PRACTICE UNIT 1

1) What type of risk can you identify for each situation below?

- **An error in a financial institution's risk estimation model leads to the regulator forcing the institution to increase its capital buffers.**

The financial loss incurred by the institution is due to an error in the construction of an internal model. This type of risk is therefore an <u>operational risk</u>.

- **The joint action of a large group of minority investors organised via internet forums, forces up the price of a company in which you had opened short positions.**

The potential losses are due to a strong movement in the price of a commodity in the financial markets. This type of risk is therefore a <u>market risk</u>.

- **An asset's very low level of trading in the financial market prevents you from closing your opened position in that market when you place the order.**

This loss may be caused by not being able to close a position at the desired price and time due to the low liquidity of the market. This type of risk is therefore a <u>liquidity risk</u>.

2) An investor decides to buy 1,000 shares in a company's stock when its BID/ASK prices are \$5.05/\$4.95. After three months, the investor sells all those shares when their BID/ASK prices are \$6.25/\$5.75. What position does the investor take and what amount of money do they pay/receive at each point in time? When does the investor open and when do they close their position?

At the time of purchase the investor opens their position by taking a long position for which they pay (1,000 * 5.05 =) \$5,050. At the time of sale, the investor closes their position by taking a short position for which they receive (1,000 * 5.75 =) \$5,750.

3) Imagine that you borrow a bond with a nominal value equal to 1,000 € with the commitment to return it within one year, and that you sell it at a price of 950 €. Before repaying, the bond makes a total of six payments, each equal to 2.5% of the bond's face value. At the time of repayment, the bond is trading at a price of $750. What is the benefit of this strategy?

By shorting the bond, we receive $950. Since it has paid coupons, a total of [6 * (1,000 * 0.025) =] $150 must be paid to its owner. To return the bond to its owner, it is bought for $750. The final profit from the transaction will therefore be (950 - 150 - 750 =) $50.

4) Imagine that TEF shares can be bought on both the Madrid Stock Exchange and the London Stock Exchange (without paying any commission), and that the price of each share is €3.25 in Madrid and €3.00 in London. What action would you take? What action would all investors take and what consequences would this have on the price of the share in both markets?

Here you would buy the shares in London at a price of €3.00 and automatically sell them in Madrid at a price of €3.25, making a risk-free profit of €0.25 per share.

As soon as this situation is identified, all investors would apply the same strategy. Increased demand for the shares in London would cause their price to rise, while increased supply in Madrid would cause their price to fall. This would eventually lead to the price in both markets being equal and cause this arbitrage opportunity to disappear.

5) Using the data from the previous exercise, assume that a commission of €0.15 is now charged for each purchase or sale of a share in either market. What implications would this have for your previous answer?

The commissions for both the purchase of the share and its subsequent sale amount to €0.30, so the result of the transaction would be a loss of €0.05 per share. Since the commissions eliminate the arbitrage opportunity, the prices would not have to be automatically rebalanced.

6) What are the differences between a hedger and a speculator?

Primarily, a speculator takes positions in the market in the expectation of making a profit in the future. A hedger, on the other hand, uses financial markets (especially derivatives) to reduce – or eliminate if possible – uncertainty about the future price they will receive or have to pay, regardless of whether the outcome of the transaction generates a profit or loss.

7) Suppose an investor opens a short position in 1,000 CFDs on a stock at a price equal to 0.0250. To open a position, the broker requires a margin of at least 20% of the exposure. Months later, the investor closes their position at a price equal to 0.0200. What is the margin the investor should have posted at the beginning and what is the result of their trade? Assume there are no costs or commissions.

The investor’s open position has a value of (0.0250 * 1,000 =) $25, so they must deposit a margin of at least (250 * 0.20 =) $5 at the outset.

On closing the position, the investor’s profit will be equal to [1,000 * (0.0250 - 0.0200) =] $5.

PRACTICE UNIT 2

Imagine you are investor A and that at t = 1 you want to open a long position in 10 BBVA futures with the following characteristics:

Maturity:	7 days (T = 7)	**Contract Size:**	100 shares
Initial Margin:	1,000.00 €	**Maintenance Margin:**	500.00 €
Future Price	$\boldsymbol{F_{1,T} = 6.55€}$		

Assume that at the close of each of the following days, the prices of this futures contract are as follows:

$\boldsymbol{F_{1,T}}$	6.60 €	$\boldsymbol{F_{5,T}}$	6.55 €
$\boldsymbol{F_{2,T}}$	6.30 €	$\boldsymbol{F_{6,T}}$	6.75 €
$\boldsymbol{F_{3,T}}$	5.75 €	$\boldsymbol{F_{7,T}}$	7.00 €
$\boldsymbol{F_{4,T}}$	6.00 €		

Finally, also assume that you keep your position open until the future matures and the following market movements occur:

t = 1	**C** buys 5 futures **B** sells 15 futures	**t = 4**	**B** buys 6 and **D** buys 4 futures **E** sells 3 and **F** sells 7 futures
t = 2	**B** buys 7 futures **C** sells 3 and **D** sells 4 futures	**t = 5**	**X** buys 4 futures **E** sells 4 futures
t = 3	**B** buys 2 futures **C** sells 2 futures	**t = 6**	**E** buys 7 and F buys 7 futures **X** sells 14 futures

Answer the following questions:

1) When you open this position, what do you expect to happen in the market with respect to the price of the underlying asset? What would you call this type of behaviour? What amount will you receive/pay when the future matures?

By opening this position, as an investor I expect the price of the underlying asset to rise, since the futures contract will allow me to pay a lower price for the shares than the market price. My behaviour would therefore be that of a bullish speculator.

Taking into account that the price of the futures contract at maturity is equal to the price of the underlying asset, the payoff of this strategy would be:

$$Payoff = 10 * 100 * (7{,}00 - 6{,}55) = 455{,}00\ €$$

2) Assume that the price of BBVA is below 6.00 € on all these days. Is the future in contango or backwardation? Why?

Since the price of futures contracts would always be above the price of the underlying asset, we would always be in a contango situation.

