SENSITIVITY AND MODE SPECTRUM OF A FREQUENCY-OUTPUT SILICON PRESSURE SENSOR*

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Abstract

The vibrational mode spectrum of a silicon vibrating pressure sensor is investigated. Particular attention is given to the analysis of the vibration shapes, quality factors and relative sensitivity of the resonance frequencies as a function of pressure. It is shown that a pressure sensitivity of a few parts per million at one atmosphere can be achieved. Some comments are also made regarding an improved design of the device.

1. Introduction

Micromachined silicon resonant sensors combine several attractive features. The silicon crystal material properties have been shown to suit the mechanical requirements of stability and lifetime of resonance sensor applications, and at the same time etching techniques allow precise micromachining to be achieved [1]. These techniques give rise to small devices that are nicely compatible with optical fibres. The combination of a mechanical frequency output [2], an optical fibre interrogating technique [3] and optical activation [4], will provide a sensor system with a high degree of reliability and immunity to electromagnetic interference.

The heart of a micromachined silicon resonance sensor is a mechanical resonator, whose frequencies of resonance are a function of a specific physical quantity through the generation of a stress. The mechanical resonator will exhibit a number of resonances, each one being characterized by a set of mechanical properties, the most important being a particular shape of vibration, a quality factor Q and a specific frequency dependence on the measurand.

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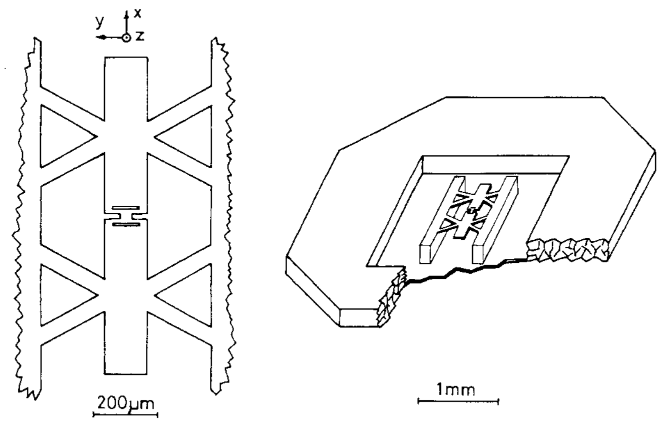
This paper presents the results of an exhaustive analysis of the spectrum of vibrational modes of a resonance sensor designed as a pressure transducer [5]. Particular attention is given to the evaluation of the intrinsic sensitivity of each mode, independent of the optics and electronics that contribute to the final sensitivity of the sensor system. The factor Qs, where s is the rate of change of resonance frequency with pressure, is proposed as a suitable parameter for comparing different resonances and different devices.

Although the results shown here are specific to a particular device, the method of analysis and the general conclusions hold for any micro-machined silicon resonance sensor. A proper knowledge of the vibrational shape is required to optimize the sensor system and to help any subsequent redesign of the device. If both optical interrogation and excitation are used, then the vibrational shape determines the location of points for the optimum interrogation and activation, and is particularly critical if both functions have to be performed by only one optical fibre [4]. It is shown that the quality factor and the sensor response to the measurand are related to the vibrational shape, and the basic features that should be taken into account in improving the sensor performance are pointed out.

2. Device and experimental arrangement

Figure 1 shows diagrams of the resonator and the complete device. The resonator consists of a coupled pair of 8 μm thick suspended rectangular plates and is integral with a thin diaphragm. The device is batch fabricated from a single crystal of silicon, following a procedure already reported [5]. When a differential pressure exists across the diaphragm, it distorts and stresses the ribs that support the plates, modifying the resonance frequency.



(a) (b)

Fig. 1. (a) Resonator diagram. (b) Complete device diagram.

Although it is possible to activate the resonator vibrations optically [4], piezoelectric activation is more efficient and has been applied to excite the large amplitudes of displacement, *e.g.*, several microns, necessary for detailed measurements of the vibrational patterns of the resonator. Figure 2 is a diagram of the device holder. The air is evacuated from the resonator side in order to increase the Q of the resonator, providing a typical vacuum of 10 Pa. This pressure establishes the reference for the test pressure that is applied to the diaphragm to enable the sensor to be calibrated. A thin sheet of glass seals the vacuum chamber while allowing optical interrogation of any particular area of the resonator to be achieved.

