taken on a very important role as the main channel that facilitates the creation of wealth and sustainable value over time without depleting resources or causing damage to the environment. It is well known that finance provides methods that are scientifically recognized and, therefore, suitable as a resource to assess sustainability in the context of circularity. However, for reasons that are unclear, few investigations in this field have put the different assessment tools available into practice.

On the one hand, the Sustainable Development Goals (SDGs) [1], also known as the 2030 Agenda and the Paris Agreement [2], play a prominent role, being a means to establish a framework of international commitment to develop policies and focus the long-term economic activity of different social and economic agents. The International Monetary Fund (IMF) argues that there is an urgent need to develop measures since, according to its estimates, the financial flows that contribute to the SDGs worldwide only add up to three trillion dollars per year, and the needs are estimated at between two and four trillion of dollars annually until the year 2030.

On the other hand, sustainable finance is currently one of the most important tools to encourage the financial system to make positive changes. Therefore, sustainable finance today is more necessary than ever, involving both the public and private sectors. To this end, reference is made to issues related to the preservation of the natural environment, social and governance aspects (Environmental, Social and Governance principles, or ESG) when making investment-related decisions. The underlying idea in ESG is that information on non-financial issues can have a transformative impact on the practices of investors and companies. On the one hand, investors can use ESGs to evaluate the corporate

behavior and future financial results of companies, thereby identifying investments that have a lower level of long-term risk. On the other hand, companies would have an incentive to improve their results and be more attractive from a financial point of view. In this context, the Principles of Responsible Investment (PRI) [3] have been developed at the initiative of the Global Compact for United Nations, which advises investors around the world on the incorporation of ESG criteria in their investment analysis and decision-making processes.

In this regard, there are already many companies that have integrated sustainability into their corporate strategy, with a notable presence in finance that provides investors with "non-financial" information through the sustainability reports prepared. Likewise, this information is an important means to attract the interest of new investors with a responsible attitude, as well as obtain a higher economic return.

To achieve the SDGs, sustainable finance will need to confront great challenges over the next decade, such as engaging with business leaders (and in particular financial managers) with regard to the crucial role they need to play; developing unified taxonomies and classifications to define which activities can be considered sustainable; incentivizing companies to make sure that the non-financial information they prepare is made with the greatest possible transparency; integrating SMEs within the scope of sustainability; allocating funds to finance the 2030 Agenda as a way to kickstart the transformation of the financial system towards sustainability, etc.

Companies have been driven towards SDGs in order to respond to the global challenges and the future sustainability of the social, economic and environmental systems. The circular economy (CE) requires a rethinking of the vision and corporate strategy of companies in order to ensure their survival and the future sustainability of the environment, which will involve a significant change in aspects related to strategy and management. In this regard, on 2 December 2015, the European Commission prepared a report entitled "Close the Circle: An Action Plan of the European Union for the Circular Economy" [4], which seeks a transition from a linear economy to a CE, as in the latter the products and resources remain in the system for a longer period and the generation of waste is reduced. Accordingly, the European Commission emphasizes that there is an association between the CE and the SDGs, as the CE helps to achieve some SDGs, in particular objective 12, which is based on the guarantee of the modalities of sustainable consumption and production. The importance of the CE has led official organisms to establish various standards in order to transform the economic and business model towards a CE.

The main difficulty found in this new framework is how to quantify the value that the CE generates in the business environment. As has been said, despite the wide variety of financial methods that are available, there are few studies that put them into practice to assess the results the CE provides. One of the most significant and widely used methods is the analysis of real options as a valuation tool. Although it has long been used as a research tool, Van Putten and MacMillan [5] claim that it still does not enjoy the same acceptance as the classic Net Present Value (NPV) decision criterion. The use of the real options methodology is appropriate when there is discretion or flexibility to act in the face of uncertainty. Given that the COVID-19 pandemic has increased uncertainty and risk in financial markets, it is critically important to implement financial tools that allow the effects produced by this uncertainty to be considered in the valuation process and, therefore, the real options method becomes especially useful in assessing sustainability in the CE environment. Accordingly, the CE will involve new business models that will bring different financial challenges in terms of, for example, cost structures or cash flows. The field of finance needs to recognize and adapt itself to this new reality and support the transition to a new economic model [6].

In this regard, the main objective of this work is to show how the new CE paradigm can be interpreted from the perspective of real options and thereby promote the development of assessment and decision-making tools adapted to that context. Clearly, the field of finance has a fundamental role to play in the transition towards a more sustainable

economy. If it is to achieve widespread acceptance that reaches beyond environmental circles, the quantification of value creation is required, especially in a business context.

The Real Options approach is more than a valuation tool that was originally developed for valuing financial derivative assets. It encompasses the strategic dimension of a business, project, or real asset. Beyond the estimation of future cash flows and the risk-adjusted discounting rate, this methodology includes the decision-making process and the inherent flexibility of discretional decisions to face challenges from uncertainty. Thus, its essence resides in strategic thinking devoted to finding, designing and valuing business opportunities to make (re)use of limited resources in a context of uncertainty. For all these reasons, the Real Options approach is a superior method to the classical NPV tool, producing a higher valuation explained by the flexibility component. After the primary use of a product, material, or resource, its life may be extended by additional investments that can earn profit from circularity. Thus, the 4Rs of the CE (Reuse, Refurbish, Remanufacture and Recycle) represented in the Value Hill model may be considered as real options whose value depends on the uncertainty about the underlying asset (product, material or resource). One of the most understandable structures of uncertainty is the binomial probability law. With two events or states (up and down) and enough flexibility, the binomial tree is also preferred by the practitioners of the Real Options approach. Additionally, the use of relative valuation principles (no arbitrage) has two advantages: 1) risk-neutral probabilities substitute natural probabilities, and 2) the risk-free interest rate substitutes the opportunity cost of capital.

The main contribution of this paper is to provide a methodological proposal that allows circularity to be assessed through the application of real options as an ideal tool, since it permits the effect of uncertainty to be incorporated into the valuation. The incremental value of this study is aimed at integrating the CE and financial valuation principles and quantifying the value creation of asset circularity as the value of real options embedded in a circular system. This paper makes a methodological proposal to determine the value creation of circularity in a general context that may be adapted to the needs of case studies. The financial literature has not examined the valuation of circularity through the real options approach, and thus it is an unpublished study providing a robust tool that can be applied with wide flexibility.

This work is structured as follows. Section 2 is devoted to the theoretical background related to the circular economy and sustainable finance. Section 3 presents the valuation methodology based on Real Options Analysis, with application to the Value Hill model of the circular economy. Section 4 concludes the paper.

2. Theoretical Background

2.1. Circular Economy

The traditional economic model, also known as a linear model, is based on the premise of an infinite increase in production and prosperity. Meadows et al. [7] argued that the linear model is not possible in a world with finite resources. Sauvé et al. [8] stated that the linear model is based on the idea of extract, produce, consume and trash (Figure 1).

Figure 1. Linear economy. Source: prepared by the authors.

In this regard, Howard [9] analyzed the historical nature of the linear model before the mid-18th century, at least in the Western world, and explained that for more than a millennium, the Western world thought that the past had been better than the present.

This idea was replaced in 1750 with the modern concept of linear progress, in which the future is by definition better than the past. The linear progress model was founded by the first industrial revolution, global expansion and the success of capitalism. This perspective argued that to obtain good economic results, it was only necessary to properly assign certain resources. For a long time, humanity occupied only a small portion of the planet and, consequently, the idea of unlimited production seemed possible; but in the decades following the Second World War, human activities represented more than 50% of the capacity of the Earth (see the webpage of the Global Footprint Foundation). As a consequence, the increase in the resources extracted from the environment drastically affected the global ecosystem. After the mid-1970s, several attempts were made to improve the relationship between human activities and the environment. In 1987, the Brundtland Commission, in its report ("Our Common Future") for the UN World Commission on Environment and Development, developed the concept of "sustainable development" based on development that meets the needs of current generations without compromising the ability of future generations to meet their own needs. According to the Global Footprint Network, in 2010 "Planet Earth needs one year and a half to produce and absorb what is consumed as raw materials and eliminated as waste in one year." A growing awareness of the environmental limits of the linear economy led to the development of a new economic model based on the goods and services necessary for improving the living standards of people. In line with this, the European Commission presented the "Manifesto for a Resource-Efficient Europe", which postulated the transition towards a resource-efficient and ultimately circular economy. In 2015, the European Commission adopted an Action Plan to transition towards a circular economy. Under this plan, the Commission looked to the European standardization organizations—CEN-CENELEC and ETSI—to devise standards to assist in this transformation process.

The concept of the circular economy (CE) has been developed by scholars, business associations and foundations, policymakers, and business consultants, among others [10,11]. The term CE has evolved over the time and has different meanings depending on various perspectives. For this reason, Kirchherr et al. [12] collected 114 definitions of CE, which are coded in 17 dimensions. In general, these authors define this term as a combination of reduce, reuse and recycle activities. Accordingly, in line with Prieto et al. [13], the CE may be defined as a cycle of the extraction and transformation of resources and the distribution, use and recovery of goods and materials, as shown in Figure 2.

Companies take resources from the environment in order to transform them into products and services. After that, companies distribute the products or services to consumers or other firms, and these are consumed in the market.

At these points, the EC proposed closing the cycle through the recovery of goods, through industrial processing or environment, instead of wasting them ([14]). In line with [15], the CE is characterized by three levels of research and implementation: micro, meso and macro. In the first level, firms are focused on the eco-innovation development and improvement process. In the second level, firms are centered on benefiting the regional economy and the natural environment ([16]). Finally, in the last level, the focus is on the development of environmental policies and institutional influences, in particular the development of eco-cities, eco-municipalities or eco-provinces ([14]). According to Prieto et al. [13], there are a set of principles that lay the foundation for the transition to the CE. In this regard, there are three principles—the 3Rs (reduce, reuse and recycle)—as cited by authors such as Ghisellini et al. [17] and Hass et al. [18], while the environmental design strategies work as catalyzers and guidelines for designing goods and services that can be reintroduced into the system over the long term as technical or biological resources [13]. National and international governments are driving the waste and resources management industry towards a more CE.
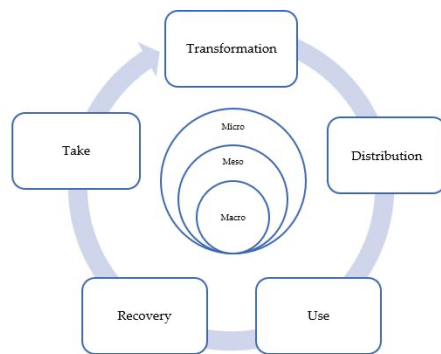
Figure 2. Circular economy circle. Source: prepared by the authors**.**

The private sector makes CE progress via public-private initiatives. In the area of R&D, the European Union assigns funding to circular economy programs through Horizon 2020. In particular, the European Fund for Strategic Investments (EFSI) makes factors of the circular economy available for deployment in private companies ([19]).