3) Find the trading volume and open interest in this market on each day:

<table>
<tr><th rowspan="2">t</th><th colspan="2">Movements</th><th colspan="2">Opened Positions</th><th rowspan="2">Trade Volume</th><th rowspan="2">Open Interest</th></tr>
<tr><th>Long</th><th>Short</th><th>Long</th><th>Short</th></tr>
<tr><td>1</td><td>A buys 10
C buys 5</td><td>B sells 15</td><td>A = 10
C = 5</td><td>B = 15</td><td>15</td><td>15</td></tr>
<tr><td>2</td><td>B buys 7</td><td>C sells 3</td><td>A = 10
C = 2</td><td>B = 8
D = 4</td><td>22</td><td>12</td></tr>
<tr><td>3</td><td>B buys 2</td><td>C sells 2</td><td>A = 10</td><td>B = 6
D = 4</td><td>24</td><td>10</td></tr>
<tr><td>4</td><td>B buys 6
D buys 4</td><td>E sells 3
F sells 7</td><td>A = 10</td><td>E = 3
F = 7</td><td>34</td><td>10</td></tr>
<tr><td>5</td><td>X buys 4</td><td>E sells 4</td><td>A = 10
X = 4</td><td>E = 7
F = 7</td><td>38</td><td>14</td></tr>
<tr><td>6</td><td>E buys 7
F buys 7</td><td>X sells 14</td><td>A = 10</td><td>X = 10</td><td>52</td><td>10</td></tr>
<tr><td>7</td><td>-</td><td>-</td><td>A = 10</td><td>X = 10</td><td>52</td><td>10</td></tr>
<tr><td>Total</td><td>-</td><td>-</td><td>A = 10</td><td>X = 10</td><td>52</td><td>10</td></tr>
</table>

4) From when you open your position until it expires, is your counterparty in the market always the same? Do you know who your counterparty is?

No, my counterparty is changing every day I have an open position in the market. Moreover, I will never know who my counterparty is exactly, as this(these) other(s) investor(s) will also not know who I am.

5) Build the marking to market corresponding to your position:

Long Position	Trade Price	Settlement Price	Daily Gain	Cumulative Gain	Margin Account Balance	Margin Call
t = 1	6.55 €	-	-	-	1.000.00 €	-
t = 1	-	6.60 €	50.00 €	50.500 €	1.050.00 €	-
t = 2	-	6.30 €	-300.00	-250.00 €	750.00 €	-
t = 3	-	5.75 €	-550.00 €	-800.00 €	200.00 €	800.00
t = 4	-	6.00 €	250.00 €	-550.00 €	1.250.00 €	-
t = 5	-	6.55 €	550.00 €	0.00 €	1.800.00 €	-
t = 6	-	6.75 €	200.00 €	200.00 €	2.000.00 €	-
t = 7	7.00 €	-	250.00 €	450.00 €	2.250.00 €	-

6) Assume that settlement is by delivery. What amount of money will you receive/pay for both delivery and mark to markets? What is the effective price paid for each share?

- A receives (10 * 100 =) 1,000 shares and pays (7.00 * 1,000 =) = 7,000.00 €
- By marking to markets, A receives (7.00 – 6.55 =) 0.45 € (450.00 €).
  - **Net Price paid:** 7.00 – 0.45 = (7,000.00 – 450.00) / 1,000 = **6.55 €**

7) Assume that settlement is by delivery and ignore contract closures before maturity. What amount of money will your counterparty receive/pay for both delivery and mark to markets? What is the effective price paid for each share?

- X delivers (10 * 100 =) 1,000 shares and receives (7.00 * 1,000 =) = 7,000.00 €
- By marking to markets, X pays (6.75 – 7.00 =) 0.25 € (250,00 €).
  - **Net Price received:** 7.00 – 0.25 = (7,000.00 – 250.00) / 1,000 = **6.75 €**

8) Imagine that, instead of BBVA shares, the futures contract has the IBEX 35 index as the underlying asset. Can the settlement be made by delivery? If not, why not and how would it be done?

If the underlying asset is the IBEX 35 index (or any other stock market index), the settlement cannot be made by delivery as it is not possible to deliver X units of an index (nor is it efficient or simple to deliver X times a portfolio containing the exact same share composition as the index).

In this and in similar cases, delivery will always be made in cash by paying or subtracting the corresponding amounts from each counterparty.

9) What is the role of the clearing house in this process and what are the advantages these institutions offer investors compared to going to over-the-counter markets?

The clearing house acts as an intermediary between the counterparties to a futures contract. It may specify any features of the futures contract it considers acceptable to the parties, or establish which party may set any conditions not previously established by the clearing house.

This procedure may be attractive for investors, firstly because of the standardisation of the contracts and the increased liquidity they offer. Secondly, the clearing house will act as a counterparty if the paying party to the contract is unable to meet its commitments (i.e., defaults), so investors will see the potential credit risk of the contract practically reduced.

Finally, the clearing house also offers investors anonymity: investors will never know who the counterparty they are dealing with is, nor will the counterparty know who they are dealing with.

PRACTICE UNIT 3

1) Can a hedger eliminate market risk? In which cases would this risk be completely eliminated and in which cases would it be partially eliminated? Indicate and give reasons for other types of risk that may still affect you.

A hedger will be able to completely eliminate market risk only when they can execute a perfect hedge. This will be the case only if they have futures available on the same underlying asset they wish to hedge that expire at the time the trade is to be made. If they do not, they may suffer basis risk, so the risk generated by price movements due to market movements would not be fully hedged.

When undertaking hedging strategies, the investor may also face credit risk (default of the counterparty in the derivative), operational risk (error in the design of the strategy), or another type of risk.

2) Suppose a speculator wishes to sell an underlying asset in the future but intends to do so using a strategy similar to a short hedge. Assuming the hedge is perfect, what will have to happen for the speculator to make a profit?

If the 'hedging' strategy is executed correctly, the speculator will expect the price of the underlying asset to be as low as possible so that they receive a significantly higher net price than they would receive in the market.

In this sense, they might even consider buying the underlying asset on the day of maturity at a lower price than they will subsequently receive for the hedged sale.

3) [Hull (2013), 3.16] The standard deviation of monthly changes in the spot price of live cattle is 1.2 (in cents per pound). The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. Obtain the covariance and beta coefficient between the changes in the futures price and the underlying asset price. Also obtain the hedge ratio using the three different methods.