Figure 3 shows the optical interrogation system, which is based on an interferometric displacement measurement technique [3] and is capable of detecting displacements of less than 1 nm. The large amplitudes of vibration in these experiments have been measured by counting the number of interferometric fringes generated. An accuracy of 1/4 of a fringe can be achieved with no special signal analysis, which corresponds to a displacement of $\lambda/8$ = 79 nm. A video system provides a magnified picture of the resonator, which helps in locating the detection point.

The actual experimental arrangement allows the fibre to be moved in order to investigate the characteristics of the sensor. However, a practical system will require the rigid attachment of the fibre to the device and proper vacuum encapsulation. The results now presented are crucial for the design of a suitable prototype.

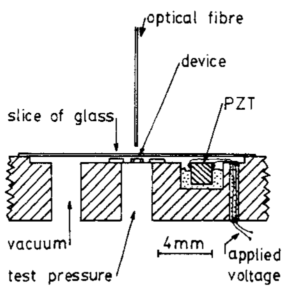


Fig. 2. Diagram of device holder.

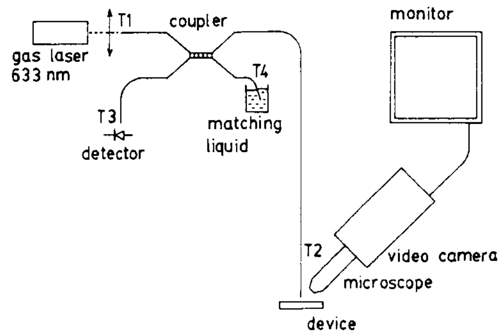


Fig. 3. Diagram of the optical arrangement.

3. Spectrum of vibrational modes

By scanning the piezoelectric transducer frequency up to 1 MHz, a number of vibrational modes have been detected. Each is a flexural mode that can be regarded as a resonance of the first-order antisymmetrical Lamb acoustic wave propagating in the plates that define the resonator [6].

The spot of light that the single mode fibre produces on the resonator is about 5 μm in diameter, while the resonator is 1 mm long and 120 μm wide. This gives good lateral resolution when optically scanning the resonator to determine the shape of vibration of a particular resonance. Modes have been designated as M_{nm} where the subscripts n and m indicate the number of nodal axes parallel to the x-axis and y-axis respectively (see Fig. 1). Only two series of modes, M_{0m} and M_{1m},.have been detected in the frequency range under investigation. Some resonances of the whole device have also been detected, and they are characterized by relatively low quality factors and small amplitudes of vibration.

Table 1 gives a summary of the detected modes of vibration and the frequencies of resonances when either atmospheric pressure or no pressure is applied to the diaphragm. The mode designation proposed for the first three modes M_{0m} requires some comment. Figure 4 shows the shape of vibration of the mode M_{00} at 128 kHz and P = 100 kPa. This M_{00} mode is a resonance in which both plates are driven up and down simultaneously by the supporting strips. If the differential pressure, P, applied to the diaphragm is decreased from atmospheric pressure down to zero, then the shape of vibration of this mode evolves and eventually two nodal axes appear at the ends of the plates, moving slowly inwards to their final position. The final shape at P = 0 and 97 kHz is shown in Fig. 4, and is referred to as M_{02}*. The results of a finite element analysis of the resonator are in good agreement with the observation just described [7]. The modes M_{01} and M_{02}, also shown in Fig. 4, cross one another at low pressures, and their shapes can hardly be distinguished at P = 0. This situation, the coupling between modes when their frequencies are very close together, is reported in more detail in Section 5. The diagrams of vibrational shapes of the remaining modes are given in Fig. 5.

TABLE 1

Modes of vibration and their frequencies of resonance

Mode	ω_r[a] (kHz)	ω_r[b] (kHz)	Mode	ω_r[a] (kHz)	ω_r[b] (kHz)
M_{02}	101	86	M_{10}	368	296
M_{01}	105	85	M_{11}	382	318
M_{00}	128	97	M_{12}	405	384
M_{03}	212	172	M_{13}	513	458
M_{04}	284	260	M_{14}	635	584
M_{05}	449	432	M_{15}	730	691
M_{06}	609	587	M_{16}	904	823
M_{07}	800	758			

[a]Differential pressure equal to atmospheric pressure.
[b]Differential pressure equal to zero.