For the circular economy to function well, trust in the quality of recycled materials is essential in existing and emerging markets, but the processing costs compared with the quality should be taken into account ([20]). In this context, Rizos et al. [21] revealed that countries such as Estonia, Belgium, Germany and Greece include several policy instruments in order to introduce CE principles into their business models, although there are a variety of barriers. In line with this, Zamfir et al. [22] documented that Bulgaria, Hungary, Poland, Romania and Slovakia do not include practices that encourage the circular economy in SMEs, in contrast with other European countries. In [23], it is argued that SMEs should make minimum investments in circular eco-innovations to obtain benefits from investing in the CE. The level of investment in the CE improves the economic performance of firms for some categories of SMEs ([22]). Flynn et al. [24] found that the transition to a CE in the UK led to the creation of new markets and economic benefits, while in China the notions of the CE were based on the reduction and recycling of materials. In both countries, governmental regulations were impediments hindering the CE.

2.1.1. Value Hill Model and Circular Business Strategies

In the linear economy, business models are mainly focused on selling new products; as a consequence, revenues come mainly from maximizing the number of sales and minimizing costs. These models encourage the design of short-lived products in order to sell new units again. In this dynamic, the manufacturer usually loses control over the product once it is sold, limiting its responsibility but also missing out on future business opportunities. To make a transition to the circular economy, companies have to question the paradigms of the linear economy and adapt their business models and strategies. To help establish business strategies compatible with the principles of the circular economy, a tool has been developed: the Value Hill model ([25]). As shown in Figure 3, the pyramid is divided into three sections: 1) Pre-use is multi-phase since it includes the phases necessary to obtain a product (extraction, manufacture, distribution); each step towards the peak of the pyramid adds value, and once at the top of the pyramid, the product is able to be used by the end user. 2) Use is the phase during which the user / buyer enjoys the product. 3) In the Post-use phase, the user / buyer has no further use for the product, and it loses value and begins its descent towards the base of the pyramid. Comparing Figure 3a,

which represents the linear model, and Figure 3b, which represents the circular model, it is worth noting the change in the post-use phase between the two models.
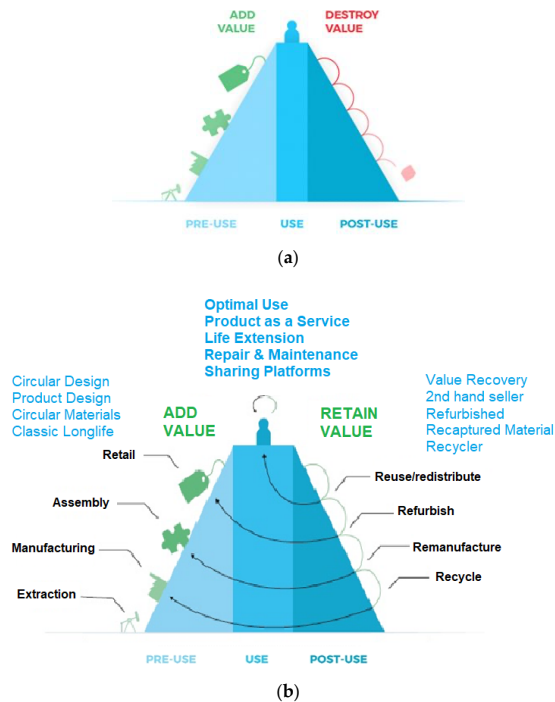

(**a**)

(**b**)

Figure 3. Value Hills by Achterberg, Hinfelaar and Bocken [25], with value hill in linear economy (**a**) and value hill in circular economy (**b**).

Whereas in the linear model, the value of the product falls rapidly to zero, the circular model implements several strategies to maintain the value of the product by reintroducing it in the previous phases on the opposite face of the pyramid. In this way, the amount of product decreases as it progresses towards the base of the pyramid (loss of value).

Within each phase, it is advisable to implement different strategies to achieve greater circularity of the business. The strategies identified [26] are the following:

• **Circular Design strategies** are suitable for activities developed in the pre-use phase, where the company can design the product in order to allow or facilitate the use and circular post-use of the product; for example, designing a product that is easy to maintain and repair.

• **Optimal Use strategies** are appropriate for the use phase of the product. This category of strategies aims to optimize the use of the product through complementary services or products that allow the value of the initial product to be maintained.

• **Value Recovery strategies** are appropriate for the post-use phase of the product. These strategies seek to recover the value of products that have lost their usefulness.

• **Network Organization strategies** do not correspond to any particular phase as they aim to facilitate the functioning and coordination of the actors and the flows of resources in the value system and between the phases.

Comentado [MDPI-071]: Please confirm whether copyright permission is needed in order to use Figure 3.

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The circular economy approach, based on maintaining the value of products through several cycles of use, requires new valuation methodologies. It is necessary to be able to assess not only the different cycles of cash flows but also the residual value of products, and this may require new accounting depreciation rules as well. There is a need to adapt financial decision-making tools to this new reality.

2.2. Sustainable Finance

2.2.1. Relevance of Sustainable Finance

In recent decades, the creation of an environment conducive to sustainable economic development has become a topic of significant interest. Many researchers have dedicated their efforts to quantifying and measuring the effect of social, environmental, institutional and financial policies as a means to guarantee sustainable economic development ([27]).

The economic model based on the maximization of private profit may be too limited to incorporate the analysis of the maximization of social returns. In line with this, the terminology "sustainability of economic development" aims to overcome the narrowness of dominant economic models and incorporate the broader aspects of economic well-being related to environmental and social values. All this could lead to investment putting the economy on a different growth path, managing to improve the standard of living, and mitigating the problems of external effects that can hinder a higher quality of social welfare.

The crisis of the welfare state and the failure of regulations that was revealed after the financial crisis of 2008 have become significant factors determining the need to seek new ways and solutions to stabilize economies and create conditions for the development of sustainable economic growth. In fact, after this financial and sovereign debt crisis, sustainable finance has offered a great opportunity for the European Union (EU) to redirect its financial system, moving from short-term stabilization to long-term growth ([28]).

Although it is true that one can easily find reliable and unified standards for measuring and evaluating financial risks and returns, this is not the case in the context of sustainability. This highlights the need to deepen the study of returns that combine the conservation of natural capital and socioeconomic well-being. Consequently, it is necessary to develop a more general theory of finance that addresses its socio-ecological nature ([29]).

In line with the above arguments, Weber [30] (p. 121) argued that "sustainable finance is finance that meets the social, environmental, and livelihood needs of the present generation without compromising the ability of future generations to meet their own needs and that creates a fair balance between societies." As a result, sustainable finance could play a significant role in enabling the establishment and development of financial standards based on sound investment principles leading to increased implementation of Environmental, Social and Governance principles (ESG). Furthermore, the financial instruments and market structures that allocate capital should be provided in such a way as to maximize the overall social return adjusted for financial and non-financial risks. Therefore, these instruments and structures should consider issues such as the environmental and social responsibility of the entities that seek to obtain financing ([31]).

In this context, socially responsible financial institutions would direct their investment and financing policy towards entities that have a sustainable profile that addresses issues such as environmental protection and social responsibility, as well as the risks that potentially have to be assumed. Similarly, socially responsible financial markets should create market structures that allow companies that adopt ESG principles to obtain financing at a lower cost due to their higher levels of transparency and commitment to sustainability issues.

2.2.2. Framework for Developing Non-financial Information in the European Union

The European Parliament adopted Directive 95/2014 [32] in order for large EU companies to draw up a report indicating the social impact of their economic activities through

the application of ESG principles, known as "non-financial performance." These regulations establish that, through their financial statements, companies must quantify the impact that their activities generate on the environment and society at the present time and in the future. They should also provide information on how the governance structure internalizes these non-financial risks and reduces the possibility of incurring losses. The EU thus creates mandatory frameworks to promote the disclosure of corporate risks related to climate ([33]). The EU legislative plans have sparked a global debate on ESG regulation and have exposed many shortcomings in the current ESG and sustainable finance landscapes.

One of the purposes of the ESG principles is that information on non-financial aspects can have a beneficial impact on the practices of both investors and companies. Thus, investors would use ESGs to assess corporate behavior and the future financial results of companies, thereby identifying potential investments that could offer a lower level of risk. On the other hand, companies would be encouraged to improve their results and increase their desirability from a financial point of view ([34]). In this way, a circular relationship is established between savers, entrepreneurs and investors, while social and environmental externalities are internalized in the process ([35]). The progressive integration of these clearly defined principles offers economic benefits such as the stimulation of innovation and economic growth, and for this, the establishment of strong climate and environmental frameworks is important ([36,37]).

As indicated above, it is very difficult to translate into accounting terms the material impact that these non-financial issues imply ([38,39]), as well as the implications that the new framework has for investors and companies, mainly with respect to the preparation and presentation of reports that give value to these parameters ([40,41]). To this end, the EU High-Level Expert Group on sustainable finance established that "the need to disclose long-term sustainability activities and metrics is a very powerful tool for fostering internal debates, ensuring proper governance and helping to promote dialogue between management, the board and stakeholders," ([28] p. 24).

Subsequently, in March 2018, the European Commission announced an ambitious action plan for sustainable financing, also based on the conclusions of the High-Level Expert Group on Sustainable Finance, which announced that redirecting investments towards sustainable projects would require "changing the investment culture and behavior of all market participants" [28] (p. 2). This change requires the involvement of higher education institutions, as well as academic research and the discipline of finance. In particular, these players will have an important role that will consist of aligning financial institutions and market agents with respect to the long-term decision-making necessary to finance sustainable economies and societies [35].

However, research and training in the field of finance has scarcely addressed the way in which sustainable finance modifies the current conception of financial theory that has its roots in empirical realism and is based mainly on a deductive approach that is developed through econometric techniques. This reveals the inability of conventional financial models to explain the reality of sustainable financing and, in this way, to be able to align financial research with true social needs. At present, the need to implement sustainable financial practices as an engine for social change is held back by the academic stream of finance itself ([42–46]).

To achieve these aims, Lawson [47] and Lagoarde-Segot [35], among others, suggest that the adoption of a critical realistic approach in finance can lead to new financial practices fostering its paradigmatic diversification, while, in turn, the appearance and dissemination of new financial research would help to reshape financial ideology and practices.

2.2.3. Sustainable Value Creation: A New Paradigm

Traditionally, the concept of value creation has been widely covered in finance. At present, this concept is evolving towards what is known as "Socially Responsible Invest-

ment", meaning that the creation of value must be "sustainable" over time. However, Weber [30] indicates that there is still no clear general strategy that encourages the financial sector to contribute to sustainable development. In turn, for the objectives established through the Principles of Responsible Investment (PRI) [3] to be successfully achieved, it will be necessary to promote and not impose the development of environmental and social principles ([27]).

Most companies today publish sustainability reports disclosing ESG factors. However, the quality of the information they publish varies substantially, and this means that investors cannot easily discriminate between sustainable initiatives that create value and those known as "greenwashing." There are clear practical difficulties in allocating funds to high-performing companies with ESG criteria for the benefit of consumers of financial products. If these issues are not addressed, it will be difficult for sustainable finance to prosper ([48]).