The covariance and beta coefficient between changes in the futures price and the underlying asset price are obtained as follows:

$$\rho_{\Delta F,\Delta S} = \frac{\sigma_{\Delta F,\Delta S}}{\sigma_{\Delta F} * \sigma_{\Delta S}} => \sigma_{\Delta F,\Delta S} = \rho_{\Delta F,\Delta S} * \sigma_{\Delta F} * \sigma_{\Delta S} = 0{,}7 * 1{,}2 * 1{,}4 = 1{,}176$$

$$\beta_{\Delta F,\Delta S} = \rho_{\Delta F,\Delta S} * \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} => 0,7 * \frac{1,2}{1,4} = 0,6$$

The minimum variance hedge ratio can be obtained in any of the following ways, all of which provide the same result:

$$h = \frac{\sigma_{\Delta F,\Delta S}}{\sigma_{\Delta F}^2} = \rho_{\Delta F,\Delta S} * \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = \beta_{\Delta F,\Delta S} = \frac{1,176}{1,4^2} = 0,7 * \frac{1,2}{1,4} = 0,6$$

It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge their risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

In this case, the producer will have to take a long position in [(200,000 / 40,000) * 0.6=] 3 futures contracts over pounds of cattle maturing in December and close them on November 15 when the purchase is made.

4) [Hull (2013), 3.29] It is now October 2013. A company anticipates that it will purchase one million pounds of copper in each of February 2014, August 2014, February 2015, and August 2015. The company has decided to use the futures contracts traded in the COMEX division of the CME Group to hedge its risk. One contract is for the delivery of 25,000 pounds of copper. The company's policy is to hedge 80% of its exposure. Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the company's needs. Devise a hedging strategy for the company.

Assume the market prices (in cents per pound) today and at future dates are as follows. What is the impact of the strategy you propose on the price the company pays for copper?

Date	10/13	02/14	08/14	02/15	08/15
S_t	372.00	369.00	365.00	377.00	388.00
$F_{03/14}$	372.30	369.10			
$F_{09/14}$	372.80	370.20	364.80		
$F_{03/15}$		370.70	364.30	376.70	
$F_{09/15}$			364.20	376.50	388.20

For each purchase, the company will have to devise a strategy to hedge [0.8 * 1,000,000 =] 800,000 pounds of copper, for which it will have to open a long position in [800,000 / 25,000 =] 32 futures. For this purpose, the process to be carried out is as follows:

- **<u>October 2013:</u>**
  - Open a <u>long position</u> in 32 $F_{3,14}$.
  - Open a <u>long position</u> in 96 $F_{9,14}$.
- **<u>February 2014:</u>**
  - <u>Close</u> the position in the 32 $F_{3,14}$.
  - <u>Buy</u> 1,000,000 pounds of copper at 369.00 /pound.
    - <u>Payment:</u>
      - $1{,}000{,}000 * 369 - (369.10 - 372.30) * 25{,}000 * 32 = 1{,}000{,}000 * \mathbf{371.56} = 371{,}560{,}000$
- **<u>August 2014:</u>**
  - <u>Close</u> the position in 96 $F_{9,14}$.
  - <u>Buy</u> 1,000,000 pounds of copper at 365.00 /pound.
    - <u>Payment:</u>
      - $1{,}000{,}000 * 365 - (364.80 - 372.80) * 25{,}000 * 32 = 1{,}000{,}000 * \mathbf{371.40} = 371{,}400{,}000$
  - Open a <u>long position</u> in 32 $F_{3,15}$.
  - Open a <u>long position</u> in 32 $F_{9,15}$.
    - <u>Result:</u>
      - $(364.80 - 372.80) * 25{,}000 * 64 * \frac{32}{64} = \mathbf{-8} * 25{,}000 * 64 * \frac{32}{64} = -6{,}400{,}000$
- **<u>February 2015:</u>**
  - <u>Close</u> the position in 32 $F_{3,15}$.
  - <u>Buy</u> 1,000,000 pounds of copper at 377.00 /pound.
    - <u>Payment:</u>
      - $1{,}000.000 * 377 - (376.70 - 364.30) * 25{,}000 * 32 + 6{,}400{,}000 = 1{,}000{,}000 * \mathbf{373.48} = 373{,}480{,}000$
- **<u>August 2015:</u>**
  - <u>Close</u> the position in 32 $F_{9,15}$.
  - <u>Buy</u> 1,000,000 pounds of copper at 388.00 /pound.
    - <u>Payment:</u>
      - $1{,}000{,}000 * 388 - (388.20 - 364.20) * 25{,}000 * 32 + 6{,}400{,}000 = 1{,}000{,}000 * \mathbf{375.20} = 375{,}200{,}000$

This strategy is successful as it manages to reduce the variability in the effective price paid, since it will be in the [371.40, 375.20] range, while the price in the market has moved to the [365, 388] range.

5) Demonstrate that the number of contracts obtained by using the minimum variance hedge ratio effectively minimises the variance of the hedged position.

The change in the position covered can be developed as follows:

$$\Delta Y = Q * \Delta S + c * m * \Delta F$$

The variance of this change is obtained from:

$$Var(\Delta y) = Var(Q * \Delta S + c * m * \Delta F)$$

or

$$Var(\Delta y) = Q^2 * Var(\Delta S) + c^2 * m^2 * Var(\Delta F) + 2 * Q * c * m * Cov(\Delta S, \Delta F)$$

By obtaining the first derivative with respect to c and equalling zero, we obtain the number of contracts, which will depend on the minimum variance coverage ratio.

$$\frac{\delta Var(y)}{\delta c} = 2 * c * m^2 * Var(\Delta F) + 2 * Q * m * Cov(\Delta S, \Delta F) = 0$$

$$c^* = -\frac{Q}{m} * \frac{Cov(\Delta S, \Delta F)}{Var(\Delta F)}$$

To check that this critical point is indeed a minimum, we derive once more and check the sign:

$$\frac{\delta^2 Var(y)}{\delta c^2} = 2 * m^2 * Var(\Delta F) > 0$$

Since the sign of this result will always be positive, we can certify that the critical point is a minimum.

