## ANALYSIS OF AN INTERFEROMETRIC OPTICAL FIBRE DETECTION TECHNIQUE APPLIED TO SILICON VIBRATING SENSORS

Indexing terms: Optical fibres, Measurement, Optical sensors

A simple and accurate analysis of a single-mode optical fibre interferometric displacement detection technique is presented. This analysis is in good agreement with experimental measurements, and shows that angular alignment between the fibre and the sensor is the most critical parameter.

Introduction: Of the several optical techniques that can be applied to interrogate silicon vibrating sensors, interferometric detection is the most suitable when small deflections, of the order of nanometres, are excited. Such interferometric techniques have been performed using either bulk optics<sup>1</sup> or single-mode optical fibres.<sup>2</sup> The option of using an optical fibre arrangement is more advantageous from the point of view of a practical sensor system.

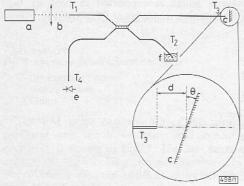


Fig. 1 Interferometric detection system

(a) Gas laser, (b) focusing lens, (c) reflecting surface, (e) detector, (f) matching liquid

 $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  = terminals of a single-mode 3 dB coupler

Fig. 1 shows a diagram of the single-mode optical fibre interferometric system mentioned above. Similar systems have also been used for velocity3 and reflectivity4 measurements. In our case, the reflecting surface (c) corresponds to the surface of a vibrating element, part of the silicon sensor, and its resonant frequency depends on the magnitude of the measurand. Two interfering signals are set up at terminal T3, the wave reflected directly from the end of the fibre, and the other, the wave coupled back into the fibre from the multiple reflections within the cavity formed by the fibre end and the reflecting surface. The resultant signal is detected by a photodetector at T<sub>4</sub>. An important characteristic of this system is that the interferometric output, determined by the relative amplitudes and phases of the two reflected waves, is independent of the effective optical length of the fibre, and is therefore unaffected by temperature and other environmental effects.

Theory: The successive contributions to the interferometric signal S, detected at  $T_4$ , can be expressed as

$$S = r_i + \sum_{m=1}^{\infty} t_i |r_s| t_e (|r_s|r_e)^{m-1} |c_m| e^{-j\phi_m}$$
 (1)

where  $r_i, r_e, t_i$  and  $t_e$  are the reflection and transmission coefficients on the fibre/air interface corresponding to internal and external incidence, respectively,  $r_s$  is the reflection coefficient of the reflecting surface, and  $c_m$  is the coupling coefficient between the reflection of order m and the  $LP_{01}$  mode.  $\phi_m$  is the phase delay associated with the path length of the multiple reflected wave and the number of reflections from the silicon surface

To provide simple and accurate expressions for the coupling coefficients  $c_m$ , we use a Gaussian approximation for the field distribution of the fundamental mode of a step-index optical fibre. The advantage of this approximation is that we can make use of the well known characteristics of the propagation of Gaussian beams within the cavity. Finally, the

orthogonality properties of the fibre modes are used to evaluate the coupling between the reflected Gaussian beams and the fundamental  $LP_{01}$  mode.

The situation is similar to calculating the coupling between a Gaussian beam and a nonaligned fibre as a function of the axial position of the fibre L, its angular misalignment  $\Psi$  and its transverse offset F. In our case these parameters are determined by the position and tilt of the reflecting surface. The reflection of order m will be defined by a total travelling distance  $L_m = 2md$ , a tilt  $\Psi_m = 2m\theta$  and an offset  $F_m = 2m^2 d\theta$ , neglecting second-order terms. The expression for  $|c_m|$  as a function of these parameters is

$$|c_m|^2 = \frac{1}{1 + \bar{L}_m^2} \exp(-\bar{\Psi}_m^2) \exp\left[-\frac{(\bar{F}_m - \bar{L}_m\bar{\Psi}_m)^2}{1 + \bar{L}_m^2}\right]$$
(2)