In recent decades, from the academic field and in the context of finance, the only goal sought was the maximization of shareholder wealth, with the share price being the sole parameter under analysis.

This conception of value maximization has led to unacceptable results on many occasions and caused the new challenges that society is facing, since the old models no longer represent reliable guides for the value creation process. To avoid this, the decision-maker must consider all the economic, social and environmental costs and benefits before deciding whether or not to undertake a business project. Therefore, the traditional focus on the maximization of shareholder wealth must be redirected towards the goal of creating sustainable value. This will involve adopting a model in which all relevant costs and benefits are properly accounted for, rather than outsourced ([49]).

Consequently, firms that follow the traditional maximization model instead of the social responsibility model will notice a negative change in demand as the detrimental effects of not incorporating social and environmental issues are made public. Only an explicit recognition of the social and environmental impact of the company's decisions will guarantee the sustainability of the value that has been generated.

There is a great amount of research that supports the proposition that companies that follow principles of sustainability and social responsibility create more value for shareholders. El Ghoul et al. [50] find that when the company's commitment is in the field of climate change or corporate governance, its market reaction is stronger. Plumlee et al. [51] show that US companies with superior performance in Corporate Social Responsibility (CSR) enjoy cheaper equity financing. The work [52] studies the cost of borrowing and finds that companies at the lower end of the CSR spectrum bear a higher financial cost. In [53], it is suggested that symbolic ESG actions in the presence of higher intangibles have a greater positive impact on the market value of the company. Finally, Eccles et al. [54] provide evidence that highly sustainable companies significantly outperform their counterparts over the long term.

Given the significance of the new challenges, it will be companies that first adopt models that consider social and environmental costs in their economic activity that will experience favorable changes in the demand for their products or services. Likewise, companies that fail to do so will see a negative change.

However, according to the study by Fatemi and Fooladi [49], it is completely consistent to implement this new conception of sustainable value with the NPV (Net Present Value) approach. What is more, this new approach also considers all incremental and opportunity costs. Therefore, in addition to incorporating traditional cash flows, this sustainable value creation approach requires explicit recognition of incremental cash flows attributable to the company's sustainability efforts. Specific examples of these new factors to consider could include increased brand value, greater customer loyalty, an improved ability to recruit and retain talent, the ability to attract new customers who demand social and environmental results, the value of being able to enter markets restricted to companies with a positive reputation for their sustainability efforts, and so on. In the same way,

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the following must also be considered: cost reductions due to less use of water and energy, the lower cost of waste disposal and healthcare for employees, the lower cost of capital due to the fact that socially responsible companies manage their risks better, among others.

3. Methodology

As shown above in Figure 3, the Value Hill is a representation of the path that the value of a product follows. At the highest point of the pyramid, the product has its maximum value and is in its primary use. In the post-use phase, to maintain the value of the product for as long as possible, it is necessary to go through different recovery cycles before reaching the point of waste, the lowest part of the pyramid. Each cycle corresponds to a value recovery strategy, which depends on the state of the product. Recall the four strategies:

- **Reuse**: If the product continues to work but has no use for the current user, it is necessary to look for a new user or an alternative use for the product. The cycles of reuse involve minimum investment since the product is still in good condition (e.g., selling second-hand washing machines).
- **Refurbish**: This allows the product to reenter the market after minimal adjustments and aesthetic improvements (e.g., painting and changing the brakes on a used bicycle).
- **Remanufacture**: This involves rebuilding the product so that it has the same characteristics as a new product. Remanufacturing cycles involve a much greater investment (e.g., changing critical components of an engine so that it has the same benefits as a new one).
- **Recycle**: When the product can no longer be used, the components and raw materials can be recovered for the production of new products. Recycling cycles do not involve any investment because they act as a residual value (e.g., recovering metal and plastic from a mobile phone).

Indeed, each cycle through which a durable product can pass throughout its life, in the sphere of the circular economy, can generate value in exchange for an investment. This circular logic can be interpreted and modelled through the real options methodology. Thus, from this perspective, one can see how the production of a good and its placement in the market also provides the option to take advantage of the value recovered in the following cycles. A manufacturer could then contemplate the possibility that the same product will generate value in each of the cycles (for example: selling new washing machines, second-hand washing machines and remanufactured machines, and then finally recycling the metal from the carcasses).

3.1. Circularity Value from the Classical NPV Method

In order to quantify the value creation of asset circularity, most authors (see [55] for a recent application) apply the well-known classical method of Net Present Value (NPV). It provides an accurate valuation when the project to be valued does not involve any flexibility or discretionary decision making, and thus the goal is to decide whether or not to take on an investment project based on its profitability. In the presence of flexibility, however, it becomes a myopic method since it allocates no value to such a strategic component. Nevertheless, this does not mean that the NPV method is not useful in a circular economy context but rather that it underestimates the circularity value because all investment opportunity is understood as a project instead of a real option.

By adopting the lens of the NPV method to estimate the value of asset circularity from the so-called 4Rs phases, each phase is seen as a piece of a chain of investment projects. Each piece or project contributes with its own value to the total value, so that the NPV corresponding to each phase—Use (0), Reuse (1), Refurbish (2), Remanufacture (3), and Recycle (4)—should be computed ($NPV_j, j = 0,1,2,3,4$). On other hand, the real asset

experiences a loss of value through the four phases, following a decreasing path. Therefore, the total profitability is computed as the sum of the NPVs:

$$Total\ Profitability\ =\ NPV_0 +\ NPV_1 +\ NPV_2 +\ NPV_3 +\ NPV_4 \tag{1}$$

Being the value creation of assets circularity given as follows:

$$Value\ creation\ of\ 4Rs =\ NPV_1 +\ NPV_2 +\ NPV_3 +\ NPV_4 \tag{2}$$

Each phase adds less value: $NPV_j > NPV_{j+1}, NPV_j \in \mathbb{R}, j = 0, 1, 2, 3, 4$

$$NPV_0 = V_0 - E_0, \ NPV_1 = \frac{V_1 - E_1}{(1+a)^{t_1}}, \ NPV_2 = \frac{V_2 - E_2}{(1+a)^{t_2}}, \ NPV_3 = \frac{V_3 - E_3}{(1+a)^{t_3}},$$

$$NPV_4 = \frac{V_4 - E_4}{(1+a)^{t_4}}$$

$$NPV_j = Net\ Present\ Value\ of\ phase\ j\ at\ t_0$$

$$V_j = Value\ of\ phase\ j\ at\ t_j \tag{3}$$

$$E_j = Initial\ outlay\ of\ phase\ j$$

$$a = Risk - adjusted\ discounting\ rate\ (opportunity\ cost\ of\ capital)$$

$$t_j = Time\ to\ the\ beginning\ of\ phase\ j$$

According to NPV criteria, a project is said to be profitable when its NPV is positive; otherwise, it is classified as unprofitable and destroys value. From a financial viewpoint, only profitable projects deserve to be undertaken, reducing the NPV rule to max (0, NPV). Nevertheless, companies investing in negative NPV projects because the real options embedded within the main project substantially increase their value do exist, resulting in a positive extended NPV. Thus, the classical NPV is surpassed by the extended NPV that includes the value of real options or strategic opportunities. In the next section, we describe a common and flexible model to value options: the known binomial method.

3.2. Circularity Value from Real Options Method

Real Options Theory has received a great deal of attention from both academics and practitioners in strategic management (see [56] for a review). Myers [57] was the first author to give the name "real option" to an opportunity to purchase real assets on favorable terms. Generally speaking, a real option is a term associated with the flexibility inherent in strategic decision making related to business or real investment projects under uncertainty. Other primary works ([58,59]) analyzed the limitations of NPV classical criterion, revising and extending it to incorporate strategic flexibility components (called extended NPV). Additionally, a typology of real options was initially available in the work by [60]: defer option, abandon option, expansion option, and contract option, among others. Far from being exhaustive, theoretical and practical applications are numerous in fields such as *mining investment* (see [61] for a review), *water utilities* ([62,63]), *power transmission investment* ([64]), *wireless network management* ([65]), *pharmaceutical R&D valuation* ([66]), *oil investments* ([67]), *carbon capture and storage investment* ([68]), *solar photovoltaic power generation* ([69]), among many others. To our knowledge, however, papers combining real options and the circular economy are still to come.

A Binomial Model

The binomial framework is inspired by the Cox, Ross, and Rubinstein (CRR) ([70]) financial option valuation model. The binomial model is a discrete time model that stands out for its theoretical simplicity and flexibility since it can be adapted to almost any type

of option ([71]). Specifically, the underlying asset value (real asset) follows a stationary multiplicative binomial process with a constant increase (up) factor *u* and a constant decrease (down) factor *d*, as well as constant probability. Although these factors can be variable depending on time and phase, for the sake of simplicity they are considered stationary over time in each phase.

On the other hand, the aforementioned authors develop a valuation model that relies on the absence of arbitrage opportunities. Such an argument has important consequences: natural probability is substituted by risk-neutral probability, and the risk-free rate of interest *rf* becomes the relevant discounted rate. Figure 4 depicts the value process for two periods.
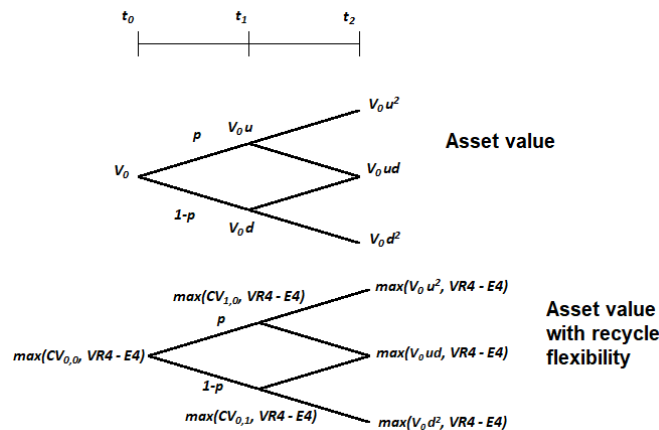

Figure 4. Two-period binomial tree. Source: prepared by the authors. V_0 represents initial value, $CV_{j,tn-j}$ —continuation value at the state *j* ups and *tn-j* downs, *VR4*—recycle value, *E4*—exercise price, *u*—constant increase factor, *d*—constant decrease factor, *p*—constant increase risk—neutral probability.

Let V_0 be the starting or initial value of the underlying asset that, in each step or period, can increase at a constant factor $u = e^{\sigma\sqrt{\Delta t}}$ or decrease at a constant factor $d = 1/u = e^{-\sigma\sqrt{\Delta t}}$ with a risk-neutral probability $p = \frac{e^{(rf-\delta)\sqrt{\Delta t}}-d}{u-d}$ and $q = 1 - p$, respectively, where σ is the percentage of volatility of the underlying asset value, rf is the risk-free rate of interest, δ is the depletion rate, and *Δt* is the stepping time that is calculated as the time scale between steps (e.g., for a one-year maturity if the binomial tree has four steps, each time-step has a stepping time of 0.25 years). Although these factors can be variable depending on the time and phase, for the sake of simplicity they are considered constant.