6) Demonstrate that the value of a forward contract at its beginning is always equal to 0:

The value of a long position in a forward contract is:

$$f = \left(F_{0,T} - K\right) * e^{-r*T}$$

where F is the price of the future to be valued, and K is the price of an analogous future traded today. In this case, the future traded today will be the same as the one to be valued. Therefore:

$$f = \left(F_{0,T} - F_{0,T}\right) * e^{-r*T} = 0$$

This can be demonstrated in an analogous way for a short position in a forward contract.

7) Consider a forward contract to purchase a coupon-bearing bond whose current price is $1,000. Suppose the forward contract matures in 12 months. Suppose that a coupon payment of $100 is expected after 4 months, and a coupon payment of $250 is expected after 8 months (assume that the 4-, 8- and 9-month risk-free interest rates continuously compounded are, respectively, 3%, 6.1% and 7.2% per annum).

HINT: Use Excel to perform the calculations.

a) Calculate the price of the future so that there are no arbitrage opportunities.

$\mathbf{100} * \boldsymbol{e}^{-\mathbf{0.03}*\frac{\mathbf{4}}{\mathbf{12}}} = \$\mathbf{99}$	$\mathbf{250} * \boldsymbol{e}^{-\mathbf{0.061}*\frac{\mathbf{8}}{\mathbf{12}}} = \$\mathbf{240}$
$\boldsymbol{F}_{\mathbf{0}} = (\mathbf{1,000} - \mathbf{99} - \mathbf{240}) * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \mathbf{661} * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \$\mathbf{710}$	

b) If the price of the future is $750, what strategy should you follow to make a profit?

	t = 0	t = 4	t = 8	t = 12
Borrow 4M	$ 99	- $ 100	$ 0	$ 0
Borrow 8M	$ 240	$ 0	- $ 250	$ 0
Borrow 12M	$ 661	$ 0	$ 0	- $ 710
Bond/Future	- $ 1,000	$ 100	$ 250	$ 750
Total	$ 0	$ 0	$ 0	$ 40

c) Repeat the previous section while assuming that the price of the future is now $600.

	t = 0	t = 4	t = 8	t = 12
Lend 4M	- $ 99	$ 100	$ 0	$ 0
Lend 8M	- $ 240	$ 0	$ 250	$ 0
Lend 12M	- $ 661	$ 0	$ 0	$ 710
Bond/Future	$ 1,000	- $ 100	- $ 250	- $ 600
Total	$ 0	$ 0	$ 0	$ 110

d) What will the non-arbitration price of the future be if the first coupon pays $150?

$\mathbf{150} * \boldsymbol{e}^{-\mathbf{0.03}*\frac{\mathbf{4}}{\mathbf{12}}} = \$\mathbf{148.51}$	$\mathbf{250} * \boldsymbol{e}^{-\mathbf{0.061}*\frac{\mathbf{8}}{\mathbf{12}}} = \$\mathbf{240}$

$$\boldsymbol{F_0} = (\mathbf{1,000} - \mathbf{148.51} - \mathbf{240}) * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \mathbf{611,49} * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \$\mathbf{656.83}$$

e) What will the non-arbitration price of the future be if the 8-month risk-free interest rate is 6.5% (considering the initial conditions)?

$\mathbf{100} * \boldsymbol{e}^{-\mathbf{0.03}*\frac{\mathbf{4}}{\mathbf{12}}} = \$\mathbf{99}$	$\mathbf{250} * \boldsymbol{e}^{-\mathbf{0.065}*\frac{\mathbf{8}}{\mathbf{12}}} = \$\mathbf{239.40}$

$$\boldsymbol{F_0} = (\mathbf{1,000} - \mathbf{99} - \mathbf{239.40}) * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \mathbf{661.60} * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \$\mathbf{710.65}$$

f) What will the non-arbitration price of the future be if the second coupon is paid in 10 months (considering the initial conditions)?

$\mathbf{100} * \boldsymbol{e}^{-\mathbf{0.03}*\frac{\mathbf{4}}{\mathbf{12}}} = \$\mathbf{99}$	$\mathbf{250} * \boldsymbol{e}^{-\mathbf{0.061}*\frac{\mathbf{10}}{\mathbf{12}}} = \$\mathbf{237.61}$

$$\boldsymbol{F_0} = (\mathbf{1,000} - \mathbf{99} - \mathbf{237.61}) * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \mathbf{663.39} * \boldsymbol{e}^{\mathbf{0.072}*\frac{\mathbf{12}}{\mathbf{12}}} = \$\mathbf{712.61}$$

PRACTICE UNIT 4

1) Explain the main differences between futures contracts and options, and how these differences are related to the prices of each of these assets.

Futures contracts oblige both parties to buy/sell, whereas options give the long position the right to buy or sell. However, the short position in an option still has the obligation to buy or sell if the buyer decides to exercise the option.

In futures contracts the obligation for both parties means that the price of the contract is zero. In options, the long party’s right to exercise is offset by the payment of a premium to the short party.

2) Explain what the investor expects to happen in the market when they take each of the four possible positions with options (buy call, sell call, buy put, sell put). Also explain what the potential profits and losses are for each of them.

When adopting a long position in a call, the investor expects the price of the stock to rise. While its potential profits are unlimited (they increase with the stock price), its maximum losses are restricted to the price of the call.

When adopting a long position in a put, the investor expects the price of the stock to decrease. While its potential benefits are high (they increase when the stock price decreases, having the barrier of a stock price equals to 0), its maximum losses are restricted to the price of the put.

When adopting a short position in a call, the investor expects the price of the stock to decrease. While its potential benefits are restricted to the price they received, its potential losses are unlimited (they increase with the stock price).

When adopting a short position in a put, the investor expects the price of the stock to rise. While its potential benefits are restricted to the price they received, its potential loses are unlimited (they increase when the stock price decreases, having the barrier of a stock price equals to zero).

3) Explain why the price of an American option is always higher that the price of a European option.

An American option can be exercised at any time until maturity, whereas a European option can be exercised only when maturity is reached. Since the former provides more opportunities to exercise the option, it must be more expensive than the latter.

4) Why is only the investor who writes the option required to maintain funds in a margin account?

