Technical memorandum

MODE CLASSIFICATION IN CYLINDRICAL SURFACE WAVEGUIDES

Indexing terms: Waveguides and waveguide components

Abstract: A new method to analyse the TM-TE field structure of hybrid modes in cylindrical surface waveguides is proposed. This method, as well as others previously suggested, is applied to a dielectric-coated wire with an intervening airgap. Our results reveal that they cannot be classified into quasi-TM and quasi-TE modes. However, a scheme of mode designation is proposed, based more on mathematical properties than on the field structure.

List of principal symbols

(ρ, ϕ, z)	= cylindrical co-ordinates
a, b, c	= radii of waveguide
ε_0, μ_0	= vacuum permittivity and permeability
ε_r, μ_r	= relative permittivity and permeability in the dielectric medium
$\mathbf{E}, \mathbf{H}$	= electric and magnetic fields
j	$= \sqrt{-1}$
Z_0	= intrinsic impedance of vacuum
R	$= jZ_0(H_z/E_z)$, when $\rho > c$
ω, k_0	= angular frequency and wave number in vacuum
f_0, f_{0c}	= normalised frequency $f_0 = k_0 c\sqrt{(\varepsilon_r \mu_r - 1)}$, and its cutoff value
β	= imaginary part of axial propagation factor in waveguide
h	= radial propagation factor in air, $h^2 = \beta^2 - k_0^2$
k	= radial propagation factor in dielectric medium, $k^2 = k_0^2 \varepsilon_r \mu_r - \beta^2$
k_c, k_f	= cutoff and far-from-cutoff values of k
S_z	= axial component of complex Poynting vector
∇_t	= transverse part of gradient operator
P	= total power flow
P_{TM}, P_{TE}, P_{HB}	= TM, TE and hybrid contributions to P

Introduction

It has been proved that the modes of inhomogeneous waveguides are hybrid modes, except for some particular solutions [1]. Surface waveguides can support different types of mode [2]. This paper deals with lossless surface waveguides and we are only concerned with guided surface waves [3], $\beta \leqslant k_0$ and $h \geqslant 0$. These nonradiating modes of cylindrical surface waveguides are usually classified into TM, TE, EH and HE modes, but the TM and TE field structures are possible only for circularly symmetrical modes. Therefore, most of them exhibit hybrid field structures.

The standard classification of hybrid surface-waveguide modes into EH (quasi-TM) and HE (quasi-TE) modes is not properly established. Inhomogeneous closed waveguide modes always exhibit a TM or TE field structure at cutoff [1], and therefore they can be classified into EH and HE modes according with their structure near cutoff, but the hybrid modes of cylindrical surface waveguides exhibit such hybrid structure even at cutoff [4]. Another standard method to classify the hybrid modes of inhomogeneous closed waveguides consists in working out their asymptotic TM or TE structure when the permittivities of the internal dielectrics tend to ε_0 [5], but this method cannot be applied to a surface waveguide.

The different designation that is given to the fundamental hybrid mode of the Goubau line is a proof of the difficulties in classifying the hybrid modes in cylindrical surface waveguides. This mode is called EH_{10} by some authors [6, 7, 8] and HE_{11} by others [9, 10]. The hybrid modes of a dielectric rod are also classified into EH and HE modes, and some authors state that this classification is based on the quasi-TM or quasi-TE field structure of each mode [11] but provide no analysis of the fields, and other authors state that such classification is arbitrary [12].

The classification of hybrid modes in cylindrical dielectric waveguides has been investigated by a number of researchers. The schemes for the classification are mainly based on mathematical properties of the characteristic equation [13, 14] and the value of some normalised ratio H_z/E_z [12].

In this paper, we discuss different methods to analyse the TM and TE contributions to the field structure of hybrid modes. These methods are applied to a dielectric-coated wire with an intervening airgap (Fig. 1), and a