3.3. Sequential Compound Options

As already shown in the Value Hill model [25, 26], it is possible to identify a number of real options that allow for a change from one phase to the next one (see Figure 5). When the phase of primary use is ended, a real option to reuse the underlying real asset still exists (e.g., washing machine, car, smart phone, etc.). Similarly, the phase of reuse includes a refurbish real option in the refurbishing phase. This phase also contains an option related to the next phase: the remanufacture option. Additionally, the remanufacturing phase provides an option with respect to the last phase, that of recycling. Furthermore, the real option in the following phase may be understood as a compound option since this phase contains, in turn, an option regarding the subsequent phase. Additionally, these options may be viewed as European-style options (there is a single date, known as the

Comentado [MDPI-075]: Figure 5 is mentioned on page 13, but only appears on page 20. Please mention in in the paragraph before the figure.

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Comentado [MDPI-077]: If you would like to start mentioning Figure 5 from this point, we suggest moving it after this highlighted paragraph.

Otherwise, the Figure can only be mentioned on page 19/20.

Comentado [AR8R7]: Figure 5 has been moved after this paragraph.

terminal date, to exercise the option) or as American-style options (which may be exercised at any time before the terminal date).
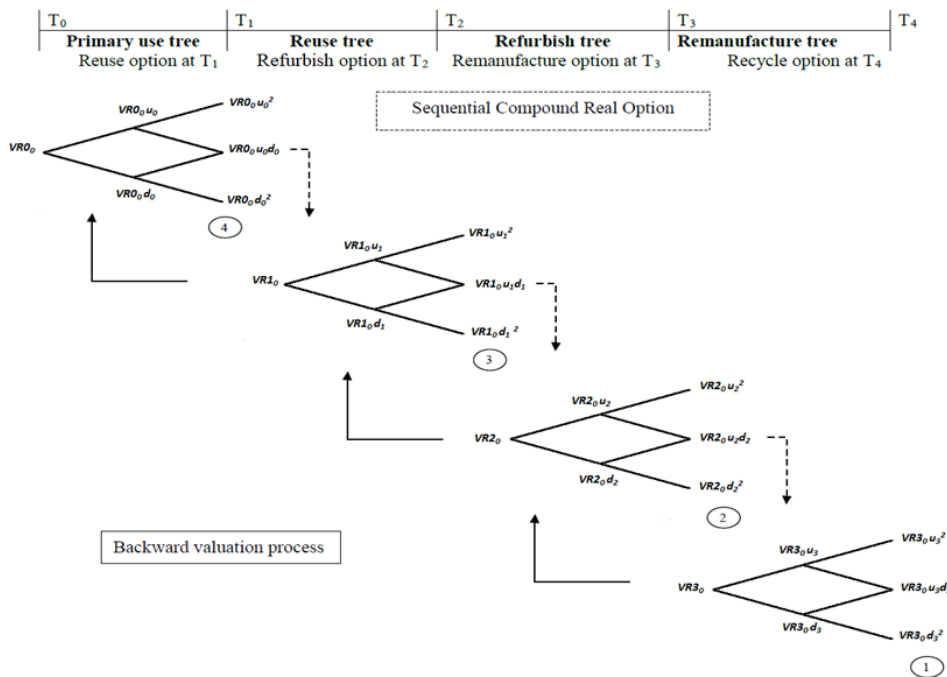

Figure 5. Sequential compound options scheme. Source: prepared by the authors.

To value options embedded in the Value Hill, a backward valuation process is conducted in the following four steps:

- **Step 1**. In the remanufacturing phase, a recycle option exists. The option value is computed as the difference between the remanufacture value with (VR3*) and without (VR3) flexibility. The valuation at the terminal date T4 (European style) is also distinguished from the valuation at a previous date $t <$ T4 (American style). Figure 5 shows the scheme of sequential compound options by considering a binomial process of the value without flexibility for two periods. Specifically, we denote $VR3_0$ as the starting remanufacture value, $u_3 = e^{\sigma_3\sqrt{\Delta t}}$ as the up factor, $d_3 = e^{-\sigma_3\sqrt{\Delta t}}$ as the down factor, σ_3 as the volatility of the remanufactured asset value, δ_3 as the depletion rate and $p_3 = \frac{e^{(rf-\delta_3)\sqrt{\Delta t}} - d_3}{u_3 - d_3}$ as the risk-neutral probability of up. In this step, the flexibility onward comes from the recycle option, which, once added, produces a similar tree to the remanufacture value with flexibility. The recycle option is viewed as a single option as it corresponds to the final phase of the Value Hill. Next, we formulate the process to compute the European option value at the terminal date T4, followed by the backward recursive process to compute the American option value at a previous date where $t <$ T4.

European Style Recycling Option

The procedure to determine the European option value begins at the terminal date T4, the only exercise date. At each node of the value tree, the value with and without flexibility are compared, and the positive difference is the option value. Thus, roughly speaking, the value of the recycle option at the terminal date is given as:

Recycle Option = Max (Remanufacture Value, Recycle Value – Exercise Price) – Remanufacture Value = Max (0, Recycle Value – Exercise Price – Remanufacture Value) (4)

Additionally, under neutral-risk valuation, an occurrence probability depending on the ups and downs of the initial process corresponds to each value, so the expected option value should be discounted at the risk-free interest rate for T4 periods to get the European option value at the beginning of the binomial tree.

Remanufacture value without flexibility at the date T_4 and node j:

$$VR3_{j,T_4} = VR3_0 u_3{}^j d_3{}^{T_4-j} \qquad j = 0, 1, \cdots, T_4 \tag{5}$$

Remanufacture value with flexibility at the date T_4 and node j:

$$VR3^*_{j,T_4} = max\left(VR3_{j,T_4}, VR4_{T_4} - E_4\right) \qquad j = 0, 1, \cdots, T_4 \tag{6}$$

$with\ VR4_{T_4} = Recycle\ value\ at\ T_4\ ,and\ \ E_4\ = Exercise\ price\ of\ recycle\ option$

Recycle option at the date T_4 and node j:

$$C_{4,j,T_4} = VR3^*_{j,T_4} - VR3_{j,T_4} = max\left(0, VR4_{T_4} - E_4 - VR3_{j,T_4}\right) \qquad j = 0, 1, \cdots, T_4 \tag{7}$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_4 = e^{-rf \cdot T_4} \sum_{=0}^{T_4} \binom{T_4}{j} C_{4,j,T_4} p_3{}^j q_3{}^{T_4-j} \tag{8}$$

- American Style Recycle Option

As in the European case, the valuation procedure of the American option begins at the terminal date T4, the last exercise date. However, the American option may be exercised at any previous date where *t* < T4. To determine the option value at a previous date, T4-h, it is necessary to first compute the continuation value at each node of the value tree, then calculate the value with flexibility and compare this with the value without flexibility, the positive difference being the option value. It is also expressed as follows:

Recycle Option = Max (Remanufacture CV – Remanufacture Value, Recycle Value – Exercise Price–Remanufacture Value) (9)

This procedure is a backward recursive one since it is repeated up to find the option value at the beginning of the value tree.

Remanufacture value without flexibility at the date $t = T_4 - h$ and node j:

$$VR3_{j,T_4-h} = VR3_0 u_3{}^j d_3{}^{T_4-h-j} \qquad j = 0, 1, \cdots, T_4 - h\ ; \quad h = 1, 2, \cdots, T_4 \tag{10}$$

Remanufacture continuation value-CV- at the date $t = T_4 - h$ and node j:

$$CVR3_{j,T_4-h} = e^{-rf\Delta t}\left(max\left(CVR3_{j+1,T_4-h+1}, VR4_{T_4-h+1} - E_4\right)p_3 + max\left(CVR3_{j,T_4-h+1}, VR4_{T_4-h+1} - E_4\right)q_3\right) \tag{11}$$

Remanufacture value with flexibility at the date $t = T_4 - h$ and node j:

$$VR3^*_{j,T_4-h} = max\left(CVR3_{j,T_4-h}, VR4_{T_4-h} - E_4\right) \tag{12}$$

Recycle option at the date t = T₄ − h and node j:

$$C_{4,j,T_4-h} = VR3^*_{j,T_4-h} - VR3_{j,T_4-h}$$
$$= max\left(CVR3_{j,T_4-h} - VR3_{j,T_4-h}, VR4_{T_4-h} - E_4 - VR3_{j,T_4-h}\right) \tag{13}$$
$$j = 0, 1, \cdots, T_4 - h$$

- **Step 2**. In the refurbishing phase, it is still possible to remanufacture the underlying real asset. Thus, a remanufacture option exists, the value of which is computed as the difference between the refurbish value with (VR2*) and without (VR2) flexibility. The valuation at the terminal date T3 (European style) is again distinguished from the valuation at a previous date where $t < T_3$ (American style). As before, Figure 5 represents the binomial process of the value without flexibility for two periods, with VR2₀ being the starting refurbish value, $u_2 = e^{\sigma_2\sqrt{\Delta t}}$ the up factor, $d_2 = e^{-\sigma_2\sqrt{\Delta t}}$ the down factor, σ_2 the volatility of the refurbished asset value, δ_2 the depletion rate, and $p_2 = \frac{e^{(rf-\delta_2)\sqrt{\Delta t}}-d_2}{u_2-d_2}$ the risk-neutral probability of up. By adding the value of the remanufacture option at each node of the refurbish tree, a new, extended refurbish tree may be drawn. The remanufacture option is a compound option, since it conveys the recycle option. Next, the process is formulated to compute the European option value at the terminal date T3, followed by the backward recursive process to compute the American option value at a previous date where $t < T_3$.

European Style Remanufacture Option

To value the European remanufacture option at the terminal date T3, it is necessary to compute the positive difference between the value with and without flexibility at each terminal node of the value tree. Once the option value is determined, its expected value is computed and discounted at the risk-free interest rate for T3 periods to get its initial value at the beginning of the binomial tree. In short, the remanufacture option is expressed as:

Remanufacture Option = Max (Refurbish Value, Remanufacture Value + Recycle Option − Exercise Price) − Refurbish Value = Max (0, Remanufacture Value + Recycle Option − Exercise Price − Refurbish Value) (14)

Refurbish value without flexibility at the date T3 and node j:

$$VR2_{j,T_3} = VR2_0 u_2{}^j d_2{}^{T_3-j} \qquad j = 0, 1, \cdots, T_3 \tag{15}$$

Refurbish value with flexibility at the date T3 and node j:

$$VR2^*_{j,T_3} = max\left(VR2_{j,T_3}, VR3^*_{T_3} - E_3\right)$$

$$VR3^*_{T_3} = Extended\ remanufacture\ value\ at\ T_3\ , and\ E_3 \tag{16}$$
$$= Exercise\ price\ of\ remanufacture\ option$$

Remanufacture option at the date T3 and node j:

$$C_{3,j,T_3} = VR2^*_{j,T_3} - VR2_{j,T_3} = max\left(0, VR3_{T_3} + C_{4,T_3} - E_3 - VR2_{j,T_3}\right) \tag{17}$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_3 = e^{-rf \cdot T_3} \sum_{j=0}^{T_3} \binom{T_3}{j} C_{3,j,T_3} p_2{}^j q_2{}^{T_3-j} \tag{18}$$

American Style Remanufacture Option

As similarly mentioned at step 1, the valuation procedure of the American remanufacture option begins at the terminal date T3, the last but not the only exercise date. To determine the option value at a previous date, T3-h, we first compute the continuation value at each node of the value tree, then compute the value with flexibility, which is

compared with the value without flexibility, and thus we obtain the option value. In other words, the remanufacture option is given as follows:

$$
\text{Remanufacture Option = Max (Refurbish Continuation Value − Refurbish Value, Remanufacture Value + Recycle Option − Exercise Price − Refurbish Value)}
\tag{19}
$$

This procedure will be repeated backward up to reach the origin of the value tree.