The investor who buys an option can only lose the amount of money they have already paid for the option, so there is no risk of default. On the other hand, the investor who sells the option has a higher risk. This is because if the counterparty exercises the option, they will have to pay an additional amount of money that depends on market conditions. Since there is a credit risk for the short party, this investor is required to maintain funds in a margin account.

5) Explain how an increase in the risk-free interest rate affects the price of a European call and put.

An increase in the risk-free interest rate reduces the present value of future payments. This implies that the value now of what I will have to pay in the future is lower (as is the value of what I will receive in the future).

A call option implies having to pay an amount of money (strike) in the future, while a put option implies receiving an amount of money in the future. Due to the increase in risk-free interest rates, the present value of this amount of money in the future is lower. With a call option, the value of my future payment is now lower, so the price of the option must increase. With a put option, the value of the money I will receive is now lower, so the price of the option must decrease.

6) Imagine a European Call option with the following characteristics:

Maturity (months)	**12**	**Risk-free interest rate (r)**	**10%**
Spot price (S_0)	**$ 100**	**Strike (K)**	**$ 50**

For this exercise, we will consider that an arbitrage opportunity exists when the profit obtained today is equal to zero (self-financed), and the profit obtained in any possible future situation may be positive or equal to zero. Are there arbitrage opportunities in the following cases?

$$c \geq S_0 - K * e^{-r*T} => c \geq 100 - 50 * e^{-0,1*\frac{12}{12}} => c \geq 55$$

a) The price of the call option (c) is equal to $ 50.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Short the stock	+ $ 100	- S_T	- S_T
Buy the call	- $ 50	$ 0	S_T - 50
Invest money	- $ 50	+ $ 55	+ $ 55
Total	$ 0	55 - S_T	+ $ 5

The minimum profit from this strategy would be $ 5, so we can affirm that there is an opportunity of arbitrage.

b) The price of the call option (c) is equal to $ 55.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Short the stock	+ $ 100	- S_T	- S_T
Buy the call	- $ 55	$ 0	S_T - 50
Invest money	- $ 45	+ $ 50	+ $ 50
Total	$ 0	50 - S_T	+ $ 0

The minimum profit from this strategy would be $ 0 but as there are no possible losses, we can affirm that there is an opportunity of arbitrage.

c) The price of the call option (c) is equal to $ 60.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Short the stock	+ $ 100	- S_T	- S_T
Buy the call	- $ 60	$ 0	S_T - 50
Invest money	- $ 40	+ $ 44	+ $ 44
Total	$ 0	44 - S_T	- $ 6

There are possible losses on this strategy, so there are no opportunities of arbitrage.

7) Imagine a European Put option with the following characteristics:

Maturity (months)	**12**	**Risk-free interest rate (r)**	**10%**
Spot price (S_0)	**$ 25**	**Strike (K)**	**$ 50**

For this exercise, we will consider that an arbitrage opportunity exists when the profit obtained today is equal to zero (self-financed) and the profit obtained in any possible future situation may be positive or equal to zero. Are there arbitrage opportunities in the following cases?

$$\boldsymbol{p \geq K * e^{-r*T} - S_0 => p \geq 50 * e^{-0,1*\frac{12}{12}} - 25 => p \geq 20}$$

a) The price of the put option (c) is equal to $ 15.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Buy the stock	- $ 25	S_T	S_T
Buy the put	- $15	50 - S_T	$ 0
Borrow money	+ $ 40	- $ 44	- $ 44
Total	$ 0	+ $ 6	S_T - 44

The minimum profit from this strategy would be $ 6, so we can affirm that there is an opportunity of arbitrage.

b) The price of the put option (c) is equal to $ 20.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Buy the stock	- $ 25	S_T	S_T
Buy the put	- $20	50 - S_T	$ 0
Borrow money	+ $ 44	- $ 50	- $ 50
Total	$ 0	$ 0	S_T - 50

The minimum profit from this strategy would be $ 0 but as there are no possible losses, we can affirm that there is an opportunity of arbitrage.

c) The price of the put option (c) is equal to $ 25.

Strategy	t_0	T	
		S_T <= 50	S_T > 50
Buy the stock	- $ 25	S_T	S_T
Buy the put	- $25	50 - S_T	$ 0
Borrow money	+ $ 50	- $ 55	- $ 55
Total	$ 0	- $ 5	S_T - 55

There are possible losses on this strategy, so there are no opportunities of arbitrage.

8) Imagine a European Call option and a European Put option with the following characteristics:

Maturity (months)	**12**	**Risk-free interest rate (r)**	**10%**
Spot price (S_0)	**$ 100**	**Strike (K)**	**$ 50**
Call price (c)	**$ 70**	**Put Price (p)**	**$ 15**

For this exercise, we will consider that an arbitrage opportunity exists when the profit obtained today is equal to zero (self-financed) and the profit obtained in any possible future situation may be positive or equal to zero (you should round up the results to avoid decimals).

a) Is the put-call parity met in this example?

$$c + K * e^{-r*T} = p + S_0$$

$$70 + 50 * e^{-0,1*\frac{12}{12}} = 15 + 100 => 115 = 115$$

b) Imagine that the spot price is reduced to \$80. Would there be any arbitrage opportunities? Design the strategy that would allow this strategy to be exploited. What price would the call have to be in order to return to equilibrium? And the put?

$$\mathbf{70 + 50 * e^{-0.1*\frac{12}{12}} = 15 + 80 => 115 \geq 95}$$

Strategy	t_0	T	
		S_T >= 50	S_T < 50
Sell the call	+ \$ 70	- (S_T – 50)	\$ 0
Sell the bond	+ \$ 45	- \$ 50	- \$ 50
Buy the put	- \$ 15	\$ 0	50 - S_T
Buy the stock	- \$ 80	S_T	S_T
Total	+ \$ 20	\$ 0	\$ 0

$$c^* = p + S_0 - K * e^{-r*T} = 15 + 80 - 50 * e^{-0.1*\frac{12}{12}} = 45$$
$$p^* = c + K * e^{-r*T} - S_0 = 70 + 50 * e^{-0.1*\frac{12}{12}} - 80 = 35$$

c) Imagine that the risk-free interest rate increases to 15% (with the initial conditions). Would there be any arbitrage opportunities? Design the strategy that would allow this strategy to be exploited. What price would the call have to be in order to return to equilibrium? And the put?

