ORIGINAL ARTICLE

Estimation of the combined effects of ageing and seasonality on mortality risk: An application to Spain

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Abstract

Despite the overwhelming evidence that shows the persistence of intra-annual variations on demographic events (deaths, birth dates and migration flows), life tables are computed and provided on an annual basis. This paper develops a new estimator for estimating sub-annual death rates that, considering the exact moment of occurrence (exact age and day) of events, concurrently accounts for ageing and calendar fluctuations. This paper also shows how modelling the intra-annual variations of death rates, through specific seasonal–ageing indexes, can be used as a tool for constructing new sub-annual tables from annual tables. This new methodology is exemplified using a real database of Spain made up of 186 million demographic events (1.5 million of which are deaths), from which seasonal–ageing indexes are estimated and conclusions drawn. First, seasonal effects are, as a rule, stronger than ageing effects. For a given integer age, season has a higher impact on increasing or decreasing the average risk of death at that age than the actual age of the exposed-to-risk. Second, the intensity of the effects varies among seasons and age-quarters. Third, neither seasonal nor ageing effects are age-stationary.

Their impact, be it to varying degrees, intensifies as people get older. Fourth, there is interaction between seasonal and ageing effects. In short, life expectancies and probabilities of dying/surviving not only depend on people's age, but also on when their birthday falls within the year. This has implications, for instance, in managing pension systems or for insurance companies.

1 | INTRODUCTION

Life tables, the invention of which can be traced back to the 17th century, are regarded as probably the most important tools for the analysis of mortality and life insurance (Benjamin & Pollard, 1986). Historically, the focus has been on properly measuring death rates and survival probabilities. In 1662, Graunt presented the first estimates of death rates by analysing data of deaths in London (Graunt, 1662). In 1693, Halley published the table of mortality of Breslau, which was used by the British government to sell life annuities with the prices adjusted depending on the age of the purchasers. In 1746, Depardieux estimated the mortality table of the French population and, around 1770, Cambert calculated the mortality table of the German population (Basulto & Garcia, 2009). The invention of the so-called Lexis diagram, a calendar-age dimensional Cartesian system that allows vital events that affect individuals belonging to different cohorts to be represented graphically (such as births and deaths), (Brasche, 1870; Lexis, 1880; Rau et al., 2018; Vandesrchirk, 2001; Zeuner, 1869) was a tipping point, although the first conceptualisations of the phenomenon through models was actually a bit earlier (Gompertz, 1825; Makeham, 1860).

Nowadays, the focus on mortality studies has moved towards analysis of its evolution and the study of longevity. Mortality forecasting is considered a fundamental pillar in different areas, such as pensions, public planning or in insurance schemes, and has been the topic of study of a number of research papers (e.g. Börger et al., 2014; Cairns et al., 2011, 2019; Dong et al., 2020; Enchev et al., 2016; Haberman & Renshaw, 2012; Lee & Carter, 1992). In this paper, we look more closely at another topic that has received less attention in the literature: the analysis of probabilities of surviving from fractional ages (or for fractional durations) and its interaction with seasonal mortality patterns. As Lledó et al. (2019) stated, death statistics show clear intra-age and calendar patterns. Although the existence of patterns between consecutive integer ages was quickly acknowledged and internalised by statistics and the actuarial literature, considering fractional age assumptions and/or continuous survival models (Hoem, 1984; Pascariu, 2018), the prevalence of seasonality patterns in death statistics has been almost forgotten (avoided) in this literature (Richards et al., 2020, is an exception), despite it being well-documented in the demographic, epidemiological and sociological literatures (e.g. Foster et al., 1998; Grant et al., 2017; Healy, 2003; Rau, 2007), as far back as the mid-19th century (Guy, 1858; Nature, 1874). The question is to what extend the death seasonality patterns impact on the sub-annual distributions of mortality

risks after taking into account the seasonality of other demographic vital events (birth dates and migration flows) and ageing effects. In this research, we answer a research query posed by Lledó et al. (2019, pp. 144–145) who point to the importance of studying 'the appropriateness of decomposing mortality rates by quarters or months', taking into account the 'age/calendar distribution of deaths'.

Life tables provide probabilities of survival and death rates for integer ages and durations. In statistics and the actuarial literature, the computation of probabilities of dying/surviving in fractional ages or in fractional durations has been traditionally performed in intra-annual mortality studies by considering the well-known fractional age assumptions (FAAs), either: (A1) a uniform distribution of deaths, (A2) a constant (intra-age) force of mortality or (A3) the hyperbolic or Balducci assumption. An FAA offers a method for interpolating between integer ages. These assumptions, however, can be misleading. On the one hand, as is well-known, the uniform distribution of deaths, assumption (A1), does not apply to zero years as its distribution concentrates on first moments (days/weeks) after birth. On the other hand, assumption (A2), constant force of mortality, entails discontinuous (forces of) mortality between contiguous ages, as also happens with assumptions (A1) and (A3). Hence, some researchers have offered different alternatives for modelling intra-annual mortality patterns (e.g. Jones & Mereu, 2000, 2002). In this vein, Hossain (2011) introduces a quadratic fractional age assumption as an alternative to assumption (A3).

The information technology revolution has made demographic microdata (births, migrations and deaths) more accessible, allowing researchers to study and incorporate the exact timing of each demographic event in the construction of death estimators. As a result of this, Lledó et al. (2019) propose a new annual period-based estimator for general populations that includes the exact time of every demographic event. Their analysis tackles an interesting debate about the three assumptions mentioned above. As their research paper showed, the intra-annual distributions of deaths do not follow a uniform distribution for some ages other than zero but present ageing and season intra-annual patterns. Therefore, some characteristics of age and calendar-related quarterly periodicity of death rates deserve a more in-depth study.

Several authors have previously studied the seasonal distribution of the deceased and the interaction of seasons with age from various perspectives. For example, Parks et al. (2018) analyse the seasonality of deaths by cause of death for several age groups. Rau et al. (2018) develop new tools to visualise mortality dynamics on the Lexis plane, including seasonal variations of deaths, and by doing so led the way to the application of, for instance, spatial statistics techniques for estimating age-season sub-annual mortality risks. Ledberg (2020) uses a model-based parametric approach for modelling cohort-based death rates taking into account both seasonal and ageing effects as a tool for studying how the decrease in seasonal fluctuations in mortality has contributed to the increase of life expectancy in Sweden. In this paper, we tackle other interesting research questions which have not yet been analysed but deserve to be addressed: (Q1) How do the intra-annual distributions of birth dates, migrations and deaths interact at different ages and shape death rates? (Q2) What would be the actual impact of the failure to meet the uniformity assumptions on the fractional estimates of death rates and on insurance computations? (Q3) Can we identify ageing and/or calendar effects for each age and gender in a given area or for a given population? (Q4) Would it be possible to obtain statistical seasonal–ageing indexes (SAIs) to be applied to central annual death rates of general population life tables or, even, to life tables used by insurance companies in order to attain properly sub-annual death probabilities?