where the normalised distance  $\bar{L}_{\rm m},$  tilt  $\bar{\Psi}_{\rm m}$  and offset  $\bar{F}_{\rm m}$  are defined by

$$\bar{L}_m = \frac{\lambda_0 L_m}{2\pi w^2} \qquad \bar{\Psi}_m = \frac{\pi w \Psi_m}{\lambda_0} \qquad \bar{F}_m = \frac{F_m}{w} \tag{3}$$

w being the width parameter of the Gaussian approximation and  $\lambda_0$  the wavelength in free space. Eqn. 2 leads to the same results for the splice losses between misaligned fibres as given by Marcuse.<sup>5</sup> In fact the angular misalignment and offset are necessarily related through their dependence on the tilt of the reflecting surface. It follows from eqns. 3 that  $\bar{F}_m = \bar{L}_m \Psi_m$  and therefore the second exponential term of eqn. 2 becomes unity, and hence  $|c_m|$  has no direct dependence on the offset. The physical explanation is that the effect of the offset is compensated by the wavefront curvature of the Gaussian beam. It should also be noted that eqn. 2 is not merely the product of the individual factors which describe the separate effects of distance, tilt and offset, as was incorrectly assumed in Reference 4.

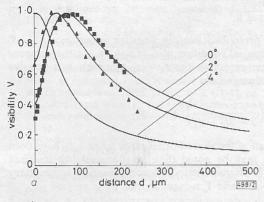
The results of eqn. 2 apply to any vibrating plane surface, but as our particular interest is in silicon structures we proceed with the calculation of numerical values for a vibrating silicon surface. The refractive index n of the fibre fixes the values of  $r_i$ ,  $r_e$ ,  $t_i$  and  $t_e$ . If we assume n = 1.46 and the reflectance of a silicon surface  $r_s r_s^* = 0.35$ , the value of the factor  $|r_s|r_e = -0.11$  shows that it will be sufficient to take into account only the first three terms of eqn. 1. Even the third term will be negligible for a fibre sensor separation greater than  $100 \, \mu \text{m}$ . The phase delay  $\phi_m$  can be approximated by  $\phi_m = m\phi$ ,  $\phi$  being the phase delay of the first reflection. Therefore, the intensity of the signal  $I = SS^*$  will fluctuate from maximum to minimum as a function of  $\phi$ . Two different parameters have been evaluated: (i) the visibility V of the interferometric signal, given by  $V=(I_{max}-I_{min})/(I_{max}+I_{min});$  (ii) the amplitude A of a fringe, given by  $A=I_{max}-I_{min},$ which is more appropriate if the detection circuit includes a highpass filter. Fig. 2 shows the calculated values of V and A as functions of d and for different values of  $\theta$ . These plots correspond to a single-mode fibre of  $4 \mu m$  core diameter and NA = 0.12, and for a wavelength in free space  $\lambda_0 = 0.633 \,\mu\text{m}$ .

Neither the visibility nor the amplitude are necessarily maximum when both d and  $\theta$  tend to zero, and their maxima are not coincident. The values of d and  $\theta$  which give a maximum visibility or amplitude depend on the relative amplitudes and phases of each reflection. The amplitude A is more critically decreased by the angular misalignment  $\theta$  than by the distance d.

Experimental results and discussion: It has been convenient to test the preceding theory against measurements using the interferometric system of Fig. 1 and the vibrating surface of a silicon sensor. The sensor was excited piezoelectrically, giving deflections greater than the  $\lambda_0/4$  necessary to produce the appropriate visibility and fringe amplitudes for the measurements

Fig. 2a shows two sets of experimental measurements. In one set the fibre was positioned by eye to be as nearly perpendicular to the surface as possible, i.e. zero tilt, while in the other the tilt was set to be small (but not zero). It is seen that the two sets of points follow appropriate tilt curves, giving good correspondence for separations up to  $200 \, \mu \text{m}$ . A pre-

vious somewhat similar analysis<sup>4</sup> produced numerical results which do not fit our experimental measurements. This shows the lack of accuracy of the expressions provided there, for the reason suggested earlier.