$$\mathbf{70 + 50 * e^{-0.15*\frac{12}{12}} = 15 + 100 => 113 \leq 115}$$

Strategy	t_0	T	
		S_T >= 50	S_T < 50
Buy the call	- \$ 70	S_T – 50	\$ 0
Buy the bond	- \$ 43	\$ 50	\$ 50
Sell the put	\$ 15	\$ 0	- (50 - S_T)x
Sell the stock	\$ 100	- S_T	- S_T
Total	+ \$ 2	\$ 0	\$ 0

$$c^* = p + S_0 - K * e^{-r*T} = 15 + 100 - 50 * e^{-0.15*\frac{12}{12}} = 72$$
$$p^* = c + K * e^{-r*T} - S_0 = 70 + 50 * e^{-0.15*\frac{12}{12}} - 100 = 13$$

9) Black and Scholes (1973) establish that the price of a European call option (c) can be defined (under certain conditions) using the following formula (simplified):

$$c = S * N(d_1) - K * e^{-r*T} * N(d_2)$$

where S is the spot price, K is the strike, T is the time to maturity, r is the risk-free interest rate and $N(\cdot)$ is the Distribution Function of a standard normal variable.

Use the above formula to obtain the price of a European put option (p) with the same characteristics [HINT: Treat $N(d_1)$ and $N(d_2)$ as two unknown numbers and bear in mind that $[N(d_1) - 1] = -[1 - N(d_1)] = -N(-d_1)$ and $[1 - N(d_2)] = N(-d_2)$].

We use the put-call parity to obtain an equation with both c and p:

$$c + k * e^{-r*T} = p + S => p = c + k * e^{-r*T} - S$$

We apply the BS formula:

$$p = S * N(d_1) - K * e^{-r*T} * N(d_2) + k * e^{-r*T} - S$$

We reorder terms and apply the results of the hints

$$p = S[N(d_1) - 1] + K * e^{-r*T} * [1 - N(d_2)]$$

$$p = K * e^{-r*T} * N(-d_2) - S * N(-d_1)$$

PRACTICE UNIT 5

1) Draw a diagram showing the variation in an investor's profit and loss depending on the terminal stock price for a portfolio consisting of:

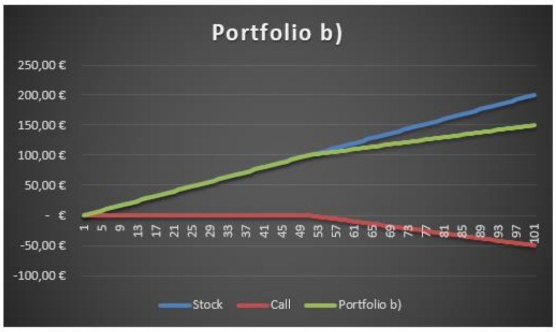
a) One share and a short position in one call option.

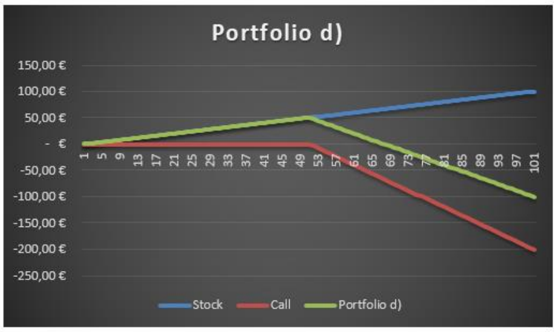
b) Two shares and a short position in one call option.

c) One share and a short position in two call options.

d) One share and a short position in four call options.

Tip: Use Excel and assume a Strike equal to 50 and a range of values for the spot price between 1 and 100.





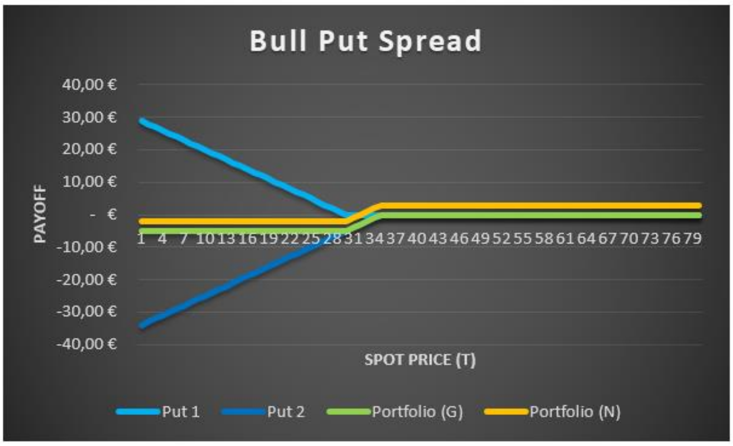
2) An investor believes there will be a big change in a stock price but is uncertain about the direction. Identify three strategies the investor can follow and explain the differences between them.

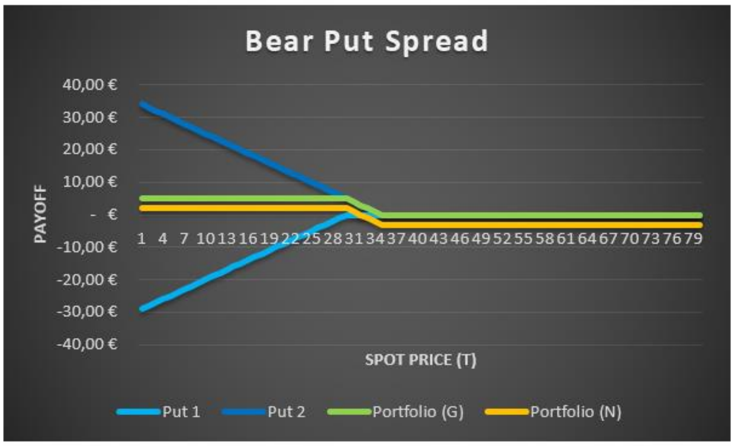
The possible strategies are: **Strangle** / **Straddle** / **Reverse Butterfly Spread**. All these strategies provide positive profits when there are large movements in the stock price. A strangle is less expensive than a straddle but it requires a greater movement in the stock price to provide a positive profit. With strangles and straddles, the profit increases as the size of the movement in the stock price increases. With reverse butterfly spread, on the other hand, the maximum potential profit does not depend on the size of the movement in the stock price.