To set about answering these four questions, this paper takes a twofold approach. On the one hand, we develop, within the period-based estimation framework, a new (non-parametric) estimator of death probabilities that captures the intra-annual (quarterly) patterns of deaths, births and migrations. On the other hand, we propose an innovative approach to model the intra-annual variations of actual mortality rates with respect to an intra-annual uniform behaviour (of deaths, births and migrations) and study their relationships with seasons and age-quarters in order to obtain new SAIs. We apply the methodological tools developed to a real database. With more than 186 million microdata events over a period of 4 years in Spain, we explore the intra-annual mortality, birth dates and migration flow patterns by gender of the Spanish population from an intra-age and quarterly calendar perspective; we develop new quarterly life tables and SAIs, and; we assess the suitability of the three FAAs for an insurance product. Thanks to the findings of this study, statisticians and insurance companies will have, for the first time in the literature as far as we know, a procedure to build age-calendar intra-annual (quarterly) statistical coefficients that can be applied to annual life tables for building sub-annual tables. Addressing this issue offers new opportunities to improve the management of pensions, public planning and insurance schemes with potential impacts on public pension systems and on the competitiveness and balance sheets of insurance companies. In our view, the coefficients attained in the application that complements this research could be used, without the need for extra calculation, in countries or for insurance companies that operate in markets with socio-economic and climatic characteristics similar to those of Spain.

The rest of the paper is structured as follows. After Section 2, in which we offer some background, Section 3 is devoted to methodological issues. We introduce the terminology utilised and the formulae employed to convert big raw micro-data into quarter (sub-annual) central death rate estimates. In Section 4, we model the intra-annual variations of mortality using age quarters (ageing effects) and calendar quarters (seasonal effects) as predictor variables. As a result of this procedure, we create an innovative seasonal-ageing (mortality) index. Section 5 explains how to derive quarterly (sub-annual) tables from annual tables using the indexes of Section 4. Section 6 presents an empirical application. The theoretical tools developed in Sections 3–5 are applied and assessed in a real database. Finally, Section 7 discusses and draws conclusions. Some supplementary material complements the paper.

2 | BACKGROUND

A (generation) life table synthesises the mortality experience of a (hypothetical) cohort of new born babies based on the assumption that, during the course of their lifetime, the members of the group experience the age-specific mortality rates of the table. Among other biometric features, a life table contains at each integer age x the probabilities of the members of the cohort of surviving to, p_x, or dying before, $q_x = 1 - p_x$, their next birthday. Traditionally, (either generation or period) life tables are built from death rates, m_x, or death probabilities, q_x, estimated from observed aggregated annual demographic events (Wilmoth et al., 2020). This entails using formulae that include certain implicit hypotheses in their construction: (H1) uniform distribution of deaths (and migrants) for each age and calendar year, (H2) a closed demographic system (or at least no explicit consideration of migratory flows) and (H3) uniform distributions of birth dates of individuals who survived the year for each age.

The more popular (annual) period-based estimators used nowadays approximate central death rates, m_x, after implicitly assuming (Lledó et al., 2019) hypotheses (H1)–(H3),

while the less hypothesis-demanding, but more data-demanding, (annual) cohort-based estimators obtain death probability estimates, q_x, by implicitly imposing (Lledó et al., 2017; Pavía et al., 2012) hypotheses (H1) and (H2). Whatever the approach, all previous studies on mortality had one thing in common: the biometric variables were computed based on annual figures and the life tables of dying and surviving probabilities were only offered for integer ages.

When we use a continuous random variable to model lifetimes, calculation of probabilities for any age and for periods of any length is fairly straightforward. However, when we use a life table only specified for integer values, some sort of assumption is needed to compute probabilities for a fractional age and/or with a fractional duration. Usually, any of the three classical Fractional Age Assumptions, (A1)–(A3), is employed. The assumption of uniform distribution of deaths (UDDs) between consecutive integer ages, (A1), translates into a survival function, $S(\cdot)$, linear between integer ages. That is, the uniform approximation of deaths entails: $S(x + t) = (1 - t)S(x) + tS(x + 1)$ and $\mu_{x+t} = \frac{q_x}{(1 - {}_tq_x)}$, for $0 < t < 1$, where μ_x accounts for the force of mortality at age x. On the other hand, when the force of mortality is assumed constant between integer ages, (A2), this leads to: $S(x + t) = S(x)^{t-1}S(x + 1)^t$ and $\mu_{x+t} = -\log p_x$, for $0 < t < 1$. Notice that (A1) and (A2) assumptions are numerically equivalent when q_x is small. Finally, the hyperbolic or Balducci assumption, (A3), leads to the relationships: $S(x + t)^{-1} = (1 - t)S(x)^{-1} + tS(x + 1)^{-1}$ and $\mu_{x+t} = \frac{q_x}{1 - (1 - t)q_x}$, for $0 < t < 1$. In short, under (A1), (A2) and (A3), the value of the survival function in an intermediate age $x + t$ between two consecutive integers x and $x + 1$ is equal to, respectively, the weighted arithmetic mean, the weighted geometric mean and the weighted harmonic mean of the values of the survival function in x and $x + 1$ with respective weights $t - 1$ and t.

Some authors have analysed and assessed the impact of the previous assumptions on life insurance products (Fernández-Duran & Gregorio-Dominguez, 2015; Frostig, 2002, 2003) concluding that they are, as a rule, inadequate under reasonable patterns of mortality. Other authors have offered alternative assumptions via new families of FAAs that, with the particular cases of (A1) to (A3) in mind, seek to overcome the shortcomings of the classical fractional age assumptions producing smoother forces of mortality (Hossain, 2011; Jones & Mereu, 2000, 2002). In a similar vein, Barz and Müller (2012) develop, within the family of linear force of mortality introduced by Jones and Mereu (2000), an algorithm to approximate the force of mortality between integer ages that, by selecting proper parameters, aims to keep the number of discontinuities small, avoiding jagged forces of mortality. All of these solutions, however, are theoretical approaches with dissimilar levels of empirical fitting. They all fail to take into account the actual behaviour of mortality between integer ages and omit seasonal patterns, despite there being extensive literature that shows that the intra-annual distribution of demographic events is non-uniform.

The study of the intra-annual distributions of demographic events (deaths, births and migrations) has been a topic of research in numerous epidemiological and social science studies (e.g. Basu & Samet, 2002; Deschênes & Greenstone, 2011; Gray & Bilsborrow, 2013; Gustafson, 2002; Herteliu et al., 2015; Lam & Miron, 1994; Rau, 2007; Simó-Noguera et al., 2020), all of them concluding that, as a rule, demographic events display calendar variations. Their non-uniformity distributions have been related to both seasonal and extraordinary events. The increase of mortality in adulthood during winter periods (Díaz et al., 2005) and heat waves in summer (Basu & Samet, 2002; Bull & Morton, 1978) have been linked to seasonal factors related to intra-annual variations of climatology and extreme temperatures (Alderson, 1985).

They have also been related to non-seasonal factors, such as wars (via both direct and indirect effects, e.g. Jawad et al., 2020), health factors (Rau, 2007) or social factors (e.g. an increase in suicides at the beginning of an economic crisis; Ruiz-Perez et al., 2017). Extensive literature has also revealed strong seasonal patterns in conceptions and births (Ellison et al., 2005; Lam & Miron, 1994), pointing to an increase in conceptions during spring and summer (Lam & Miron, 1996) and as a response to the religious calendar (Herteliu et al., 2015). They have also happened in response to extraordinary events, such as the end of a war (as was the case, for instance, after the end of the Spanish Civil War; Lledó et al., 2017). The hypotheses of uniformity of migratory flows have been equally analysed and, as a rule, rejected (Lledó et al., 2017, 2019).

The supplementary material includes a graphical presentation of the intra-annual, non-uniform behaviour of the demographic events corresponding to the data sets used in this paper to illustrate our theoretical proposals. These non-uniformities can be observed in Figures S1–S19 available in Sections S1–S5 of the supplementary material where, for each age, the quarterly intra-age and seasonal distributions of the proportions of deaths and migrants recorded in Spain during the years 2005–2008 as well as the standardised monthly distributions of birth dates are presented.