Refurbish value without flexibility at the date $t = T_3 − h$ and node j:

$$
VR2_{j,T_3-h} = VR2_0 u_2{}^j d_2{}^{T_3-h-j} \qquad j = 0,1,\cdots,T_3-h \; ; \quad h = 1,2,\cdots,T_3
\tag{20}
$$

Refurbish continuation value-CV- at the date $t = T_3 − h$ and node j:

$$
CVR2_{j,T_3-h} = e^{-rf\Delta t}\big(max\big(CVR2_{j+1,T_3-h+1},VR3_{T_3-h+1}+C_{4,T_3-h+1}-E_3\big)p_2 \\
+ max\big(CVR2_{j,T_3-h+1},VR3_{T_3-h+1}+C_{4,T_3-h+1}-E_3\big)q_2\big)
\tag{21}
$$

Refurbish value with flexibility at the date $t = T_3 − h$ and node j:

$$
VR2_{j,T_3-h}^{*} = max\big(CVR2_{j,T_3-h},VR3_{T_3-h}+C_{4,T_3-h}-E_3\big)
\tag{22}
$$

Remanufacture option at the date $t = T_3 − h$ and node j:

$$
C_{3,j,T_3-h} = VR2_{j,T_3-h}^{*} - VR2_{j,T_3-h} \\
= max\big(CVR2_{j,T_3-h}-VR2_{j,T_3-h},VR3_{T_3-h}+C_{4,T_3-h}-E_3-VR2_{j,T_3-h}\big)
\tag{23}
$$

- **Step 3.** In the reuse phase, an option exists in the next refurbishing phase. The option value is computed as the difference between the reuse value with (VR1*) and without (VR1) flexibility. We also distinguish the valuation at the terminal date T2 (European style) from the valuation at a previous date where $t < T_2$ (American style). For brevity, Figure 5 depicts the binomial process of the reuse value without flexibility for only two periods, where $VR1_0$ is the starting reuse value, $u_1 = e^{\sigma_1\sqrt{\Delta t}}$ the up factor, $d_1 = e^{-\sigma_1\sqrt{\Delta t}}$ the down factor, σ_1 the volatility of the reused asset value, δ_1 the depletion rate, and $p_1 = \frac{e^{(rf-\delta_1)\sqrt{\Delta t}}-d_1}{u_1-d_1}$ the risk-neutral probability of up. By adding the refurbish option value at each node of the reuse tree, a new extended reuse tree may be drawn. The refurbish option is also a compound option that conserves both the remanufacture and recycle options. Next, we formulate the process to compute the European option value at the terminal date T2, followed by the backward recursive process to compute the American option value at a previous date where $t < T_2$.

European Style Refurbish Option

To value the European refurbish option at the terminal date T2, the positive difference between the value with and without flexibility at each terminal node of the value tree must be computed.

$$
\text{Refurbish Option = Max (Reuse Value, Refurbish Value + Remanufacture Option − Exercise Price) − Reuse Value = Max (0, Refurbish Value + Remanufacture Option − Exercise Price − Reuse Value)}
\tag{24}
$$

The option value is determined by discounting the expected value of the option value at the risk-free interest rate for the T2 periods.

Reuse value without flexibility at the date T_2 and node j:

$$
VR1_{j,T_2} = VR1_0 u_1{}^j d_1{}^{T_2-j} \qquad j = 0,1,\cdots,T_2
\tag{25}
$$

Reuse value with flexibility at the date T_2 and node j:

$$
VR1_{j,T_2}^{*} = max\big(VR1_{j,T_2},VR2_{T_2}^{*}-E_2\big)
$$

$$
VR2_{T_2}^{*} = Extended\ refurnish\ value\ at\ T_2\ ,and\ E_2 \\
= Exercise\ price\ of\ refurnish\ option
\tag{26}
$$

Refurbish option at the date T_2 and node j:

$$C_{2,j,T_2} = VR1^*_{j,T_2} - VR1_{j,T_2} = max\left(0, VR2_{T_2} + C_{3,T_2} - E_2 - VR1_{j,T_2}\right) \quad (27)$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_2 = e^{-rf \cdot T_2} \sum_{j=0}^{T_2} \binom{T_2}{j} C_{2,j,T_2} p_1^{\ j} q_1^{\ T_2 - j} \quad (28)$$

American Style Refurbish Option

As already known, the recursive valuation procedure of the American refurbish option begins at the terminal date T_2. To determine the option value at a previous date, T_2-h, the continuation value at each node of the value tree is first computed, then the value with flexibility is calculated and compared with the value without flexibility to obtain the option value. Roughly speaking, the refurbish option is expressed as:

Refurbish Option = Max (Reuse Continuation Value − Reuse Value, Refurbish Value + Remanufacture Option − Exercise Price − Reuse Value) (29)

That procedure should be repeated backward up to reach the origin of the value tree to get the initial value of an American option.

Reuse value without flexibility at date t = T_2 − h and node j:

$$VR1_{j,T_2-h} = VR1_0 u_1^{\ j} d_1^{\ T_2-h-j} \qquad j = 0,1,\cdots,T_2 - h; \quad h = 1,2,\cdots,T_2 \quad (30)$$

Reuse continuation value at the date t = T_2 − h and node j:

$$CVR1_{j,T_2-h} = e^{-rf\Delta t}\left(max\left(CVR1_{j+1,T_2-h+1}, VR2_{T_2-h+1} + C_{3,T_2-h+1} - E_2\right)p_1 \\ + max\left(CVR1_{j,T_2-h+1}, VR2_{T_2-h+1} + C_{3,T_2-h+1} - E_2\right)q_1\right) \quad (31)$$

Reuse value with flexibility at the date t = T_2 − h and node j:

$$VR1^*_{j,T_2-h} = max\left(CVR1_{j,T_2-h}, VR2_{T_2-h} + C_{3,T_2-h} - E_2\right) \quad (32)$$

Refurbish option at the date t = T_2 − h and node j:

$$C_{2,j,T_2-h} = VR1^*_{j,T_2-h} - VR1_{j,T_2-h} \\ = max\left(CVR1_{j,T_2-h} - VR1_{j,T_2-h}, VR2_{T_2-h} + C_{3,T_2-h} - E_2 - VR1_{j,T_2-h}\right) \quad (33)$$

- **Step 4**. In the primary use phase, an option exists in the reuse phase. The option value is computed as the difference between the use value with (VR0*) and without (VR0) flexibility. The valuation at the terminal date T_1 (European style) is also distinguished from the valuation at a previous date where $t < T_1$ (American style). Figure 5 shows the binomial process of the primary use value without flexibility for two periods, where $VR0_0$ is the starting primary use value, $u_0 = e^{\sigma_0\sqrt{\Delta t}}$ the up factor, $d_0 = e^{-\sigma_0\sqrt{\Delta t}}$ the down factor, σ_0 the volatility of in-use asset value, δ_0 the depletion rate, and $p_0 = \frac{e^{(rf-\delta_0)\sqrt{\Delta t}} - d_0}{u_0 - d_0}$ the risk-neutral probability of up. By adding the option value at each node of the primary use tree, a new extended use tree is drawn. The reuse option is a compound option that keeps the options on refurbish, remanufacture and recycle. Next, we first formulate the process to compute the European option value at the terminal date T_1, followed by the backward recursive process to compute the American option value at a previous date where $t < T_1$.

European Style Reuse Option

As a European option, the reuse option is first valued at each terminal node of the value tree by the difference between the Equation (36) and the Equation (35), shown in the Equation (37), then its expected value is calculated and discounted at the risk-free interest rate for the option lifetime.

Reuse Option = Max (Use Value, Reuse Value + Refurbish Option − Exercise Price) − Use Value = Max (0, Reuse Value + Refurbish Option − Exercise Price − Use Value) (34)

Primary use value without flexibility at the date T₁ and node j:

$$VR0_{j,T_1} = VR0_0 u_0{}^j d_0{}^{T_1-j} \qquad j = 0, 1, \cdots, T_1$$ (35)

Primary use value with flexibility at the date T₁ and node j:

$$VR0^*_{j,T_1} = max\left(VR0_{j,T_1}, VR1^*_{T_1} - E_1 \right)$$ (36)

$VR1^*_{T_1} = Extended\ reuse\ value\ at\ T_1\ , and\ \ E_1\ = Exercise\ price\ of\ reuse\ option$

Reuse option at the date T₁ and node j:

$$C_{1,j,T_1} = VR0^*_{j,T_1} - VR0_{j,T_1} = max\left(0, VR1_{T_1} + C_{2,T_1} - E_1\ - VR0_{j,T_1}\right)$$ (37)

European option value under risk-neutral probability at the beginning of value tree:

$$C_1\ = e^{-rf \cdot T_1} \sum_0^{T_1} \binom{T_1}{j} C_{1,j,T_1} p_0{}^j q_0{}^{T_1-j}$$ (38)

American Style Reuse Option

As an American option, the reuse option is valued first at the terminal date T₁, then a backward procedure is performed by recursively computing the option value at previous dates, T₁-h, up to reach the origin of the value tree.

Reuse Option = Max (Use Continuation Value − Use Value, Reuse Value + Refurbish Option − Exercise Price − Use Value) (39)

Primary use value without flexibility at the date t = T₁ − h and node j:

$$VR0_{j,T_1-h} = VR0_0 u_0{}^j d_0{}^{T_1-h-j} \qquad j = 0, 1, \cdots, T_1 - h\ ; \quad h = 1, 2, \cdots, T_1$$ (40)

Primary use continuation value without flexibility at the date t = T₁ − h and node j:

$$CVR0_{j,T_1-h} = e^{-rf\Delta t}\left(max\left(CVR0_{j+1,T_1-h+1}, VR1_{T_1-h+1} + C_{2,T_1-h+1} - E_1\right)p_0 \right.$$
$$\left. + max\left(CVR0_{j,T_1-h+1}, VR1_{T_1-h+1} + C_{2,T_1-h+1} - E_1\right)q_0\right)$$ (41)

Primary use value with flexibility at the date t = T₁ − h and node j:

$$VR0^*_{j,T_1-h} = max\left(CVR0_{j,T_1-h}, VR1_{T_1-h} + C_{2,T_1-h} - E_1\right)$$ (42)

Reuse Option at the date t = T₁ − h and node j:

$$C_{1,j,T_1-h} = VR0^*_{j,T_1-h} - VR0_{j,T_1-h}$$
$$= max\left(CVR0_{j,T_1-h}\ - VR0_{j,T_1-h}, VR1_{T_1-h} + C_{2,T_1-h} - E_1\ - VR0_{j,T_1-h}\right)$$ (43)

At this point, it may be said that circularity adds value to a real asset, providing an extended value that results from the aggregation of two components: the underlying real asset and the real options. Consequently, the value creation of circularity may be interpreted and computed as the value of real options that are embedded in the aforementioned Value Hill model of the CE. More precisely, the total value of the real option component will be computed at the beginning of the primary use phase. Specifically, the option to choose carries a 'right' to select from among several subsequent phases, which enhances the value of the real option component or, in other words, the value creation of circularity. The distinction between the European and American options is an issue of exercise rights. A European option is characterized by the right to exercise at a terminal date only, whereas an American option may be exercised in any date during its lifetime by invoking an anticipated exercise right. This difference of rights makes American options more valued than European ones.