3) Imagine that put options on a stock with strike prices $30 and $35 cost $4 and $7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

Strategy	Product	t_0	T		
			$S_T \leq 30$	$30 < S_T < 35$	$S_T \geq 35$
Bull Put Spread	**Long Put**	- $ 4	30 - S_T	$ 0	$ 0
	Short Put	$ 7	- (35 - S_T)	- (35 - S_T)	$ 0
	Total	**$ 3**	**- $ 5**	**- (35 - S_T)**	**$ 0**

Strategy	Product	t_0	T		
			$S_T \leq 30$	$30 < S_T < 35$	$S_T \geq 35$
Bear Put Spread	**Short Put**	$ 4	- (30 - S_T)	$ 0	$ 0
	Long Put	- $ 7	35 - S_T	35 - S_T	$ 0
	Total	**- $ 3**	**$ 5**	**35 - S_T**	**$ 0**



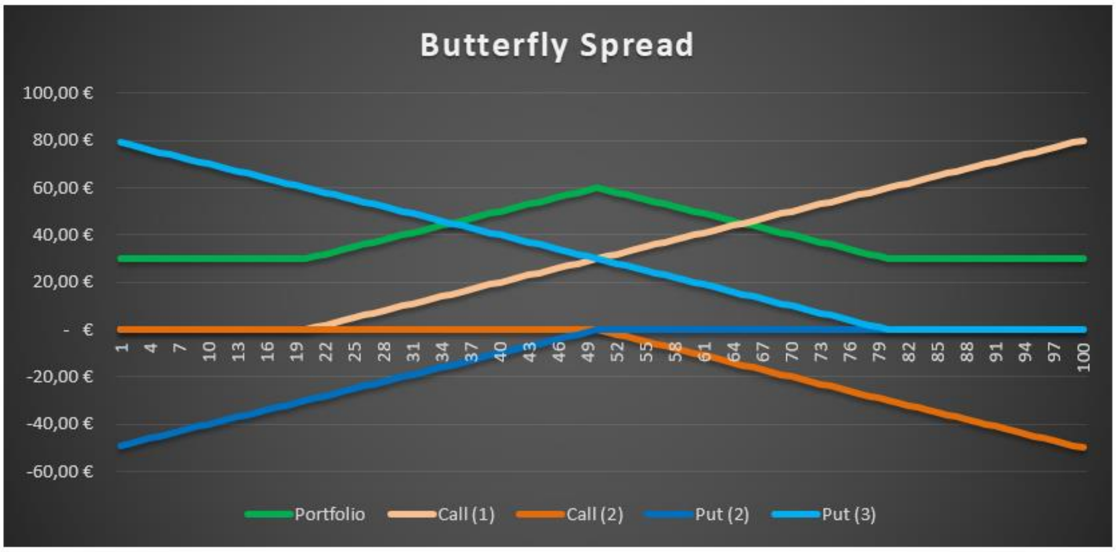


4) What trading position is created from a long strangle and a short straddle when both have the same time to maturity? Assume that the strike price in the straddle is halfway between the two strike prices in the strangle. Use a payoff table to complement your answer.

$$\frac{K_1 + K_3}{2} = K_2 => K_1 + K_3 = 2K_2 => K_2 - K_1 = 0,5 * K_3$$

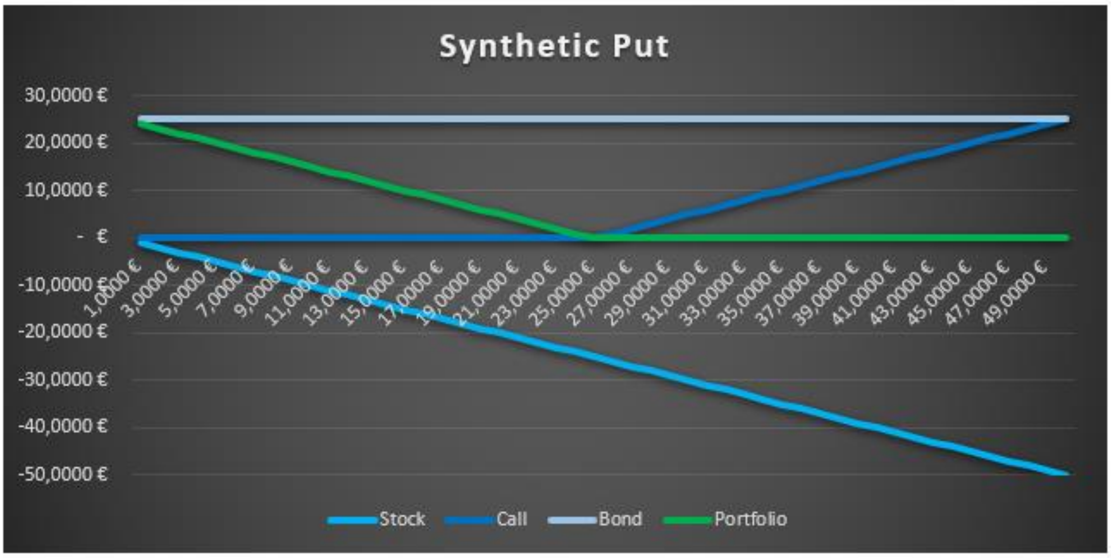
$$K_3 - K_2 = 2K_2 - \ K_1 - K_2 = K_2 - K_1 = 0,5 * K_3$$

Combination	Strategy	Product	t_0	T			
				$S_T \leq K_1$	$K_1 < S_T \leq K_2$	$K_2 < S_T < K_3$	$S_T \geq K_1$
Butterfly Spread	Long Strangle	Long Call$_3$	$- c_3$	0	0	0	$S_T - K_3$
		Long Put$_1$	$- p_1$	$K_1 - S_T$	0	0	0
	Short Straddle	Short Call$_2$	c_2	0	0	$-(S_T - K_2)$	$-(S_T - K_2)$
		Short Put$_2$	p_2	$-(K_2 - S_T)$	$-(K_2 - S_T)$	0	0
	Total		$c_2 + p_2 - c_3 - p_1$	$K_1 - K_2$	$S_T - K_2$	$K_2 - S_T$	$K_2 - K_3$



5) Build a synthetic put. To do so, make a combination of different assets so that its payoff is replicated at all possible times and in all possible scenarios. Construct a table in which you show the payoff of the put on one hand and the payoff of each instrument on the other, and demonstrate that both results are always the same (HINT: Remember the put-call parity).