In summary, demographic events do not behave uniformly throughout the year and this should be considered, along with the non-uniform behaviour of mortality between integer ages, for a proper management of risks. Further investigation of the impact of these intra-annual variations on mortality rates and on insurance products is still necessary, and that is the purpose of this research.

3 | METHODOLOGY

Considering the need to build sub-annual life tables to answer the research questions posed in this paper and given the overwhelming evidence showing the presence of intra-annual variations in demographic events, this section discusses the new formulae developed that, taking into account the exact moment of occurrence (in terms of both exact age and exact day) of the demographic events, enables quarterly life tables to be estimated. Without loss of generality, quarterly tables are created by dividing each age-year and calendar-year into four quarters, making a total of 16 subgroups. The expressions used and posterior computations can of course be generalised and adapted to other frequencies.

Although the classic (meteorological) definition of seasons (see, e.g. Boja et al., 2018) delimits calendar quarters in a different fashion (in the Northern Hemisphere: spring, March to May; summer, June to August; autumn, September to November; and winter, December to February), we divide the calendar-year into four equal parts starting at the beginning of the year. This division of the year is really close to the astronomical delimitations of seasons (Trenberth, 1983) and makes comparisons easier between sub-annual mortality risks. But, more importantly, it allows our additional methodological proposals to be introduced, that is, the computation of SAIs and their posterior use to derive quarterly tables from annual tables, which could not be implemented if a season straddles two years. The raw estimation of quarterly probabilities of death, nevertheless, is still possible with other definitions of quarters. Indeed, with the data of our empirical application, we have verified that if we had employed the classic definition of seasons, we would have attained estimates of quarterly probabilities of death quite similar to the ones that we have obtained with our definition of seasons (see Tables S1 and S2 of the supplementary material).

First, we define the notation required for our computations. As time variables, we define τ as the time elapsed in years ($0 < \tau < 1$) between the start of the year (0:00 AM on 1 January of annum a) and the moment of the occurrence of the event (date of death/birth/migration) within the year; x as the exact integer age completed by the subject when the event occurs; t as the fractional age in years ($0 < t < 1$) of the subject at the moment of the occurrence of the event; and y as the exact age of the subject when the event occurs. Note that $y = x + t$ and $x = \lfloor y \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function (i.e. the function that for any real number z computes the greatest integer number less than or equal to it). To place each event on a quarterly basis, we use $s = \lfloor 4\tau \rfloor + 1$ and $r = \lfloor 4t \rfloor + 1$ to denote, respectively, the season (year-quarter: winter, spring, summer and autumn, in the Northern Hemisphere) and the age-quarter (1Q, 2Q, 3Q and 4Q) in which the event occurs. Note that the correspondence between astronomical calendar seasons and our seasons (year-quarters) is not exact. For instance, we calculate winter (in a non-leap year) as a period of 91.25 days that ranges from 0:00 AM on 1 January to 6:00 AM on 2 April. For simplicity, we still use the names of the seasons to identify the year-quarters.

It should also be noted that although t and τ have been defined in continuous time, the exact ages and specific moments of the events are actually observed in a discrete fashion, at daily intervals. The date (day) of the birth of the person and the specific day of the year in which the (death or migration) event occurs are available, but not the exact instants within the day in which they occur. The exact ages and specific moments in which the events happen are therefore known with a maximum error of $1/T$ and $1/2T$ years, respectively, where $T = 365$ ($T = 366$ when a is a leap year). In our application, we have assumed for calculation purposes that the events (births, deaths and migrations) are evenly distributed throughout each day. Hence, on the one hand, we generate a random number δ from a uniform distribution in the interval $\left(-T^{-1}, T^{-1}\right)$ for each death or migration event—except for those corresponding to people who were born and died on the same day, in which case the interval used is $\left(0, T^{-1}\right)$. On the other hand, for those people who survive, we randomly generate a moment of birth within the day of their birth date. This strategy introduces a negligible random effect in the estimates but avoids biases by solving the problem of deciding in which quarter to place an event that falls between the limit of two quarters on a particular day.

For instance, for a person born on 2 April 2005 and deceased on 20 September 2007, we have (i) a completed age $x = 2$ when the death occurs, (ii) $t = \frac{171 + \delta}{365}$ years of exposure to the risk of dying with an age of x and (iii) $\tau = \frac{262.5 + .5\delta}{365}$ years of exposure to the risk of dying during 2007; with δ being a (really small) number from the interval $\left(\frac{-1}{365}, \frac{1}{365}\right)$. The figures 171 and 262.5 correspond to the number of days elapsed since, respectively, noontime 2 April 2007 and midnight 31 December 2006 to noontime 20 September 2007. In this example, the age-quarter and season-quarter in which the event occurs are expressed, respectively, as $r = \lfloor 4t \rfloor + 1 = 2$ (2Q) and $s = \lfloor 4\tau \rfloor + 1 = 3$ (summer).

Second, to note population variables and number of events, we use C_{x+t}^{a} to represent the total population with exact age $x + t$ ($x \in \mathbb{N}$, $0 < t < 1$) on 1 January of year a (at midnight 31 December of year $a - 1$) and B_{τ}^{a} to represent the number of births registered after τ years ($0 < \tau < 1$), or equivalently τT days, have elapsed since the beginning of the year a. Likewise, we denote by $d^{a}(x + t, \tau)$, $e^{a}(x + t, \tau)$ and $i^{a}(x + t, \tau)$, respectively, the number of deaths, emigrants and immigrants recorded in year a with exact age $x + t$ ($x \in \mathbb{N}$, $0 < t < 1$) after τT days have elapsed since the beginning of the year. At this point and denoting the indicator function by $I(\cdot)$, which takes the value 1 if the condition is met and 0 otherwise, we can derive new variables: ${}_{s}^{r}D_{x}^{a}$, ${}_{s}^{r}E_{x}^{a}$ and ${}_{s}^{r}I_{x}^{a}$ defined, respectively, as the number of deaths, emigrants and immigrants with completed age

(age at last birthday) x in age-quarter r and season-quarter s of year a, whose relationship with the previous variables is given by the following expressions:

$$
{}_s^rD_x^a = \sum_{0<t<1} \sum_{0<\tau<1} d^a(x+t,\tau)I\left(\frac{r-1}{4} < t < \frac{r}{4}\right)I\left(\frac{s-1}{4} < \tau < \frac{s}{4}\right)
$$

$$
{}_s^rE_x^a = \sum_{0<t<1} \sum_{0<\tau<1} e^a(x+t,\tau)I\left(\frac{r-1}{4} < \tau < \frac{r}{4}\right)I\left(\frac{s-1}{4} < t < \frac{s}{4}\right)
$$

$$
{}_s^rI_x^a = \sum_{0<t<1} \sum_{0<\tau<1} i^a(x+t,\tau)I\left(\frac{r-1}{4} < \tau < \frac{r}{4}\right)I\left(\frac{s-1}{4} < t < \frac{s}{4}\right).
$$

Furthermore, we denote the total number of deaths, emigrants and immigrants with completed age x in year a by $D_x^a = \sum_{r=1}^4\sum_{s=1}^4 {}_s^rD_x^a$, $E_x^a = \sum_{r=1}^4\sum_{s=1}^4 {}_s^rE_x^a$ and $I_x^a = \sum_{r=1}^4\sum_{s=1}^4 {}_s^rI_x^a$, respectively.