3.4. Options to Choose

The phase options embedded in the Value Hill may also be interpreted as options to choose among the several subsequent phases. Thus, in the primary use phase, an option to choose among reuse, refurbish, remanufacture, and recycle exists. Similarly, the reuse phase contains an option to choose among refurbish, remanufacture, and recycle, and so on. Figure 6 represents a simplified scheme of the option to choose.
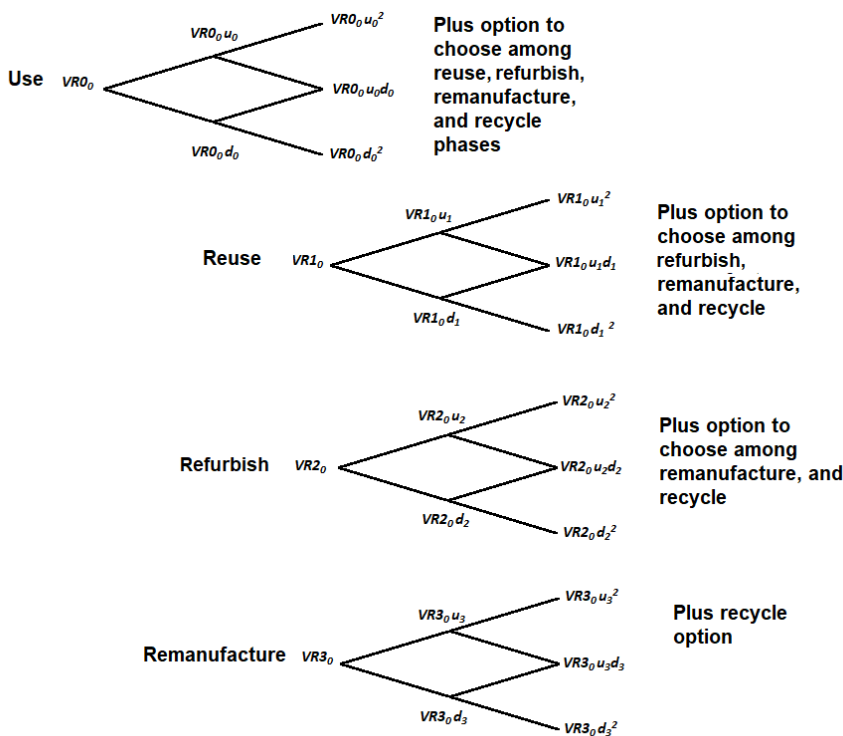
Use $VR0_0$

$VR0_0 u_0$ → $VR0_0 u_0^2$

$VR0_0 u_0 d_0$

$VR0_0 d_0$ → $VR0_0 d_0^2$

Plus option to choose among reuse, refurbish, remanufacture, and recycle phases

Reuse $VR1_0$

$VR1_0 u_1$ → $VR1_0 u_1^2$

$VR1_0 u_1 d_1$

$VR1_0 d_1$ → $VR1_0 d_1^2$

Plus option to choose among refurbish, remanufacture, and recycle

Refurbish $VR2_0$

$VR2_0 u_2$ → $VR2_0 u_2^2$

$VR2_0 u_2 d_2$

$VR2_0 d_2$ → $VR2_0 d_2^2$

Plus option to choose among remanufacture, and recycle

Remanufacture $VR3_0$

$VR3_0 u_3$ → $VR3_0 u_3^2$

$VR3_0 u_3 d_3$

$VR3_0 d_3$ → $VR3_0 d_3^2$

Plus recycle option

Figure 6. Option to choose scheme. Source: prepared by the authors.

Therefore, the value of circularity is determined by a backward valuation process in four steps, as follows:

- **Step 1**. In the remanufacturing phase, a recycle option exists (Choose Option 4). The option value is computed as the difference between the remanufacture value with (VR3*) and without (VR3) flexibility. The valuation at the terminal date T4 (European style) is also distinguished from the valuation at a previous date where $t < T_4$ (American style). For clarity, Figure 6 shows the binomial process of the value without flexibility for two periods, with $VR3_0$ being the starting remanufacture value, $u_3 = e^{\sigma_3 \sqrt{\Delta t}}$ the up factor, $d_3 = e^{-\sigma_3 \sqrt{\Delta t}}$ the down factor, σ_3 the volatility of the remanufactured asset value, δ_3 the depletion rate, and $p_3 = \frac{e^{(rf - \delta_3)\sqrt{\Delta t}} - d_3}{u_3 - d_3}$ the risk-neutral

Comentado [MDPI-0713]: Figure 6 is mentioned on page 19, but only appears on page 27. Please mention it in the paragraph before the figure.

Comentado [AR14R13]: I've added "on page 26"

Comentado [MDPI-0715]: Once again, if you would like to start mentioning Figure 6 at this point, it must be added after this paragraph.

Comentado [AR16R15]: I've added it after the first paragraph of this section

probability of up. At this step, the flexibility onward comes from the recycle option, which, once added, produces a similar tree of the remanufacture value with flexibility. The recycle option is a single option as it corresponds to the final phase of the Value Hill. Next, we formulate the process to compute the European option value at the terminal date T4, followed by the backward recursive process to compute the American option value at a previous date where $t <$ T4.

European style Choose Option 4

The procedure to determine the European option value begins at the terminal date T4, the only exercise date. At each node of the value tree, we compare the value with and without flexibility, as well as the positive difference being the option value. Under neutral risk, an occurrence probability depending on the ups and downs of the initial process corresponds to each value, so the expected option value is discounted at the risk-free interest rate for T$_4$ periods to reach the European option value at the beginning of the binomial tree. In short, at the terminal date T$_4$, the value of the option is expressed as follows:

$$\text{Choose Option 4} = Max\ (Remanufacture\ Value, Recycle\ Value - Exercise\ Price\ 4) - Remanufacture\ Value \tag{44}$$

Remanufacture value without flexibility at the date T$_4$ and node j:

$$VR3_{j,T_4} = VR3_0 u_3{}^j d_3{}^{T_4 - j} \qquad j = 0, 1, \cdots, T_4 \tag{45}$$

Remanufacture value with flexibility at the date T$_4$ and node j:

$$VR3_{j,T_4}^* = max\left(VR3_{j,T_4}, VR4_{T_4} - E_4\right) \qquad j = 0, 1, \cdots, T_4$$

$$\text{with } VR4_{T_4} = Recycle\ value\ at\ T_4\ , and\ \ E_4\ = Exercise\ price\ of\ recycle\ option \tag{46}$$

Choose Option 4 at the date T$_4$ and node j:

$$C_{4,j,T_4} = VR3_{j,T_4}^* - VR3_{j,T_4} = max\left(0, VR4_{T_4} - E_4\ - VR3_{j,T_4}\right) \qquad j = 0, 1, \cdots, T_4 \tag{47}$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_4\ = e^{-rf \cdot T_4} \sum_{j=0}^{T_4} \binom{T_4}{j} C_{4,j,T_4} p_3{}^j q_3{}^{T_4 - j} \tag{48}$$

American style Choose Option 4

As in the European case, the valuation procedure of the American option begins at the terminal date T4, the last exercise date. However, the American option may be exercised at any previous date where $t <$ T4. To determine the option value at a previous date, T4-h, it is necessary to first compute the continuation value at each node of the value tree, then compute the flexibility value and compare this to the value without flexibility, the positive difference being the option value. This procedure is a backward recursive one since is repeated up to find the option value at the beginning of the value tree. Therefore, the value of the option at a date before the terminal date is:

$$\text{Choose Option 4} = Max\ (Remanufacture\ Continuation\ Value, Recycle\ Value\ - Exercise\ Price\ 4) - Remanufacture\ Value \tag{49}$$

Remanufacture value without flexibility at the date t = T$_4$ − h and node j:

$$VR3_{j,T_4 - h} = VR3_0 u_3{}^j d_3{}^{T_4 - h - j} \qquad j = 0, 1, \cdots, T_4 - h\ ; \quad h = 1, 2, \cdots, T_4 \tag{50}$$

Remanufacture continuation value-CV- at the date t = T$_4$ − h and node j:

$$CVR3_{j,T_4 - h} = e^{-rf\Delta t} \left(max\left(CVR3_{j+1,T_4 - h+1}, VR4_{T_4 - h+1} - E_4\ \right)p_3 + max\left(CVR3_{j,T_4 - h+1}, VR4_{T_4 - h+1} - E_4\ \right)q_3\right) \tag{51}$$

Remanufacture value with flexibility at the date t = T₄ − h and node j:

$$VR3^*_{j,T_4-h} = max\left(CVR3_{j,T_4-h}, VR4_{T_4-h} - E_4 \right) \tag{52}$$

Choose Option 4 at the date t = T₄ − h and node j:

$$C_{4,j,T_4-h} = VR3^*_{j,T_4-h} - VR3_{j,T_4-h}$$
$$= max\left(CVR3_{j,T_4-h} - VR3_{j,T_4-h}, VR4_{T_4-h} - E_4 - VR3_{j,T_4-h}\right) \tag{53}$$
$$j = 0, 1, \cdots, T_4 - h$$

- **Step 2**. In the refurbishing phase, an option to choose between remanufacturing and recycling exists (Choose Option 3). The option value is computed as the difference between the refurbish value with (VR2*) and without (VR2) flexibility. The valuation at the terminal date T3 (European style) is also distinguished from the valuation at a previous date where $t < T_3$ (American style). Figure 6 shows the binomial process of the value without flexibility for two periods, where VR2₀ is the starting refurbish value, $u_2 = e^{\sigma_2\sqrt{\Delta t}}$ the up factor, $d_2 = e^{-\sigma_2\sqrt{\Delta t}}$ the down factor, σ_2 the volatility of the refurbished asset value, δ_2 the depletion rate, and $p_2 = \frac{e^{(rf-\delta_2)\sqrt{\Delta t}}-d_2}{u_2-d_2}$ the risk-neutral probability of up. In this phase of the Value Hill, the flexibility onward is due to the option to choose among the next two phases, remanufacturing and recycling. By adding the option value at each node of the refurbish tree, a new extended refurbish tree may be drawn. The Choose Option 3 is a compound option that corresponds to an intermediate phase of the Value Hill and keeps the recycle option. Next, we formulate the process to compute the European option value at the terminal date T3, followed by the backward recursive process to compute the American option value at a previous date where $t < T_3$.