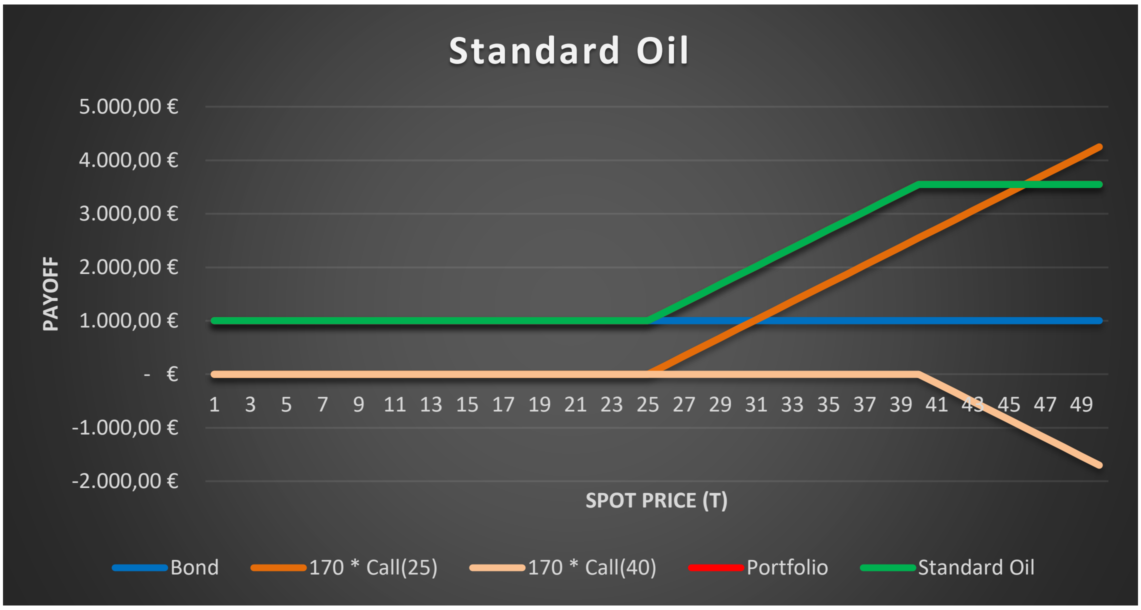
Product	**t_o**	**T**	
		$S_T \leq K$	**$S_T > K$**
Long Put	- p	K - S_T	0
Long Call	- c	0	S_T - K
Short Stock	S_0	- S_T	- S_T
Long Bond	- $K * e^{-r*T}$	K	K
Total	**S_0 - c - $\boldsymbol{K * e^{-r*T}}$**	**K - S_T**	**0**



6) (Exercise 1.31 Hull (2000)) In 1986, Standard Oil issued bonds promising to pay $1,000 at maturity plus an additional amount based on the oil price on that date. The additional amount was set as the product of 170 and the excess (if any) of the price of a barrel of oil over USD 25 on the maturity date. The maximum additional amount was set at USD 2,550 (which corresponded to a price of USD 40 per barrel). Demonstrate that the bond described above is the combination of a normal bond, a long position in call options on oil with an exercise price of USD 25 (per barrel), and a short position in call options on oil with an exercise price of USD 40.

Product	$S_T \leq 25$	$25 < S_T < 40$	$S_T \geq 40$
Standard Oil	**$ 1.000**	**1.000 + 170·(S_T – 25)**	**$ 3.550**
Long Bond	$ 1.000	$ 1.000	$ 1.000
170 * Long Call$_{25}$	$ 0	170·(S_T – 25)	170·(S_T – 25)
170 * Short Call$_{40}$	$ 0	$ 0	- 170·(S_T – 40)
Total	**$ 1.000**	**1.000 + 170·(S_T – 25)**	**$ 3.550**

$$\mathbf{1.000 + 170(S_T - 25) - 170(S_T - 40) = 1.000 + 170(40 - 25)}$$
$$\mathbf{= 1.000 + 2.550 = 3.550}$$



7) Catastrophe bonds, or cat bonds, promise a high return to investors in exchange for the possibility of losing much of their principal or interest, or both, in the event of certain natural disasters. When a disaster occurs and its cost (losses on insured assets) exceeds a certain quantity in monetary units, investors lose substantial portions of their investment and the issuer of the bonds (an insurance company) uses the cash saved to meet the claims payments.

Consider the following catastrophe bonds issued by an insurance company. These are non-coupon bonds, which promise to pay \$50 million after one year, less an amount dependent on Florida's hurricane claims payments. The amount to be subtracted is equal to the excess (if any) in claims payments over \$1 billion. In addition, there is a maximum amount that can be subtracted: \$50 million (which corresponds to a claims payment level of \$1.05 billion by the insurance company).

Demonstrate that the promised payment pattern can be synthetically created at the maturity date of the bonds through risk-free assets and a combination of European call options on claims payments.

Product	**$S_T \leq 1,000$**	**$1.000 < S_T < 1,050$**	**$S_T \geq 1,000$**
Cat Bond	**\$ 50**	**50 - (S_T – 1,000)**	**\$ 0**
50 * Long Bond	\$ 50	\$ 50	\$ 50
Short Call$_{1.000}$	\$ 0	- (S_T – 1,000)	- (S_T – 1,000)
Long Call$_{1.050}$	\$ 0	\$ 0	S_T – 1,050
Total	**\$ 50**	**50 - (S_T – 1,000)**	**\$ 0**

$$\mathbf{50 + S_T - 1,050 - (S_T - 1,000) = 50 + 1,000 - 1,050 = 1,050 - 1,050 = 0}$$