Third, in order to quantify the total population at risk of dying (or the total number of 'person-years' at risk of dying), we introduce a new expression. We denote, by $P^a(x+t,\tau)$, the population alive in year a with exact age $x+t$ ($x \in \mathbb{N}$, $0 \le t \le 1$) after τ years (τT days, $0 \le \tau \le 1$) have elapsed since the beginning of the year. These new quantities can be derived from the previous ones through the equations: $P^a(x+t,\tau) = C_{x+t-\tau}^a + \sum_{0<\zeta<\tau}\left[i^a(x+t-\tau+\zeta,\zeta) - d^a(x+t-\tau+\zeta,\zeta) - e^a(x+t-\tau+\zeta,\zeta)\right]$ when $x > 0$ and $P^a(t,\tau) = B_{\tau-t}^a + \sum_{t<\zeta<\tau-t}\left[i^a(t-\tau+\zeta,\zeta) - d^a(t-\tau+\zeta,\zeta) - e^a(t-\tau+\zeta,\zeta)\right]$ when $x = 0$ and $t \le \tau$.

We now have the definitions for all the statistics necessary to estimate the quarterly (central) death rates. These statistics could be derived from the available microdata that current official statistical systems produce. On the one hand, we can compute $d^a(x+t,\tau)$, $e^a(x+t,\tau)$, $i^a(x+t,\tau)$ from the dates of births and of deaths, immigration and emigration recorded in year a. On the other hand, B_τ^a and C_{x+t}^a can be attained, respectively, from official statistics of year a of births and stocks of populations (census) that include birth dates.

With the above notation, in a given quarter (r,s) of year a, the total time of exposure to risk of dying of the population is obtained as the sum of (i) the time at risk of dying in the quarter of the subjects counted in $P^a\left(x+t,\frac{s}{4}\right)$, for $t \in \mathscr{F}_s = \left[\frac{s-1}{4},\frac{s}{4}\right]$, (ii) the time at risk of dying in the quarter of the subjects counted in $P^a\left(x+\frac{r}{4},\tau\right)$, for $\tau \in \mathscr{F}_r = \left[\frac{r-1}{4},\frac{r}{4}\right]$, (iii) the time alive in the quarter of the subjects counted in ${}_r^sD_x^a$, which are not counted either in $P^a\left(x+t,\frac{s}{4}\right)$ or in $P^a\left(x+\frac{r}{4},\tau\right)$, (iv) the time at risk of dying in the quarter as a member of the target population of the subjects counted in ${}_r^sE_x^a$, which are not counted either in $P^a\left(x+t,\frac{s}{4}\right)$ or in $P^a\left(x+\frac{r}{4},\tau\right)$, minus (v) the time not at risk of dying in the quarter as a member of the study population of the subjects counted in ${}_r^sI_x^a$, which have been counted in either $P^a\left(x+t,\frac{s}{4}\right)$ or $P^a\left(x+\frac{r}{4},\tau\right)$. In particular, the total number of 'person-years' at risk of dying with age x in the quarter (r,s) of the year a, ${}_r^sI_x^a$, is obtained through the equation:

$$
{}_s^rL_x^a = \sum_{\tau\in\mathscr{F}_s}\left(\tau - \frac{s-1}{4}\right)P^a\left(x+\frac{r}{4},\tau\right) + \sum_{t\in\mathscr{F}_r}\left(t - \frac{r-1}{4}\right)P^a\left(x+t,\frac{s}{4}\right)
$$

$$
+ \sum_{\{4\tau\}>\{4t\}}\left(t - \frac{s-1}{4}\right)\left(d^a(x+t,\tau) + e^a(x+t,\tau) - i^a(x+t,\tau)\right)
$$

$$
\times I\left(\frac{r-1}{4} < \tau < \frac{r}{4}\right)I\left(\frac{s-1}{4} < t < \frac{s}{4}\right)
$$

$$+ \sum_{\{4\tau\}<\{4t\}} \left(\tau - \frac{r-1}{4}\right)\left(d^a(x+t,\tau) + e^a(x+t,\tau) - i^a(x+t,\tau)\right)$$

$$\times I\left(\frac{r-1}{4} < \tau < \frac{r}{4}\right) I\left(\frac{s-1}{4} < t < \frac{s}{4}\right),$$

where $\{\cdot\}$ denotes the fractional part function. That is, $\{z\} = z - \lfloor z \rfloor$ for any real number $z \geq 0$.

For instance, considering a woman born on 31 March 1972 and emigrating from our target population on 29 September 2005, we calculate, assuming that both birth and emigration moments occurred at midday, that she has a completed age $x = 33$, with $t \approx 0.498630$ and $\tau \approx 0.743836$, in the moment of her emigration, which means that the event occurred in the age-season quarter $(2,3)$, $r = \lfloor 4t \rfloor + 1 = 2$ and $s = \lfloor 4\tau \rfloor + 1 = 3$. Likewise, we calculate (working with six decimals) that during 2005 and with completed age 33, this person was also a member of the following populations: $P^{2005}\left(33 + 0.004795, \frac{1}{4}\right)$, $P^{2005}\left(33 + \frac{1}{4}, 0.495205\right)$ and $P^{2005}\left(33 + 0.254795, \frac{2}{4}\right)$. Therefore, this emigrant has been exposed to the risk of dying during 2005 with a completed age 33 in the following (r,s), age-season, quarters: 0.004795 years in quarter $(1,1)$, 0.245205 years in quarter $(1,2)$, 0.004795 years in quarter $(2,2)$ and 0.243836 years in quarter $(2,3)$. Note that this woman was also exposed to the risk of dying as a member of our target population during 2005 but with a completed age $x = 32$ and, in particular, as a member of the population $P^{2005}\left(32 + \frac{4}{4}, 0.245205\right)$, which means 0.245205 years of exposure with completed age 32 during quarter $(4,1)$.

Once the total number of 'person-years' at risk of dying for each completed age x in each quarter (r, s) of year a has been computed, a crude estimate of the central death rate, ${}^r_s m^a_x$, of the quarter can be easily obtained. Just as with annual figures, we obtain this by dividing the total number of deaths recorded in the quarter, ${}^r_s D^a_x$, by ${}^r_s L^a_x$.

$$\substack{r\\s} m^a_x = \frac{{}^r_s D^a_x}{{}^r_s L^a_x}.$$

Note that the crude annual central rate of mortality ($m^a_x = D^a_x / L^a_x$, where $L^a_x = \sum_{r=1}^4 \sum_{s=1}^4 {}^r_s L^a_x$) is no more than a weighted average of the quarterly rates, with weights being the aggregates of times of exposure to risk of dying in each quarter.

$$m^a_x = \frac{D^a_x}{L^a_x} = \frac{\sum_{r=1}^4 \sum_{s=1}^4 {}^r_s D^a_x}{\sum_{r=1}^4 \sum_{s=1}^4 {}^r_s L^a_x} = \sum_{r=1}^4 \sum_{s=1}^4 \frac{{}^r_s D^a_x}{\sum_{r'=1}^4 \sum_{s'=1}^4 {}^{r'}_{s'} L^a_x}$$

$$= \sum_{r=1}^4 \sum_{s=1}^4 \frac{{}^r_s L^a_x}{\sum_{r'=1}^4 \sum_{s'=1}^4 {}^{r'}_{s'} L^a_x} \cdot \frac{{}^r_s D^a_x}{{}^r_s L^a_x} = \sum_{r=1}^4 \sum_{s=1}^4 \frac{{}^r_s L^a_x}{\sum_{r'=1}^4 \sum_{s'=1}^4 {}^{r'}_{s'} L^a_x} \cdot \substack{r\\s} m^a_x$$