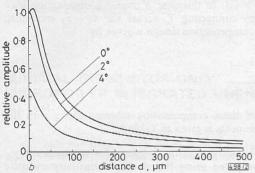


Fig. 2 Visibility and amplitude as functions of distance d for different values of tilt  $\boldsymbol{\theta}$ 

■, ▲ experimental points corresponding to different visual alignments of fibre

Conclusions: The analysis which has been carried out is in good agreement with the experimental measurements, and provides a proper understanding of the properties of the system. The system is proved to be tolerant of longitudinal separation and more critically dependent on angular misalignment.

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## PRECISE REALISATION OF AN INSENSITIVE FLOATING NEGATIVE ADMITTANCE CONVERTOR

Indexing terms: Circuit theory and design, Current conveyors

A new floating negative admittance convertor (FNAC) network using the second-generation current conveyor (CCII) active devices has been proposed. The novelties of the scheme are no component matching requirements, use of only the nominal termination admittance under ideal device conditions, and practically active-insensitive network characteristics. Even with nonideal but matched devices, no extra matching component is required for a true FNAC realisation. However, with nonideal and nonidentical devices, insertion of a single compensating admittance yields a precise bilateral FNAC.

Introduction: The negative immittance convertor (NIC) building block is useful for various functional applications in electronic circuits and systems. A most appropriate application of the NIC is in the simulation of the active admittance functions Y(s) = K/s and  $Y(s) = k/s^2$ , wherein their positive lossy components are cancelled by an identical negative component generated by the NIC. These applications may require the use of both grounded and floating types of convertor.

The use of operational amplifiers (OAs) for a grounded NIC, obtainable with two extra matched components in addition to the termination immittance, is well known.<sup>3</sup> It has been reported previously<sup>4</sup> that this matching design constraint, along with the concomitant sensitivities in the OA version, may be conveniently obviated by an insensitive ground-NIC design utilising the relatively new CCII devices<sup>5</sup> connected appropriately. The CCII is now well established as a functional active circuit element, and its numerous applications, along with its integrable implementation schemes, are available in the current literature.<sup>6</sup> The active realisation of floating negative immittance convertors using OAs has also been proposed,<sup>7</sup> and it may be seen that similar design constraints involving additional component matching are unavoidable.

In this letter a new circuit scheme for the realisation of an FNAC block using CCII devices has been proposed. It is shown that a true FNAC is realisable with two same-polarity CCIIs and only the nominal termination admittance Y. Under the consideration that the CCII characteristics are matched, whether ideal  $(h_1 = h_2 = 1)$  or nonideal  $(h_1 \neq 1, h_2 \neq 1, \text{ but})$  $h_1 = h_2$ ), no extra matching component is required for the realisation. However, with nonideal and unmatched CCIIs  $(h_1 \neq 1, h_2 \neq 1, h_1 > h_2 \text{ or } h_2 > h_1)$  a simple compensation scheme precisely re-establishes a bilateral admittance matrix, giving rise to a true FNAC. The compensation requires the insertion of a single admittance Y at the appropriate terminals of the CCIIs for either type of device mismatch  $(h_1 > h_2)$  or  $h_2 > h_1$ ). The sensitivities of the elements of the Y-matrix relative to active device deviations have been shown to be extremely low.

Realisation of ideal CCII: The CCII is a three-port groundednetwork element having the terminal characteristics

$$i_{v} = 0 \quad v_{x} = v_{v} \quad i_{z} = hi_{x} \tag{1}$$

where h represents the current transfer ratio between the x and z ports. Ideally h=1, which ensures tracking port currents  $i_x=i_z$ .

Analysis of the proposed network of Fig. 1 with ideal positive-polarity CCIIs  $(h_{1,2} = 1)$  and disregarding  $Y_c$  now yields the short-circuit admittance matrix

$$[Y] = \begin{bmatrix} -Y & Y \\ Y & -Y \end{bmatrix} \tag{2}$$

This represents a true FNAC realisation simulating an admittance of -Y across the floating x-ports, after terminating the z-ports by the nominal component (Y).