European style Choose Option 3

To value the European Choose Option 3 at the terminal date T3, it is necessary to calculate the positive difference between the value with and without flexibility at each terminal node of the value tree. Notice that the Equation (56) involves both the remanufacture value with flexibility and the recycle value. Once the option value is determined, its expected value is computed and discounted at the risk-free interest rate for T3 periods to get its initial value at the beginning of the binomial tree. In short, the value of the option at the terminal date T3 is:

$$\text{Choose Option 3 = Max (Refurbish Value, Remanufacture Value + Choose}$$
$$\text{Option 4} - \text{Exercise Price 3, Recycle Value} - \text{Exercise Price 4)} - \text{Refurbish Value} \tag{54}$$

Refurbish value without flexibility at the date T3 and node j:

$$VR2_{j,T_3} = VR2_0 u_2{}^j d_2{}^{T_3-j} \qquad j = 0, 1, \cdots, T_3 \tag{55}$$

Refurbish value with flexibility at the date T3 and node j:

$$VR2^*_{j,T_3} = max\left(VR2_{j,T_3}, VR3^*_{T_3} - E_3, VR4_{T_3} - E_4\right)$$
$$\text{with } VR4_{T_3} = \text{Recycle value at } T_3, \ E_4 = \text{Exercise price of recycle option}$$
$$VR3^*_{T_3} = \text{Extended remanufacture value at } T_3, \text{and } E_3$$
$$= \text{Exercise price of remanufacture option} \tag{56}$$

Choose Option 3 at the date T3 and node j:

$$C_{3,j,T_3} = VR2^*_{j,T_3} - VR2_{j,T_3}$$
$$= max\left(0, VR3_{T_3} + C_{4,T_3} - E_3 - VR2_{j,T_3}, VR4_{T_3} - E_4 - VR2_{j,T_3}\right) \tag{57}$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_3 = e^{-rf \cdot T_3} \sum_{j=0}^{T_3} \binom{T_3}{j} C_{3,j,T_3} p_2{}^j q_2{}^{T_3 - j} \tag{58}$$

American Style Choose Option 3

As similarly mentioned in step 1, the valuation procedure of the American Choose Option 3 begins at the terminal date T_3, the last but not the only exercise date. To determine the option value at a previous date, T_3-h, we first compute the continuation value at each node of the value tree, then the value with flexibility, which is compared with the value without flexibility, thereby obtaining the option value. Notice that Equation (61) and Equation (62) involve both the remanufacture value with flexibility and the recycle value. This procedure is repeated backward up to reach the origin of the value tree. At a previous date, the value of the option is:

Choose Option 3 = Max (Refurbish Continuation Value, Remanufacture Value
+ Choose Option 4 − Exercise Price 3, Recycle Value − Exercise Price 4) −   (59)
Refurbish Value

Refurbish value without flexibility at the date t = $T_3 - h$ and node j:

$$VR2_{j,T_3 - h} = VR2_0 u_2{}^j d_2{}^{T_3 - h - j} \qquad j = 0, 1, \cdots, T_3 - h \; ; \quad h = 1, 2, \cdots, T_3 \tag{60}$$

Refurbish continuation value-CV- at the date t = $T_3 - h$ and node j:

$$CVR2_{j,T_3 - h} = e^{-rf \Delta t} \big(max \big(CVR2_{j+1,T_3 - h + 1}, VR3_{T_3 - h + 1} + C_{4,T_3 - h + 1} - E_3, VR4_{T_3 - h + 1} \\ - E_4 \big) p_2 \\ + max \big(CVR2_{j,T_3 - h + 1}, VR3_{T_3 - h + 1} + C_{4,T_3 - h + 1} - E_3, VR4_{T_3 - h + 1} \\ - E_4 \big) q_2 \big) \tag{61}$$

Refurbish value with flexibility at the date t = $T_3 - h$ and node j:

$$VR2_{j,T_3 - h}^* = max \big(CVR2_{j,T_3 - h}, VR3_{T_3 - h} + C_{4,T_3 - h} - E_3, VR4_{T_3 - h} - E_4 \big) \tag{62}$$

Choose Option 3 at the date t = $T_3 - h$ and node j:

$$C_{3,j,T_3 - h} = VR2_{j,T_3 - h}^* - VR2_{j,T_3 - h} = \\ = max \big(CVR2_{j,T_3 - h} - VR2_{j,T_3 - h}, VR3_{T_3 - h} + C_{4,T_3 - h} - E_3 \\ - VR2_{j,T_3 - h}, VR4_{T_3 - h + 1} - E_4 - VR2_{j,T_3 - h} \big) \tag{63}$$

- **Step 3.** In the reuse phase, an option to choose between refurbish, remanufacture and recycle exists (Choose Option 2). The option value is computed as the difference between the reuse value with (VR1*) and without (VR1) flexibility. The valuation at the terminal date T2 (European style) is also distinguished from the valuation at a previous date where $t < T_2$ (American style). Figure 6 shows the binomial process of the value without flexibility for two periods, where VR1$_0$ is the starting reuse value, $u_1 = e^{\sigma_1 \sqrt{\Delta t}}$ the up factor, $d_1 = e^{-\sigma_1 \sqrt{\Delta t}}$ the down factor, σ_1 the volatility of the reused asset value, δ_1 the depletion rate, and $p_1 = \frac{e^{(rf - \delta_1) \sqrt{\Delta t}} - d_1}{u_1 - d_1}$ the risk-neutral probability of up. In this phase of the Value Hill, the flexibility onward is due to the option to choose among the next three phases: refurbish, remanufacture and recycle. By adding the option value at each node of the reuse tree, a new extended reuse tree may be drawn. The Choose Option 2 is a compound option that involves both the remanufacture and recycle options. Next, we formulate the process to compute the European option value at the terminal date T2, followed by the backward recursive process to compute the American option value at a previous date where $t < T_2$.

European Style Choose Option 2

To value the European Choose Option 2 at the terminal date T_2, it is necessary to compute the positive difference between the value with and without flexibility at each terminal node of the value tree. Notice that Equation (66) involves three subsequent phases—refurbishing, remanufacturing and recycling—and the flexibility inherent in each phase. The option value is determined in Equation (68) by discounting the expected value of the option value at the risk-free interest rate for T_2 periods.

At the terminal date T_2:

$$Choose\ Option\ 2 = Max\ (Reuse\ Value, Refurbish\ Value + Choose\ Option\ 3 -$$
$$Exercise\ Price\ 2, Remanufacture\ Value + Choose\ Option\ 4 - Exercise\ Price\ 3, \quad (64)$$
$$Recycle\ Value - Exercise\ Price\ 4) - Reuse\ Value$$

Reuse value without flexibility at the date T_2 and node j:

$$VR1_{j,T_2} = VR1_0 u_1{}^j d_1{}^{T_2-j} \qquad j = 0,1,\cdots,T_2 \tag{65}$$

Reuse value with flexibility at the date T_2 and node j:

$$VR1^*_{j,T_2} = max\left(VR1_{j,T_2}, VR2^*_{T_2} - E_2 , VR3^*_{T_2} - E_3 , VR4_{T_2} - E_4\right)$$

$$with\ VR4_{T_2} = Recycle\ value\ at\ T_2\ ,\ E_4\ = Exercise\ price\ of\ recycle\ option$$

$$VR3^*_{T_2} = Extended\ remanufacture\ value\ at\ T_2\ , and\ E_3 \tag{66}$$
$$= Exercise\ price\ of\ remanufacture\ option$$

$$VR2^*_{T_2} = Extended\ refurnish\ value\ at\ T_2\ , and\ E_2$$
$$= Exercise\ price\ of\ refurnish\ option$$

Choose Option 2 at the date T_2 and node j:

$$C_{2,j,T_2} = VR1^*_{j,T_2} - VR1_{j,T_2}$$
$$= max\left(0, VR2_{T_2} + C_{3,T_2} - E_2\ - VR1_{j,T_2}, VR3_{T_2} + C_{4,T_2} - E_3 \right. \tag{67}$$
$$\left. - VR1_{j,T_2}, VR4_{T_2} - E_4\ - VR1_{j,T_2}\right)$$

European option value under risk-neutral probability at the beginning of value tree:

$$C_2\ = e^{-rf \cdot T_2} \sum_{j=0}^{T_2} \binom{T_2}{j} C_{2,j,T_2} p_1{}^j q_1{}^{T_2-j} \tag{68}$$

American Style Choose Option 2

As we already know, the recursive valuation procedure of the American Choose Option 2 begins at the terminal date T_2. To determine the option value at a previous date, T_2-h, we first compute the continuation value at each node of the value tree, then the value with flexibility, which is then compared with the value without flexibility to obtain the option value. Note that Equation (71) and Equation (72) involve both refurbish and remanufacture values with flexibility, as well as the recycle value. As before, the procedure must be repeated backward up to reach the origin of the value tree and find the initial value of an American option. Before the terminal date, the value of the option is given as:

$$Choose\ Option\ 2 = Max\ (Reuse\ Continuation\ Value, Refurbish\ Value + Choose$$
$$Option\ 3 - Exercise\ Price\ 2, Remanufacture\ Value + Choose\ Option\ 4 - Exercise \quad (69)$$
$$Price\ 3, Recycle\ Value - Exercise\ Price\ 4) - Reuse\ Value$$

Reuse value without flexibility at date $t = T_2 - h$ and node j:

$$VR1_{j,T_2-h} = VR1_0 u_1{}^j d_1{}^{T_2-h-j} \qquad j = 0,1,\cdots,T_2-h; \quad h = 1,2,\cdots,T_2 \tag{70}$$

Reuse continuation value at the date t = T₂ − h and node j:

$$CVR1_{j,T_2-h} = e^{-rf\Delta t}\big(max\big(CVR1_{j+1,T_2-h+1}, VR2_{T_2-h+1}+C_{3,T_2-h+1} \\ -E_2 \, , VR3_{T_2-h+1}+C_{4,T_2-h+1}-E_3 \, , VR4_{T_2-h+1}-E_4 \, \big)p_1 \\ + max\big(CVR1_{j,T_2-h+1}, VR2_{T_2-h+1}+C_{3,T_2-h+1} \\ -E_2 \, , VR3_{T_2-h+1}+C_{4,T_2-h+1}-E_3 \, , VR4_{T_2-h+1}-E_4 \, \big)q_1\big) \tag{71}$$

Reuse value with flexibility at the date t = T₂ − h and node j:

$$VR1^*_{j,T_2-h} = max\big(CVR1_{j,T_2-h}, VR2_{T_2-h}+C_{3,T_2-h}-E_2 \, , VR3_{T_2-h}+C_{4,T_2-h}-E_3 \, , VR4_{T_2-h} \\ -E_4 \, \big) \tag{72}$$