In the same fashion that (r, s) quarterly rates are estimated, we can also estimate marginal r or s rates. That is, seasonal and ageing central rates of mortality can be obtained either as a rate between deaths and exposed-to-risk or as weighted averages of (r, s) rates once the corresponding age-quarter or season-quarter is kept fixed:

$$_s m^a_x = \frac{_s D^a_x}{_s L^a_x} = \frac{\sum_{r=1}^4 {}^r_s D^a_x}{\sum_{r=1}^4 {}^r_s L^a_x} = \sum_{r=1}^4 \frac{{}^r_s D^a_x}{\sum_{r'=1}^4 {}^{r'}_s L^a_x} = \sum_{r=1}^4 \frac{{}^r_s L^a_x}{\sum_{r'=1}^4 {}^{r'}_s L^a_x} \cdot \frac{{}^r_s D^a_x}{{}^r_s L^a_x} = \sum_{r=1}^4 \frac{{}^r_s L^a_x}{\sum_{r'=1}^4 {}^{r'}_s L^a_x} \cdot \substack{r\\s} m^a_x$$

$$^r m^a_x = \frac{^r D^a_x}{^r L^a_x} = \frac{\sum_{s=1}^4 {}^r_s D^a_x}{\sum_{s=1}^4 {}^r_s L^a_x} = \sum_{s=1}^4 \frac{{}^r_s D^a_x}{\sum_{s'=1}^4 {}^r_{s'} L^a_x} = \sum_{s=1}^4 \frac{{}^r_s L^a_x}{\sum_{s'=1}^4 {}^r_{s'} L^a_x} \cdot \frac{{}^r_s D^a_x}{{}^r_s L^a_x} = \sum_{s=1}^4 \frac{{}^r_s L^a_x}{\sum_{s'=1}^4 {}^r_{s'} L^a_x} \cdot \substack{r\\s} m^a_x$$

Death rates are only used as intermediate tools for building life tables. Once estimates of crude death rates are available, (raw) death probability estimates, q_x, are obtained from them. Quarterly estimates of death probabilities corresponding to a (r, s) quarter can be estimated either (S1) assuming a uniform distribution of deaths within the period under study (in this case a quarter), or (S2) taking into account the average number of quarters lived, ${}_s^r f_x$, in the quarter s for those who died with an exact age between $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$. Under (S1), probability estimates are obtained through ${}_{\frac{1}{4}} q^{(s)}_{x + \frac{(r-1)}{4}} = \frac{{}_s^r m_x}{4 + \frac{1}{2} {}_s^r m_x}$, whereas under (S2) the proper expression is ${}_{\frac{1}{4}} q^{(s)}_{x + \frac{(r-1)}{4}} = \frac{{}_s^r m_x}{4 + (1 - {}_s^r f_x) {}_s^r m_x}$; where ${}_{\frac{1}{4}} q^{(s)}_{x + \frac{(r-1)}{4}}$ denotes the probability of dying in quarter s with an exact age between $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$.

4 | ESTIMATING SEASONAL–AGEING INDEXES

A death rate can be seen as a discrete indicator of the strength of the force of mortality during a given period. Indeed, when the period considered is short (a year or, ideally, even less), the central rate of mortality is approximately equal to the average force of mortality over the period. Therefore, m_x^a can be thought of as the average force of mortality at age x last birthday during year a and ${}_s^r m_x^a$ as the average force of mortality between ages $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$ during the quarter s of year a. In the previous section, we developed an original quarter period-based estimator for constructing quarterly life tables. In this section, we analyse and model the intra-annual variations of mortality rates, ${}_s^r m_x^a$, by comparing them to the mortality rate of the whole period, m_x^a, which is assumed to be constant through both the age-year and the calendar-year when constructing annual tables.

One of the rewards of working with death rates lies in the fact that, irrespective of the period considered, they are all expressed on the same scale. That is to say, m_x^a and ${}_s^r m_x^a$ can be compared directly. For example, when ${}_s^r m_x^a > m_x^a$ for a given (r, s)-quarter, it means that during that quarter the risk of dying per unit of time of a person with an age between ages $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$ is on average higher than the average risk of dying of an average person with completed age x per unit at any time during that age. Hence, by systematically comparing ${}_s^r m_x^a$ and m_x^a, we can find out (for each age x and year a) what the combinations of age-quarters and season-quarters are with higher and lower risks. Furthermore, by exploiting the seasonal patterns and ageing patterns that mortality (see, e.g. Figures S1–S10 in the supplementary material) and other demographic events (see, e.g. Figures S11–S19 in the supplementary material) show, we can also estimate SAIs. These indexes can tell us about the increase or decrease of the risk of dying that a person with integer age x faces in a given (r, s) quarter compared to average risk at that age.

In our approach, as is usual in seasonal adjustment of time series, we consider that the seasonal-ageing variations are stationary. That is, we assume that the coefficients, $\gamma_{rs}^{(x)}$, that capture the (underlying) variations in the risk of mortality that a person with completed age x experiences during season s between ages $x + \frac{r-1}{4}$ and $x + \frac{r}{4}$ does not depend on a. In other words, for the estimation of the $\gamma_{rs}^{(x)}$ coefficients, we should not consider years where the intra-annual patterns of mortality are broken due to exceptional circumstances (e.g. a war, a revolution or a pandemic). Likewise, given that the risk of mortality is not stationary (i.e. it evolves over time), we also assume that the variations induced by the $\gamma_{rs}^{(x)}$ coefficients are relative, that is, that they impact in a proportional fashion. In short, when no exceptional circumstances are happening, we consider that ${}_s^r m_x^a \approx m_x^a \gamma_{rs}^{(x)}$. Hence, as a consequence of the natural fluctuations that mortality (and other demographic events) experience through the years (over time), these indexes should

be computed by smoothing the observed variations of several years, $a = 1, 2, \ldots, A$, where A is assumed not to be excessively large since, as Ledberg (2020) shows, seasonal fluctuations also evolve in the long-term.

A way of estimating the SAIs is to model the log of the ratio between the quarterly and annual rates as a function of the corresponding age-quarter (ageing effect) and calendar-quarter (seasonal effect) and some random effects, $_s^r\varepsilon_x^a$, and to estimate the coefficients of the model by OLS.

$$log\left(\frac{_s^r m_x^a}{m_x^a}\right) = log\left(\gamma_{rs}^{(x)}\right) + _s^r\varepsilon_x^a \quad \text{for} \quad a = 1, 2, \ldots, A$$

In the above model, we are implicitly assuming that there is interaction between the ageing and the seasonal effects. For example, we assume that on average the effect of winter on the risk of dying is not the same for an 80-year-old person as it is for an 80.75-year-old person. Looking at Figures S1–S10 in the supplementary material, this seems to be the case, at least for the newly born and the most elderly people. To estimate the marginal effects, we can use the models $log\left(\frac{_r m_x^a}{m_x^a}\right) = log\left(\alpha_r^{(x)}\right) + {}^r\varepsilon_x^a$ and $log\left(\frac{_s m_x^a}{m_x^a}\right) = log\left(\beta_s^{(x)}\right) + _s\varepsilon_x^a$, where $\alpha_r^{(x)}$ and $\beta_s^{(x)}$ represent, respectively, the marginal ageing and seasonal effects in age x. Under the hypothesis of non-interaction effects, this gives us: $\gamma_{rs}^{(x)} = \alpha_r^{(x)} \cdot \beta_s^{(x)}$.