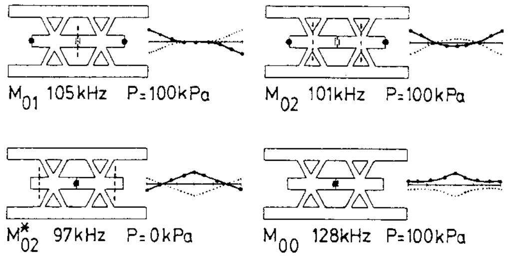


Fig. 4. Vibration shapes of M_{01}, M_{02} and M_{00} - M_{02}* modes. Top views include nodal axes (— — —) and points of maximum deflection (•). Lateral views give a magnified cross-section of the vibration shapes.

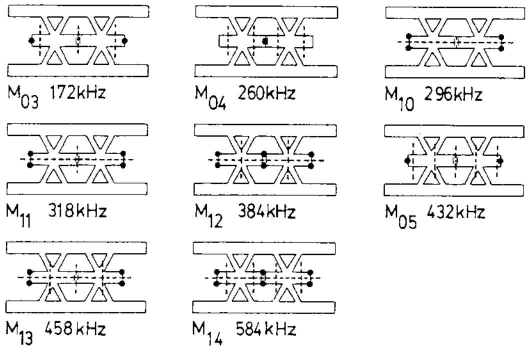


Fig. 5. Diagrams of vibration shapes showing the nodal axes (— — —) and the points of maximum deflection (•).

4. Sensitivity

The sensitivity of this sensor is a function of several factors. First, the degree of coupling between the resonator and the differential pressure determines the change of the resonance frequency ω_r as a function of P. The efficiency of this coupling can be measured by means of the relative slope s:

$$s = \frac{1}{\omega_r} \frac{\partial \omega_r}{\partial P} \tag{1}$$

Secondly, the resonance curve determines the maximum change of the amplitude of vibration ξ that can be produced by a given change of the applied frequency ω. Assuming that a feedback loop keeps the excitation frequency ω at the optimum point, *i.e.*, at the point where the Lorentzian curve of resonance exhibits the maximum slope, then the relative change of amplitude $\delta\xi/\xi$ produced by a small change of ω, $\delta\omega$, will be given by

$$\frac{\delta \xi}{\xi} = \frac{2\sqrt{2}}{3} Q \delta \omega \tag{2}$$

where Q is the quality factor of the resonance. Substituting eqn. (2) into eqn. (1), the sensitivity of the system, *i.e.*, the minimum change of pressure that can be measured, δP_{min}, is determined by the minimum relative change of amplitude $(\delta\xi/\xi)_{min}$ that the interrogation system can detect:

$$\delta P_{min} = \frac{3}{2\sqrt{2}} \frac{1}{Qs} \left[\frac{\delta\xi}{\xi}\right]_{min} \tag{3}$$

The term $(\delta\xi/\xi)_{min}$ is a function of the optics and electronics of the interrogation system, while the product Qs is a characteristic of the sensor. We have used the factor Qs as a figure of merit to characterize the intrinsic sensitivity of the resonance sensor when exciting a particular mode of vibration. In this paper the factor Qs is used to compare different modes, but it may be used to compare different devices and different resonance sensors.

Figure 6 shows the response curve, ω_r *versus* P, for the M_{00} mode. The values of Q and s change with the magnitude of P, both increasing when P decreases. For the M_{00} mode $Q = 13\,000$ and $s = 1.7 \times 10^{-6}$/Pa at atmospheric pressure, and $Q = 21\,000$ and $s = 4.1 \times 10^{-6}$/Pa at low pressure. The factor Qs changes from 2.2×10^{-2}/Pa to 8.6×10^{-2}/Pa as the differential pressure decreases from one atmosphere down to zero. If $(\delta\xi/\xi)_{min}$ is estimated as 1/20, then the expected sensitivity of this M_{00} resonance will lie in the range 0.6 - 2.4 Pa, which gives a relative sensitivity better than 25 ppm at atmospheric pressure.

Table 2 gives a summary of quality factors Q and relative slopes s measured at low differential pressures. It is possible to point out a qualitative relation between the vibrational shapes and the characteristic parameters Q and s. The mode M_{12} exhibits a relatively low slope s, and this can be related to the fact that this mode has the two nodal axes exactly over the supports, which decreases the sensitivity of the resonator to the stress induced by the diaphragm. At the same time the Q of this mode, M_{12}, is relatively high because the position of the nodal axes decrease the losses of acoustic wave energy through the supports. A similar discussion can be carried out for the M_{02} mode, which exhibits a higher Q than the M_{00}, M_{01} and M_{03} modes. This mode, M_{02}, also exhibits a relatively small slope s, but this cannot be shown at low differential pressure because of the coupling with the M_{01} mode (Section 5). However, if we compare the slopes at atmospheric pressure, $s(M_{00}) = 1.72$, $s(M_{01}) = 1.17$, $s(M_{02}) = 0.93$ and $s(M_{03}) = 1.64 \times 10^{-6}$/Pa, then the previous comment is made clear.