Choose Option 2 at the date t = T₂ − h and node j:

$$C_{2,j,T_2-h} = VR1^*_{j,T_2-h} - VR1_{j,T_2-h} \\ = max\big(CVR1_{j,T_2-h} - VR1_{j,T_2-h}, VR2_{T_2-h}+C_{3,T_2-h}-E_2 \\ -VR1_{j,T_2-h}, VR3_{T_2-h}+C_{4,T_2-h}-E_3 \, -VR1_{j,T_2-h}, VR4_{T_2-h}-E_4 \\ -VR1_{j,T_2-h}\big) \tag{73}$$

- **Step 4**. In the primary use phase, an option to choose between reuse, refurbish, re-manufacture and recycle exists (Choose Option 1). The option value is computed as the difference between the use value with (VR0*) and without (VR0) flexibility. The valuation at the terminal date T₁ (European style) is also distinguished from the valuation at a previous date where $t < T_1$ (American style). Figure 6 shows the binomial process of the value without flexibility for two periods, where VR0₀ is the starting primary use value, $u_0 = e^{\sigma_0\sqrt{\Delta t}}$ the up factor, $d_0 = e^{-\sigma_0\sqrt{\Delta t}}$ the down factor, σ_0 the volatility of in-use asset value, δ_0 the depletion rate, and $p_0 = \frac{e^{(rf-\delta_0)\sqrt{\Delta t}}-d_0}{u_0-d_0}$ the risk-neutral probability of up. In this phase of the Value Hill, the flexibility onward is due to the option to choose among the next four phases: reuse, refurbish, remanufacture and recycle. By adding the option value at each node of the primary use tree, a new extended use tree is drawn. The Choose Option 1 is a compound option that involves the refurbish, remanufacture and recycle options. Next, we formulate the process to compute the European option value at the terminal date T₁, followed by the backward recursive process to compute the American option value at a previous date where $t < T_1$.

European Style Choose Option 1

As a European option, Choose Option 1 is first valued at each terminal node of the value tree by the difference between ==Equation (76)== and ==Equation (75),== shown in ==Equation (77),== then its expected value is computed and discounted at the risk-free interest rate for the option lifetime. Notice that ==Equation (77)== involves the four subsequent phases—reuse, refurbish, remanufacture and recycle—and the flexibility inherent in each phase. At the terminal date T₁, the value is determined as follows:

Choose Option 1 = Max (Use Value, Reuse Value + Choose Option 2 − Exercise Price 1, Refurbish Value + Choose Option 3 − Exercise Price 2, Remanufacture Value + Choose Option 4 − Exercise Price 3, Recycle Value − Exercise Price 4) − Use Value (74)

Primary use value without flexibility at the date T₁ and node j:

$$VR0_{j,T_1} = VR0_0 u_0{}^j d_0{}^{T_1-j} \qquad j = 0,1,\cdots,T_1 \tag{75}$$

Primary use value with flexibility at the date T₁ and node j:

$$VR0^*_{j,T_1} = max\left(VR0_{j,T_1}, VR1^*_{T_1} - E_1 , VR2^*_{T_1} - E_2 , VR3^*_{T_1} - E_3 , VR4^*_{T_1} - E_4 \right)$$

with $VR4_{T_1}$ = *Recycle value at* T_1 , E_4 = *Exercise price of recycle option*

$VR3^*_{T_1}$ = *Extended remanufacture value at* T_1 , *and* E_3 = *Exercise price of remanufacture option*　　(76)

$VR2^*_{T_1}$ = *Extended refurbish value at* T_1 , *and* E_2 = *Exercise price of refurbish option*

$VR1^*_{T_1}$ = *Extended reuse value at* T_1 , *and* E_1 = *Exercise price of reuse option*

Choose Option 1 at the date T_1 and node j:

$$\begin{aligned}C_{1,j,T_1} &= VR0^*_{j,T_1} - VR0_{j,T_1}\\ &= max\left(0, VR1_{T_1}+C_{2,T_1} - E_1 - VR0_{j,T_1}, VR2_{T_1}+C_{3,T_1} - E_2 - VR0_{j,T_1}, VR3_{T_1}+C_{4,T_1} - E_3\right.\\ &\quad \left. - VR0_{j,T_1}, VR4_{T_1} - E_4 - VR0_{j,T_1}\right)\end{aligned}$$　(77)

European option value under risk-neutral probability at the beginning of value tree:

$$C_1 = e^{-rf\cdot T_1}\sum_{j=0}^{T_1}\binom{T_1}{j}C_{1,j,T_1}p_0{}^j q_0{}^{T_1-j}$$　(78)

American Style Choose Option 1

As an American option, the Choose Option 1 is valued first at the terminal date T_1, then a backward procedure is performed by recursively computing the option value at previous dates, T_1-h, up to reach the origin of the value tree. At a previous date:

Choose Option 1 = Max (Use Continuation Value, Reuse Value + Choose Option 2 − Exercise Price 1, Refurbish Value + Choose Option 3 − Exercise Price 2, Remanufacture Value + Choose Option 4 − Exercise Price 3, Recycle Value − Exercise Price 4) − Use Value　(79)

Primary use value without flexibility at the date t = T_1 − h and node j:

$$VR0_{j,T_1-h} = VR0_0 u_0{}^j d_0{}^{T_1-h-j}\qquad j = 0, 1, \cdots, T_1 - h ; \quad h = 1, 2, \cdots, T_1$$　(80)

Primary use continuation value without flexibility at the date t = T_1 − h and node j:

$$\begin{aligned}CVR0_{j,T_1-h} = e^{-rf\Delta t}&\left(max\left(CVR0_{j+1,T_1-h+1}, VR1_{T_1-h+1}+C_{2,T_1-h+1} - E_1 , VR2_{T_1-h+1}+C_{3,T_1-h+1} - E_2 , VR3_{T_1-h+1}+C_{4,T_1-h+1}\right.\right.\\ &\left. - E_3 , VR4_{T_1-h+1} - E_4 \right)p_0\\ &+ max\left(CVR0_{j,T_1-h+1}, VR1_{T_1-h+1}+C_{2,T_1-h+1} - E_1 , VR2_{T_1-h+1}+C_{3,T_1-h+1} - E_2 , VR3_{T_1-h+1}+C_{4,T_1-h+1}\right.\\ &\left.\left. - E_3 , VR4_{T_1-h+1} - E_4 \right)q_0\right)\end{aligned}$$　(771)

Primary use value with flexibility at the date t = T_1 − h and node j:

$$VR0^*_{j,T_1-h} = max\left(CVR0_{j,T_1-h}, VR1_{T_1-h}+C_{2,T_1-h} - E_1 , VR2_{T_1-h}+C_{3,T_1-h} - E_2 , VR3_{T_1-h}+C_{4,T_1-h} - E_3 , VR4_{T_1-h} - E_4 \right)$$　(82)

Choose Option 1 at the date t = T_1 − h and node j:

$$\begin{aligned}C_{1,j,T_1-h} &= VR0^*_{j,T_1-h} - VR0_{j,T_1-h}\\ &= max\left(CVR0_{j,T_1-h} - VR0_{j,T_1-h}, VR1_{T_1-h}+C_{2,T_1-h} - E_1 - VR0_{j,T_1-h}, VR2_{T_1-h}+C_{3,T_1-h} - E_2\right.\\ &\quad \left. - VR0_{j,T_1-h}, VR3_{T_1-h}+C_{4,T_1-h} - E_3 - VR0_{j,T_1-h}, VR4_{T_1-h} - E_4 - VR0_{j,T_1-h}\right)\end{aligned}$$　(83)

From a circular economy viewpoint, the option to choose either European- or American-style adds flexibility to decision making, which is translated into value creation. Specifically, the option to choose carries a 'right' to select from among several subsequent phases, which enhances the value of the real option component, that is to say, the value creation of circularity. Consequently, the value of a European (American) option to choose will be greater than the value of a European (American) compound option, as defined in Subsection 3.3.

4. Conclusions

The circular economy (CE) is considered to be an appropriate approach for achieving national and international sustainability. For this reason, it has drawn increased attention from multinational firms, academics, researchers and policy makers in industrialized countries (European Commission, 2015). In general terms, the CE is defined as a combination of eco-design, reduce, reuse and recycle activities. The transition is from a linear model based on the optimization of economic performance to a circular model where all business decision-making and governance processes are based on the association among the economic, social and environmental dimensions. The change towards the CE requires an extensive transformation of corporate strategy, focusing on a culture of sustainability and modifying the corporate vision. In this regard, the CE can be considered as a tool that can be used by different countries, social agents, and institutions to achieve some Sustainable Development Goals (SDGs).

With the appearance of SDG principles, new opportunities are presented to those interested in participating in sustainable finance or who wish to engage with shareholders to convince them of the importance of taking these principles into account in their investment strategies to guarantee long-term viability of their investments. On the other hand, proponents of these SDG principals can also take advantage of legislation in those countries that has introduced sanctions for non-compliance with the disclosure obligations set forth in the European Directive 95/2014. Consequently, critical engagement with the underlying assumptions of the dissemination of SDG principles and sustainable finance is offered as a great opportunity for civil society to embark on a path that will enable the transformation of the global economy. To this end, the integration of mandatory regulation and increased transparency could also play a transformative role in promoting sustainable financing.

On the other hand, one cannot manage what one cannot currently measure. Although SDG principles have allowed companies to disclose important information about sources of environmental and social risk, their usefulness is very limited. There is little standardization in disclosure and no strong evidence that SDG investments outperform traditional portfolios.

A new approach to scientific research that promotes the application of multi-method research in the field of finance, in which data analysis can be combined with case studies, interviews or other ethnographic sources, is needed. Furthermore, the research results should be interpreted from a multidisciplinary perspective, while paying specific attention to the institutional, political and historical context of the research. This will facilitate greater awareness of sustainability issues among researchers and the finance industry.

This paper attempts to integrate the circular economy and financial valuation through a real options-based approach. On the one hand, the Value Hill model is used to identify the 4R real options inherent in the circular economy: Reuse, Refurbish, Remanufacture and Recycle. On the other hand, a binomial model for valuing real options is developed, using relative valuation principles. The real options embedded in the Value Hill may be defined as European or American options to choose among the subsequent phases. The distinction between European and American options is due to the exercise right. In a European option, there is one and only one exercise date that may be interpreted as the end of the current phase. However, an American option may be exercised on any previous date up to the maturity date, that is, the change of phase may take place at any moment before the end of the current phase.

This paper represents an initial approach to value creation from the perspective of asset circularity, which is flexible enough to be adapted to any specific case study. The value of circularity is determined as the value of a Compound option as well as the value of a Choose option, which eventually depends on the circular system under analysis.

This work will be extended along two lines of inquiry in the future. One line will be aimed at broadening and improving the methodological design and its implementation through a financial tool that incorporates several additions, such as alternative models to

represent uncertainty, Monte Carlo simulation, sensibility analysis, etc. The other line will be devoted to the application of real options methodology to real cases, adapted to the firm's situation or needs in order to provide a valuation of asset circularity. It deserves to be mentioned that data availability is a frequent limitation for empirical studies based on real data.

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