Estimating coefficients of the above models by OLS is mathematically equivalent to taking geometric means: $\hat{\gamma}_{rs}^{(x)} = \sqrt[A]{\prod_{a=1}^A \frac{_s^r m_x^a}{m_x^a}}$, $\hat{\alpha}_r^{(x)} = \sqrt[A]{\prod_{a=1}^A \frac{_r m_x^a}{m_x^a}}$ and $\hat{\beta}_s^{(x)} = \sqrt[A]{\prod_{a=1}^A \frac{_s m_x^a}{m_x^a}}$. Hence, as an alternative, the most classical approach used in time series analysis of taking arithmetic means of the ratios (see, e.g. Newbold et al., 2019) can be also used. Note that this alternative approach is equivalent to assuming that relationship $\frac{_s^r m_x^a}{m_x^a} = \gamma_{rs}^{(x)} + _s^r\varepsilon_x^a$ holds. This implies that the annual irregular (random) fluctuations, $_s^r\varepsilon_x^a$, impact additively on the SAIs. In our main specification, we are implicitly assuming that they impact multiplicatively, which we consider an issue with better theoretical foundations. Furthermore, the geometric mean has the property of being ratio-invariant: with geometric averages the average ratio and the ratio of averages coincide. We have, however, found almost no differences in using either geometric or arithmetic means in our application.

Whatever the mean used, the estimated coefficients are tied by aggregation constraints. For each age x, the sum of the $\hat{\gamma}_{rs}^{(x)}$ coefficients must be 16, whereas the aggregation of either $\hat{\alpha}_r^{(x)}$ or $\hat{\beta}_s^{(x)}$ must equal 4. In practice, after estimation, some small differences are observed. Thus, as is usual practice in business and economics, we propose adjusting the estimated raw intra-annual variation indexes so that their average for the year is 1.

5 | BUILDING SUB-ANNUAL LIFE TABLES USING SAIs

Using the strategies and estimators introduced in Section 3, sub-annual (e.g. quarterly) life tables can be built in a similar fashion to annual tables. Sub-annual crude central death rates for any frequency can be directly estimated from observed demographic data and, from them, sub-annual tables constructed. However, due to the exponential dividing effect of increasing frequencies, this approach requires large populations to yield proper results. Just to build quarterly tables, and in comparison to an annual table, we need to split out the statistics of deaths and exposures into 16

subgroups. This number grows even more, to 144, if we are interested in constructing monthly tables.

These reductions in the sample sizes of exposed-to-risk populations directly impact on the uncertainty of the sub-annual estimates. When the populations are not that large (as is the case, for instance, in the portfolios of insurance companies, at least for some ages), the strategy of directly building sub-annual tables leads to data estimates that are too noisy. Thus, as an alternative, we propose applying to the annual life table of the population of interest the SAIs obtained from a reference population, a significantly larger population exposed to similar climatic and socio-economic conditions as our population of interest. This will have no impact on the average levels of the risks of dying of the members of our population, which have been estimated using significantly larger samples, but allows to intra-annually distribute the risks.

As occurs with death probabilities or rates, SAIs show age trends. All of them, (r, s) SAIs and marginal SAIs (see Figures 2 and 3; and also Figures S32–S35 in the supplementary material), tend to increase with age under adverse climatic conditions and to decrease when the climatic conditions are less harsh. Both the ageing effects and the seasonal effects have a relatively higher impact as people get older, in line with a scenario where health conditions worsen with age. Although in empirical applications these trends can be clearly observed in crude estimates of SAIs, in the same fashion as with crude estimates of death probabilities or rates, they may show saw-tooth patterns at certain ages, such as young ages or very old ages, as a consequence of a lower representation of deaths in the empirical data. Hence, before applying crude SAIs, we propose a smoothing of the patterns as a function of time. This is not new in this framework; graduation techniques are routinely employed in mortality analysis with the aim of producing smoother death estimates from initial crude death estimates (e.g. Debón et al., 2005). In the light of the results obtained in our empirical application, a linear trend would be enough.

Once smoother SAIs, $\tilde{\gamma}_{rs}^{(x)}$, are obtained, estimates of quarterly (sub-annual) central death rates, to generate quarterly tables, are calculated through the equation ${}_s^r \tilde{m}_x = m_x \tilde{\gamma}_{rs}^{(x)}$, and from them estimates of the dying probabilities for the quarterly table by ${}_{\frac{1}{4}} \tilde{q}_{x+\frac{(r-1)}{4}}^{(s)} = \frac{{}_s^r \tilde{m}_x}{4 + \frac{1}{2} {}_s^r \tilde{m}_x}$. The 4 in the denominator comes from the fact a year has four quarters, as can be deduced from the following biometric relationships:

$$
{}_s^r m_x = \frac{{}_s^r D_x}{{}_s^r L_x} = \frac{{}_s^r D_x}{\frac{1}{4} \cdot \ell_{x+\frac{r-1}{4}}^{(s)} - \frac{1}{8} \cdot {}_s^r D_x} = \frac{4 \cdot {}_s^r D_x}{\ell_{x+\frac{r-1}{4}}^{(s)} - \frac{1}{2} \cdot {}_s^r D_x} = \frac{4 \cdot {}_{\frac{1}{4}} q_{x+\frac{(r-1)}{4}}^{(s)}}{1 - \frac{1}{2} \cdot {}_{\frac{1}{4}} q_{x+\frac{(r-1)}{4}}^{(s)}} \Rightarrow {}_{\frac{1}{4}} q_{x+\frac{(r-1)}{4}}^{(s)} = \frac{{}_s^r m_x}{4 + \frac{1}{2} {}_s^r m_x}
$$

where we have denoted by $\ell_{x+\frac{r-1}{4}}^{(s)}$ the number of persons surviving to exact age x at the beginning of season s and we have used in the third mathematical expression that a person who survives the whole quarter lives $\frac{1}{4}$ of a year and that, under uniform distribution of deaths, a person who dies in a quarter lives on average half of the quarter (i.e. $\frac{1}{8}$ of a year). Likewise, we have divided by $\ell_{x+\frac{r-1}{4}}^{(s)}$ and used that, by definition, ${}_{\frac{1}{4}} q_{x+\frac{(r-1)}{4}}^{(s)} = \frac{{}_s^r D_x}{\ell_{x+\frac{r-1}{4}}^{(s)}}$ to reach the fifth expression. In general, if we denote by F the frequency of the sub-annual table, the correcting factor to pass from death rates to death probabilities is F.

It should be noted that to build the quarterly (sub-annual) tables the estimates probabilities must be combined in the proper way, depending on the season when the birthday occurs, and that the time unit of measure is now the quarter. Denoting winter by w, spring by sp, summer by su and autumn by au, we can express the sequential probabilities of dying associated with a

person born, for instance, in spring as: $\frac{1}{4}q_0^{sp}, \ldots, \frac{1}{4}q_{x-\frac{1}{4}}^{w}, \frac{1}{4}q_x^{sp}, \frac{1}{4}q_{x+\frac{1}{4}}^{su}, \frac{1}{4}q_{x+\frac{2}{4}}^{au}, \frac{1}{4}q_{x+\frac{3}{4}}^{w}, \frac{1}{4}q_{x+\frac{5}{4}}^{sp}, \ldots$.
We now have four quarterly life tables, depending on the season of birth.