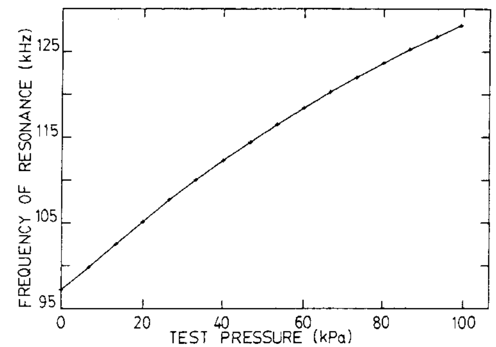


Fig. 6. Mode M_{00}. Frequency of resonance ω_r as a function of test pressure.

TABLE 2

Quality factors Q and relative slopes s measured at low differential pressures

Mode	Q 10^3	s 10^{-6}/Pa	Qs 10^{-2}/Pa	Mode	Q 10^3	s 10^{-6}/Pa	Qs 10^{-2}/Pa
M_{02}	28.0	3.21	9.0	M_{10}	26.0	3.60	9.4
M_{01}	6.3	3.55	2.3	M_{11}	28.0	3.22	9.0
M_{00}	21.0	4.07	8.6	M_{12}	49.0	0.77	3.8
M_{03}	21.0	2.65	5.6	M_{13}	33.0	1.37	4.5
M_{04}	18.3	1.03	1.9	M_{14}	11.0	0.80	0.9
M_{05}	14.6	0.45	0.68	M_{15}	27.0	0.61	1.7
M_{06}	13.4	0.32	0.45	M_{16}	15.9	1.12	1.8
M_{07}	6.2	0.83	0.60				

These results are particularly useful when thinking about improving the sensor design. The resonator discussed here does not allow simultaneous increases of Q and s, because both rely on the relative strain at the supports. The former parameter requires the position of the supports to be matched to the nodal axes, while the second is improved when the supports vibrate with large amplitudes. It would be desirable to concentrate the stress induced on the resonator in a position away from the supports, thereby making it possible to increase the parameter s of a given mode without decreasing the Q and vice versa.

5. Coupling between modes

It has been found that some modes exhibit very close frequencies of resonance within a certain range of differential pressure, leading to coupling between them. Figure 7 shows the coupling of the M_{14} and M_{06} modes over the pressure range 0 - 25 kPa.

If k is the coupling coefficient and ω_1 and ω_2 are the uncoupled resonance frequencies, then the frequencies of resonance of the coupled system are given [8] by the solutions Ω of

$$(\Omega - \omega_1)(\Omega - \omega_2) = k^2 \tag{4}$$

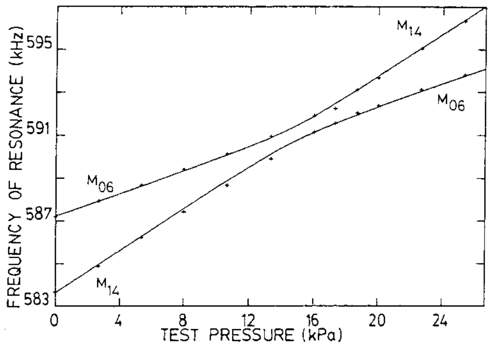


Fig. 7. Coupling between modes M_{14} and M_{06} in the range 0 to 25 kPa.

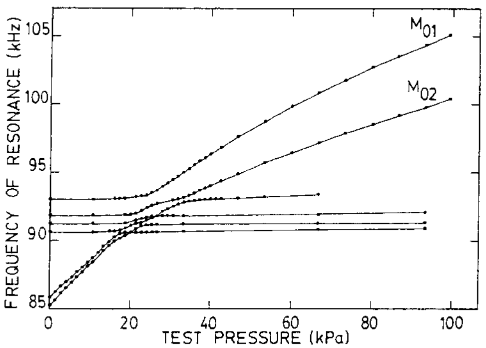


Fig. 8. Coupling between modes M_{01} and M_{02}, and four resonances of the whole device. Frequencies of resonance as a function of test pressure.