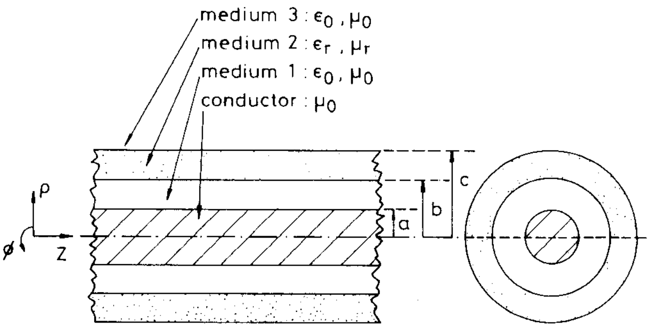


Fig. 1 *Geometry of cylindrical surface waveguide*

nomenclature is proposed for the whole spectrum of nonradiating modes of this surface waveguide. The discussion is carried out using a provisional nomenclature, where each hybrid mode (HM) is referred to with two subscripts. The first subscript is the integer n that determines the angular dependence of the fields through the function $\exp(jn\phi)$. When the solutions for a given n are arranged in sequence from those with large h values to those with small h values, the second subscript corresponds to the integer which specifies the position in the sequence, starting from 1 for the first solution. If $n = 0$, the standard TM-TE nomenclature is used. It has been shown that the surface waveguide under investigation exhibits two modes without cutoff frequency [4], these are named TM_{00} and HM_{11} with this provisional nomenclature.

Axial components of fields and parameter *R*

Hybrid modes are solutions of the wave equation that exhibit both electric and magnetic axial components that are nonzero. It has been suggested that the normalized relation between the axial components of the fields can be useful to analyse the TM and TE contributions to the field structure of a given hybrid mode [7, 10].

Applying a surface impedance dyadic method to the surface waveguide under investigation [4], the fields and the characteristic parameters of the nonradiating modes have been derived. From these results, we have investigated the type of information that the parameter $jZ_0(H_z/E_z)$ provides. Fig. 2 shows this parameter, as a function of the radial distance ρ, for the first two modes of the group of symmetry $n = 1$, including the fundamental mode HM_{11}. The main conclusion from Fig. 2 that we want to point out, is the fact that the normalised relation $jZ_0(H_z/E_z)$ depends strongly on the particular point where it is evaluated. As a consequence, we can include that $jZ_0(H_z/E_z)$ is not a characteristic parameter of a given mode, and it can produce quite misleading results in the evaluation of the TM and TE contributions to the field structure. The parameter $jZ_0(H_z/E_z)$ is independent of ρ only in the external medium that surrounds the waveguide. This property suggests that the value of this parameter when $\rho > c$ may contain some useful information. We have defined the parameter R as

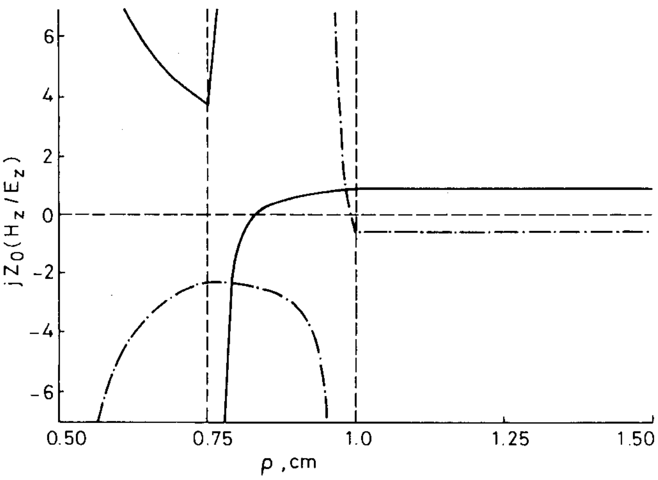


Fig. 2 *$jZ_0(H_z/E_z)$ as a function of ρ*

$a = 0.5$ cm; $b = 0.75$ cm; $c = 1$ cm; $\varepsilon_r = 2.56$; $\mu_r = 1$, radial propagation factor $h = 1\ \mathrm{cm}^{-1}$
—— HM_{11}
-·- HM_{12}

$$R = jZ_0(H_z/E_z)_{\rho > c} \tag{1}$$

It has been found that the asymptotic values of this parameter R at cutoff ($h \to 0$) and far from cutoff ($h \to \infty$) are constant values [4]:

$$\lim_{h\to 0} R = 1 \qquad \lim_{h\to\infty} R = \pm\sqrt{\frac{\varepsilon_r}{\mu_r}} \tag{2}$$

These results confirm that the hybrid structure of these modes is persistent even at cutoff, and show that two different sets of hybrid modes can be distinguished as a function of the sign of their asymptotic R values far from cutoff.

We observe in Fig. 2 that the axial components of the fields exhibit one zero in the dielectric medium. The plots corresponding to HM_{13} and HM_{14} would show that they exhibit two zeros, HM_{15} and HM_{16} exhibit three zeros, and so on. A similar behaviour can be observed when $n > 1$. This shows that each group of symmetry n is a double series of modes which correspond to the two degrees of freedom provided by the two axial components H_z and E_z. When $n = 0$, the two series of modes can be identified as the TM_{0m} and TE_{0m} modes.