6 | EXEMPLIFYING THE METHODOLOGY. AN APPLICATION TO SPAIN

6.1 | Data and software

In the previous sections, we have detailed a new methodology for constructing (sub-annual) quarterly life tables and for estimating SAIs. In this section, we demonstrate its use by applying it to a real database. We use a database comprising of microdata of population and demographic events of Spain for the years 2005–2008. The microdata have been provided by the Spanish National Institute of Statistics (henceforth, INE) by payment in advance. The database consists of detailed statistics of births, deaths, emigrations and immigrations recorded in Spain during the years 2005–2008 as well as stocks of people residing in Spain as of 1 January for each of these years.

Birth statistics, which include gender and date of birth, are available free in the section Vital Statistics of INE. Emigrant and immigrant microdata come from the Statistics of Residential Variation, which compile ins and outs from foreign countries by gender, including the exact dates of birth and of migration of each migrant. Death microdata include nationality, gender and dates of birth and of death of each deceased and where provided by INE. Stocks of population come from the Population Now Cast estimates, which is a synthetic statistic developed by INE from administrative registers intended to determine at any given time the profile of the resident population in Spain, broken down by sex and age. The microdata of population supplied by INE also include the dates of birth of all Spanish residents. Overall, we have handled, processed and analysed about 180.15 million population inputs, 1.5 million death inputs, 0.7 million emigrant inputs, 3.2 million immigrant inputs and around 1 million birth inputs for the period 2005–2008. In total, more than 186 million demographic events were dealt with individually.

Demographic data available in official statistical agencies are not perfect (Cairns et al., 2016; Kelly, 1987; Lledó et al., 2017), as is the case with INE data. INE agents pointed out that when an immigrant does not know the exact day of his/her birth, this date is administratively set to be 1 January. This provokes an artificial peak on that day. The same occurs when one deals with emigrant data, as a significant portion of Spanish emigrants are former immigrants. To solve these issues, we followed recommendations made by Lledó et al. (2017) and we randomly assigned a date of birth to a randomly selected number of first-of-January-born immigrants (emigrants) equal to the yearly average excess. All the computations described and presented in this research have been performed using ad-hoc scripts in the statistical software R, version 4.0.2 (R Core Team, 2020).

Some graphical statistical summaries of the data analysed are available in Sections S1–S5 of the supplementary material attached to this paper. As can be seen in Figures S1–S10, where the intra-annual (quarterly age/calendar) distributions of deaths recorded in Spain during the period 2005–2008 are displayed by gender, death events do not occur uniformly, neither along each age nor over the year. Both biological and seasonal issues impact on the intra-annual distributions of deaths at each age. On the one hand, as is no surprise, the deaths are concentrated into the first quarter of life (mainly during the first days/weeks) at age 0. On the other hand, the combining effects of season and ageing are chiefly observed for adult ages. Within each calendar

year and from 50 years of age, the number of deaths is relatively higher during winter and, to a lesser extent, also during autumn. This calendar effect interacts with the ageing effect. During these seasons the number of deaths is, as a rule, higher over the second half of each age. Likewise, from 70 years of age, we also observe an increase of deaths in winter during the first age-quarter, maybe due to an interaction between climatology and psychological factors. For other patterns, although not so sharp, we can hazard a guess as to the reason for their occurrence. We see some relative increase in deaths of masculine teenagers and in their early twenties during summer, likely as a consequence of an increase of risk behaviour of this group of people during the summer holidays. This is in line with the results reported by Rau et al. (2018) and Parks et al. (2018).

The non-uniformity of intra-annual distributions is not just seen in the number of deaths; it is also evident for other demographic events. In Figures S11 and S12, where the intra-annual distributions of emigrants are displayed by gender, we can see that emigration in Spain is highly seasonal; it is mainly concentrated in autumn, after the summer period (when many jobs related to the summer holiday season are terminated and the Spanish unemployment rate escalates). Analysing immigrant figures (see Figures S13 and S14 in the supplementary material), we see quite uniform distributions, with slightly higher intra-annual concentrations during the first two quarters of the calendar year for people aged from 20 to 50 and in summer for teenagers. It seems that, on the one hand, a significant portion of Spanish immigrants between 20 and 50 years old are job seekers that come to Spain at the beginning of the calendar year and, on the other hand, there is some immigration flow related to family reunions during summer, once the academic year has finished in the country of origin of the immigrants and the enrolment of children in Spanish schools at the beginning of the academic year starts.

Finally, as Figures S15–S19 show, birth dates also show seasonal patterns. These patterns however have evolved over time. While the standardised distribution of birthdays is quite uniform over the calendar-year for younger people, we observe how progressively, in the older population, more and more people have their birthdays during the first two quarters of the calendar year, and mainly in winter. These results are in line with the expected behaviour of a society that during the 20th century evolved from a society governed by a natural scheme of fertility to a secularised society in which an effective control of fertility is exercised. For the oldest people, from 90 years and beyond, the monthly distributions of birthdays seem to reflect some ageing effects, as the relative proportions of people born during the second half of the calendar-year tend to increase for each age. Furthermore, some administrative delay is observed in the registration of births and this has an impact on the statistics of stocks of population at 1 January in the form of smaller proportions of people recorded with completed age 0 born in December (and also in November).

6.2 | Seasonal and ageing estimation of mortality risks

After applying the strategies and mathematical expressions introduced in Section 3 to the data described in the above subsection, crude estimates of quarterly death rates and raw estimates of probabilities of mortality are obtained for each age x ($x = 0, 1, \ldots, 100$), year a ($a = 2005, \ldots, 2008$), and (r, s)-quarter, ($r, s = 1, \ldots, 4$). Given that the aim of this section is just to show how quarterly tables can be built and SAIs estimated, we prefer to omit the issues related to the estimation and graduation of rates for centenarians (Li & Liu, 2019). A graphical summary of the raw estimates of the quarterly probabilities of death by season of birth is presented
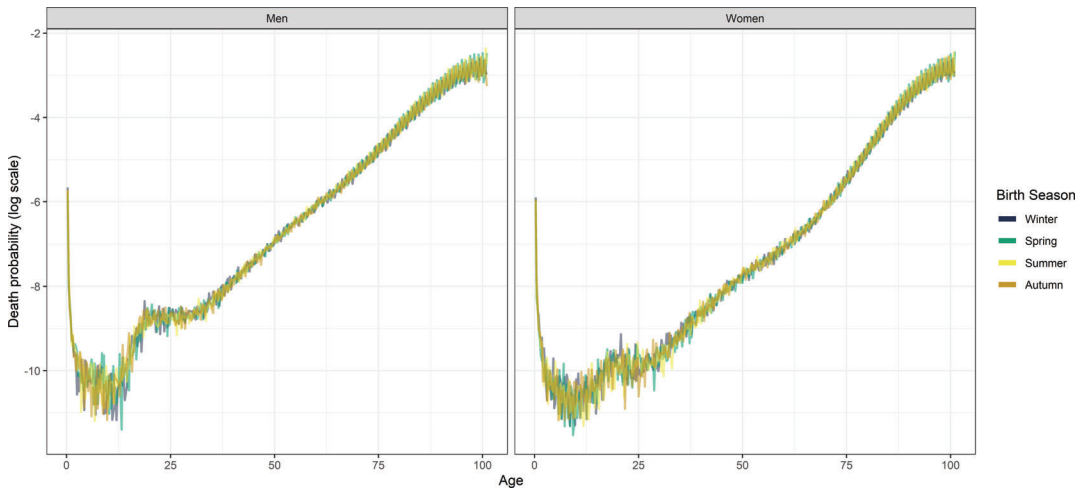
FIGURE 1 Raw estimates of quarterly probabilities of death by season in which the birthday falls. Aggregation 2005–2008. Men (left panel) and women (right panel). The sequential raw probabilities presented (in log-scale) in the panels have been obtained, assisted by the mathematical expressions introduced in Section 3, by mapping crude rates into raw probabilities after computing, for each (r, s) age-season quarter, the number of deaths and the total 'person-years' at risk of dying by aggregating the number of deaths and exposed-to-risk of the four years: ${}^r_s D_x = \sum_{a=2005}^{2008} {}^r_s D_x^a$ and ${}^r_s L_x = \sum_{a=2005}^{2008} {}^r_s L_x^a$