The curves fitting the experimental points of Fig. 7 are solutions of this equation with a coupling coefficient k = 346 Hz. We have assumed a linear change of ω_1 and ω_2 with pressure over this small range: $\omega = \omega_0 + \gamma P$. The curves shown in Fig. 7 correspond to the values $\omega_0(M_{14})$ = 583.661 kHz, $\gamma(M_{14})$ = 0.502 Hz/Pa, $\omega_0(M_{06})$ = 587.198 kHz and $\gamma(M_{06})$ = 0.262 Pa. Above P = 25 kPa, the higher and lower resonances have vibration shapes M_{14} and M_{06} respectively, just the opposite to the situation at P = 0.

Figure 8 shows the same qualitative behaviour, but in a rather complex situation with six resonances interacting. Four of them are vibrations of the whole device and their frequencies of resonance are nearly independent of the differential pressure. These resonances induce large vibration amplitudes of the resonator only when they are coupled to the modes M_{01} and M_{02}. The two modes M_{01} and M_{02} are very close at P = 0, but exhibit different slopes at low differential pressure, and effectively cross over each other.

These features limit the use of some particular modes of vibration because the electronics could easily be misled in the coupling regions.

Further work on the design of the device should be carried out if an increase of the separation between a selected pair of modes is wished.

6. Conclusions

The analysis of the spectrum of vibrational modes of a silicon vibrating sensor is necessary to select the optimum resonance for a sensor application, as a function of the interrogation and excitation techniques as well as the sensitivity. A factor Qs is proposed to evaluate the intrinsic sensitivity of a particular mode, which allows a comparison to be made between different vibrational modes, different devices and different resonance sensors. The coupling between modes is a feature that may prevent the use of some modes in a practical system.

The results shown here provide the basic information necessary for an improved design of the pressure sensor, and show that it is possible to achieve a sensitivity better than a few parts per million at one atmosphere.

References

1 K. E. Peterson, Silicon as a mechanical material, *Proc. IEEE, 70* (1982) 420 - 457.
2 R. M. Langdon, Resonator sensors — a review, *J. Phys. E: Sci. Instrum., 18* (1985) 103 - 115.
3 M. V. Andres, M. J. Tudor and K. W. H. Foulds, Analysis of an interferometric optical fibre detection technique applied to silicon vibrating sensors, *Electron. Lett., 23* (1987) 774 - 775.
4 M. V. Andres, K. W. H. Foulds and M. J. Tudor, Optical activation of a silicon vibrating sensor, *Electron. Lett., 22* (1986) 1097 - 1099.
5 J. C. Greenwood, Etched silicon vibrating sensor, *J. Phys. E: Sci. Instrum., 17* (1984) 650 - 652.
6 B. A. Auld, *Acoustic Fields and Waves in Solids*, Vol. 2, Wiley, New York, 1973, pp. 76 - 88.
7 J. C. Greenwood, Resonant silicon sensors at STL, *Eurosensors '87, First European Conference on Sensors and their Applications, Cambridge, U.K., September 22 - 24 1987.*
8 A. B. Pippard, *The Physics of Vibration*, Vol. 1, Cambridge University Press, Cambridge, 1978, p. 366.

Biographies

Miguel V. Andres was born in Valencia, Spain, in 1957. He received the Licenciatura en Ciencias Fisicas in 1979, and the Ph.D. degree in 1985, both from the Universidad de Valencia, Spain.

He has been engaged in the past in research on microwave surface waveguides, and his current interest is in optical fibre sensors. He is at present with the Departamento de Fisica Aplicada, Universidad de Valencia, Spain, and has been working for several periods in the Department of Physics, University of Surrey, U.K.

K. W. H. Foulds was born in London in 1928. He was awarded the B.Sc. (Eng.) in 1949 and the Ph.D. in 1953, both from University College, London.

After working on high powered radar at the Marconi Co., Chelmsford, he joined the Electrical Engineering Department at Imperial College of Science and Technology, London, in 1955 and moved to the Physics Department at Battersea College of Technology, the forerunner of the University of Surrey, in 1960, where he is a Reader and the Deputy Head of Department. Until 1981 his research interests lay in microwaves, particularly microwave active devices, but he is now concerned with the application of optical fibres to sensors.

Dr. Foulds is a Fellow of the Institute of Electrical Engineers, and a Member of the Institute of Electrical and Electronic Engineers.

M. John Tudor received the B.Sc. (Eng.) Honours degree in electronic and electrical engineering from University College, London, in 1983. Since 1984 he has been studying for the Ph.D. at the University of Surrey in collaboration with Standard Telecommunication Laboratories, Harlow. His research interests are in micromachined resonators, optical fibres and sensing.