It is interesting to prove that each one of these series corresponds to one of the two possible asymptotic values of R far from cutoff. The far-from-cutoff equation that the radial propagation factor k has to satisfy splits into two conditions, and the different asymptotic values of R depend on which condition is satisfied by a particular mode [4]. Table 1 shows the k_f values for the first hybrid modes that belong to the groups of symmetry $n = 1$ and $n = 2$. Such k_f values are alternately the solutions of the two far-from-cutoff conditions. Therefore, each of the modes that defines a pair with the same number of zeros for the axial components H_z and E_z exhibits a different sign for the asymptotic value of R far from cutoff.

Table 1 : k_f values

modes $n = 1$	k_f (1/cm)	R*	modes $n = 2$	k_f (1/cm)	R*
HM_{11}	10.455	+	HM_{21}	10.522	+
HM_{12}	10.719	−	HM_{22}	11.042	−
HM_{13}	20.935	+	HM_{23}	20.969	+
HM_{14}	21.071	−	HM_{24}	21.239	−
HM_{15}	31.410	+	HM_{25}	31.433	+
HM_{16}	31.501	−	HM_{26}	31.614	−

* Asymptotic value of R far from cutoff: $+ = \sqrt{(\varepsilon_r/\mu_r)}$; $- = -\sqrt{(\varepsilon_r/\mu_r)}$
$b = 0.70$ cm, $c = 1$ cm

After this discussion, one can conclude that the relative values of the axial components H_z and E_z, i.e. the normalised relation $jZ_0(H_z/E_z)$, cannot be used to ascertain whether the field structure of a hybrid mode is quasi-TM or quasi-TE. However, it has been shown that some characteristic properties of the modes can be derived from this analysis, specifically the asymptotic values R and the number of zeros of the axial components H_z and E_z in the dielectric medium.

TM and TE contributions to power flow

The real part of the axial component of the complex Poynting vector S_z can be written in terms of E_z and H_z [1]:

$$\mathrm{Re}\,(S_z) = \frac{1}{2g^4}\{\omega\beta(\varepsilon\nabla_t E_z \cdot \nabla_t \mathbf{E}_z^* + \mu\nabla_t H_z \cdot \nabla_t H_z^*) + (\beta^2 + \omega^2\varepsilon\mu)\,\mathrm{Re}\,(\mathbf{u}_z \cdot (\nabla_t E_z \times \nabla_t H_z^*))\} \tag{3}$$

where g is the radial propagation factor, equal to h in the air and to k in the dielectric medium. This expression shows that one can distinguish three terms, when integrating the Poynting vector to calculate the total power flow. The first two terms are proportional to $\nabla_t E_z \cdot \nabla_t E_z^*$ and $\nabla_t H_z \cdot \nabla_t H_z^*$, and their integrals over a cross-section can be regarded as the TM and TE contributions to the power flow (P_{TM}, P_{TE}). The third term, proportional to $\mathrm{Re}\,(\mathbf{u}_z \cdot (\nabla_t E_z \times \nabla_t H_z^*))$, is a mixed term and it is present owing to the hybrid nature of these modes. The relative values of P_{TM} and P_{TE} with respect to the total power flow can be used to evaluate the TM and TE contributions to the field structure of a hybrid mode. We proposed this analysis as a new method to investigate whether a hybrid mode can be classified into quasi-TM or quasi-TE modes. This method has the advantage that it provides an average evaluation of the TM and TE contributions, instead of the local information provided by the parameter $jZ_0(H_z/E_z)$.

It has been found that the relative values of P_{TM}, P_{TE} and P_{HB} depend strongly on the geometry of the waveguide. This is illustrated in Fig. 3, where these parameters

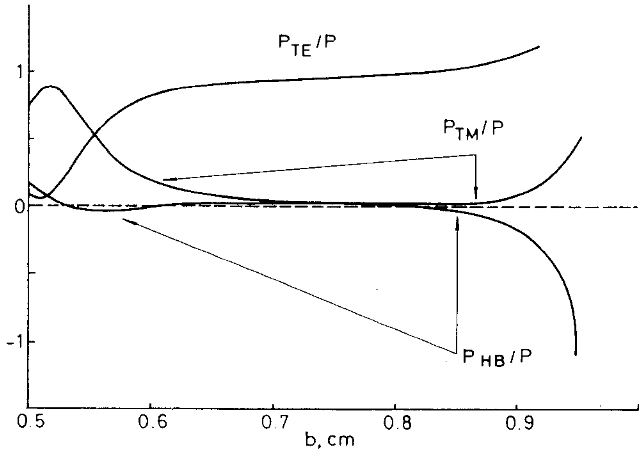


Fig. 3 *Relative P_{TM}, P_{TE} and P_{HB} contributions for HM_{11} mode*
$a = 0.5$ cm; $c = 1$ cm; $\varepsilon_r = 2.56$; $\mu_r = 1, f_0 = 10$

are plotted as a function of radius b for the HM_{11} mode. It can also be shown that for a given geometry the relative values of P_{TM}, P_{TE} and P_{HB} also depend on the frequency.

These results demonstrate that the hybrid modes of the surface waveguide under investigation cannot be classified into quasi-TM and quasi-TE modes. We have found it very interesting to relate these transitions of the field structure with qualitative changes of the diagrams of the electric ($\mathbf{E}_t$) and magnetic ($\mathbf{H}_t$) transverse components. Fig. 4 shows that when the field structure changes from quasi-TM to quasi-TE as a function of radius b, the position of the intensity maxima shifts at the same time that the relative values of the transverse components E_ρ and E_ϕ change from $E_\rho \gg E_\phi$ to $E_\phi \gg E_\rho$.

Nomenclature

It has been shown in the previous Section that the hybrid modes of the surface waveguide under investigation cannot be classified into quasi-TM and quasi-TE modes, therefore the nomenclature EH-HE cannot be established as a function of this classification. However, the use of EH and HE to denote the hybrid modes is attractive because they imply the hybrid structure of these modes, and they may designate each of the two series of modes that define a group of angular symmetry n.

We have found it useful to work out the asymptotic values of the cutoff frequencies to decide the designation of each series as EH or HE. The cutoff conditions previously given [4] for the TM_{0m} and TE_{0m} modes, as a function of the radial propagation factor k, can be approximated when $k \to \infty$. Such approximations determine the asymptotic values of the cutoff frequencies for large values:

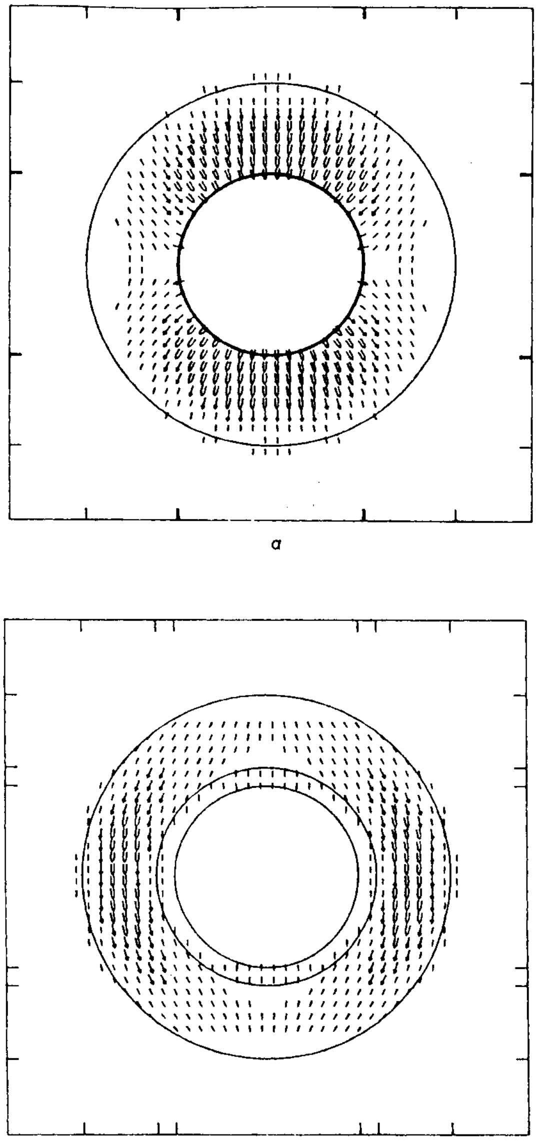
$n = 0$: TM_{0m} modes ($m = 0, 1, 2, \ldots$):

$$f_{0c} \to m \frac{\pi}{1 - b/c}$$

TE_{0m} modes ($m = 1, 2, 3, \ldots$):

$$a \neq b, f_{0c} \to (m-1) \frac{\pi}{1 - b/c} \qquad (4)$$

$$a = b, f_{0c} \to (m - \tfrac{1}{2}) \frac{\pi}{1 - b/c}$$



a

b

Fig. 4 *Transverse components of electric field*
Mode HM_{11}, $a = 0.5$ cm; $c = 1$ cm; $\varepsilon_r = 2.56$; $\mu_r = 1; f_0 = 10$.
Double arrow = amplitude between maximum and 1 dB smaller than maximum
Single arrow = amplitude between 1 dB and 3 dB smaller than maximum
Short single line = amplitude between 3 dB and 10 dB below maximum
a $b = 0.51$ cm
b $b = 0.6$ cm

The cutoff condition for the TE_{0m} modes is a function of radius a [4], and when $k \to \infty$ such a dependence gives rise to two different asymptotic values of f_{0c}, as a function of whether $a \neq b$ or $a = b$. The convergence of f_{0c} towards its asymptotic values can be slow, particularly when a and b are different but their difference is small.

The cutoff conditions of the hybrid modes HM_{nq} [4], $n > 0$, split into two different approximated expressions for large cutoff frequencies. These expressions give rise to different asymptotic values for f_{0c} which can be written as functions of an integer $m = 1, 2, 3, \ldots$:

$$f_{0c} \to (m-1) \frac{\pi}{1 - b/c} \qquad (5a)$$

$$\left.\begin{aligned} a \neq b, f_{0c} &\to (m-1) \frac{\pi}{1 - b/c} \\ a = b, f_{0c} &\to (m - 1/2) \frac{\pi}{1 - b/c} \end{aligned}\right\} \qquad (5b)$$

One of the approximated expressions for large cutoff frequencies exhibits a particular dependence on radius a, which gives rise to two different asymptotic values of f_{0c} as a function of whether $a \neq b$ or $a = b$. This property establishes a straightforward relationship between the hybrid modes HM_{nq} and the symmetric modes TM_{0m} and TE_{0m}, which suggests the designation of the series of hybrid modes that satisfies eqn. 5a as EH, and the other as HE.

Table 2 provides some numerical values of k_c for $n = 1$

Table 2: k_c values and mode designation

modes $n=1$	k_c (1/cm)		modes $n=2$	k_c (1/cm)	
	$b=0.5$	$b=0.7$		$b=0.5$	$b=0.7$
HM_{11}-EH_{10}	0	0	HM_{21}-EH_{21}	2.963	3.626
HM_{12}-HE_{11}	3.917	5.127	HM_{22}-HE_{21}	4.988	4.401
HM_{13}-EH_{11}	6.393	10.522	HM_{23}-EH_{22}	7.459	11.265
HM_{14}-HE_{12}	9.714	13.544	HM_{24}-HE_{22}	10.218	12.077
HM_{15}-EH_{12}	12.625	20.970	HM_{25}-EH_{23}	13.234	21.358
HM_{16}-HE_{13}	15.883	23.003	HM_{26}-HE_{23}	16.195	21.727

$a = 0.5$ cm; $c = 1$ cm; $\varepsilon_r = 2.56$; $\mu_r = 1$.

and $n = 2$, as well as the mode designation EH_{nm} HE_{nm} that is proposed. These values have been calculated solving the cutoff conditions [4]. The modes whose asymptotic values of f_{0c} depend on whether $a \neq b$ or $a = b$ can be identified ($f_{0c} = k_c c$).

We propose to designate the HM_{nq} modes as EH_{nm} if the subscript q is odd, taking $m = (q + 1)/2$, and as HE_{nm} if q is even, now taking $m = q/2$. Only when $n = 1$ will we start the series EH_{nm} with $m = 0$, which corresponds to $m = (q - 1)/2$. As a consequence, the fundamental hybrid mode belongs to a EH series and we propose to name it EH_{10}. Such a designation of the fundamental hybrid mode agrees with the nomenclature introduced by other authors [*6*, *7*, *8*] when investigating the Goubau line.

At the same time, each one of the EH and HE series will be characterised by its asymptotic value of R for large frequencies, in accordance with the comments on Table 1.

Conclusions

The TM and TE contributions to the field structure of a hybrid mode cannot be evaluated using the normalised relation $jZ_0(H_z/E_z)$. However, the TM and TE contributions to the power flow appear to be suitable parameters to analyse the TM-TE field structure of hybrid modes. The analysis of these contributions in a dielectric-coated wire with an intervening airgap shows that the hybrid modes of this waveguide cannot be classified into quasi-TM and quasi-TE modes. A scheme for the designation of the hybrid modes of this particular waveguide has been proposed, but based on mathematical properties such as the asymptotic values of cutoff frequencies and the R parameter far from cutoff.

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