in Figure 1. The estimates in Figure 1 represent a numerical synthesis of the corresponding estimates of the 4 years, obtained after aggregating the number of deaths and exposed-to-risk of the 4 years. In Section S6 of the supplementary material, interested readers can also consult the equivalent figures for each of the years (see Figures S20–S23), as well as details of the estimates displayed in Figure 1 for, respectively, the age ranges 0 to 50 and 51 to 100 (see Figures S24 and S25). Furthermore, just for explanatory purposes, in Figures S26–S31 we show the raw estimates of the quarterly probabilities in a different fashion and in Tables S1 and S2 we summarise, in relative absolute terms and for the age ranges 50–89, the differences between the estimates obtained and those that would have been attained if a classic meteorological scheme had been used to divide the calendar-year. In Figures S26–S31, the time sequences used to present death probabilities within ages are hypothetical and have no meaning in the real world; they are just used to help the reader to better appreciate the impact of seasonal and ageing effects. These figures show the probabilities as if the exposed-to-risk were always living during age x either in the same season (Figures S26–S28) or in the same quarter of age (Figures S29–S31), conditioned to having had a normal life before and after age x.

In addition to obtaining raw estimates of the probabilities of death, ${}_{\frac{1}{4}} \hat{q}^{(s)}_{x+\frac{(r-1)}{4}}$, we have also used the estimates of crude mortality, ${}^r_s \hat{m}^a_x$ (for $x = 0, 1, \ldots, 100$, $a = 2005, \ldots, 2008$ and $r, s = 1, \ldots, 4$) for estimating, using the approach detailed in Section 4, the SAIs, $\hat{\gamma}^{(x)}_{rs}$, of the Spanish population. Figure 2 shows the raw indexes for men aged between 50 and 100, obtained using geometric means, as well as their smoothing with linear trends. Figure 3 presents the same data for women. In order not to disturb the readers with noise and distract them away from the main message of these graphical representations, in Figures 2 and 3, we have not included the indexes for ages 0–49 due to their high volatility. At these ages, the force of mortality is weak and so the number of deaths tends to be small unless working with very large populations. In these
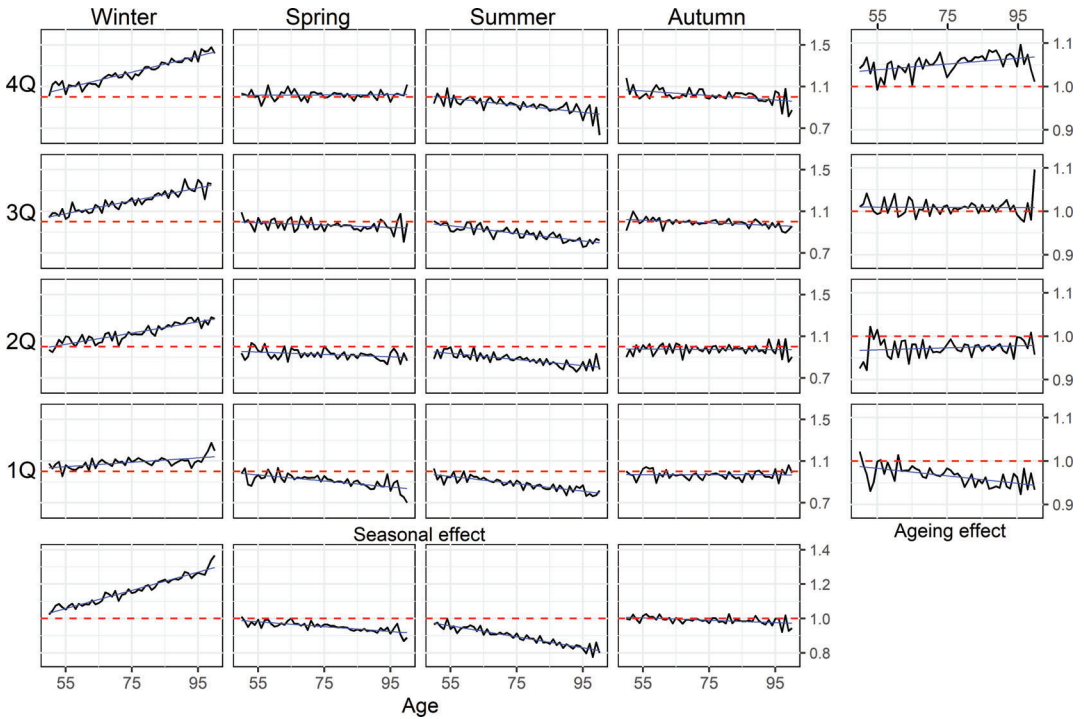
FIGURE 2 Estimates of seasonal–ageing indexes for men aged between 50 and 100. The top 16 panels on the left show, for each age-quarter and season-quarter, the combined seasonal–ageing effects, $\gamma_{rs}^{(x)}$. The four rightmost panels show the ageing (marginal) effects, $\alpha_{rs}^{(x)}$, and the four lowest panels show the seasonal (marginal) effects, $\beta_{rs}^{(x)}$. Dashed red lines have been included as reference of no effect. Age trend blue lines of the indexes have also been included in the plots. Note the difference in scale between the central and marginal panels

circumstances, estimates of death rates and consequently of SAIs are more volatile. The Killick et al. (2012) test for detecting structural changes in variance was used for determining the change point, which appears to be around age 50 in this data set.

The overall picture regarding SAIs is given in Sections S7–S9 of the supplementary material, where we offer a more comprehensive presentation of the estimated SAIs: graphically in Section S7 and numerically in Sections S8 and S9. In Section S7, Figures S32 and S33 show the indexes estimated using geometric means for, respectively, men and women of ages between 1 and 100. Figures S34 and S35 display the same indexes calculated employing arithmetic means. We have not included in any of these figures age 0 due to the different behaviour of mortality and of SAIs at this age. Indeed, we do not recommend using the values of age 0 to smooth the series of SAIs. The actual numbers obtained for the raw and smoothed estimated SAIs, using geometric means, are presented, respectively, in Section S8 (see Tables S3–S103) and in Section S9 (see Tables S104–S204). As an example, Table 1 shows the smoothed estimated indexes attained for ages 0 and 65.

Several interesting findings regarding the intra-annual fluctuations of the risks of mortality emerge when analysing Figures 2 and 3 (and also Figures S32–S35 and Tables S3–S204). First, seasonal effects are, as a rule, stronger than ageing effects. For a fixed completed age x, the season has a higher impact on the risk of death than the exact age of the exposed-to-risk, early or late x within the age-year. Age 0 is the exception. At age 0, the higher risks are
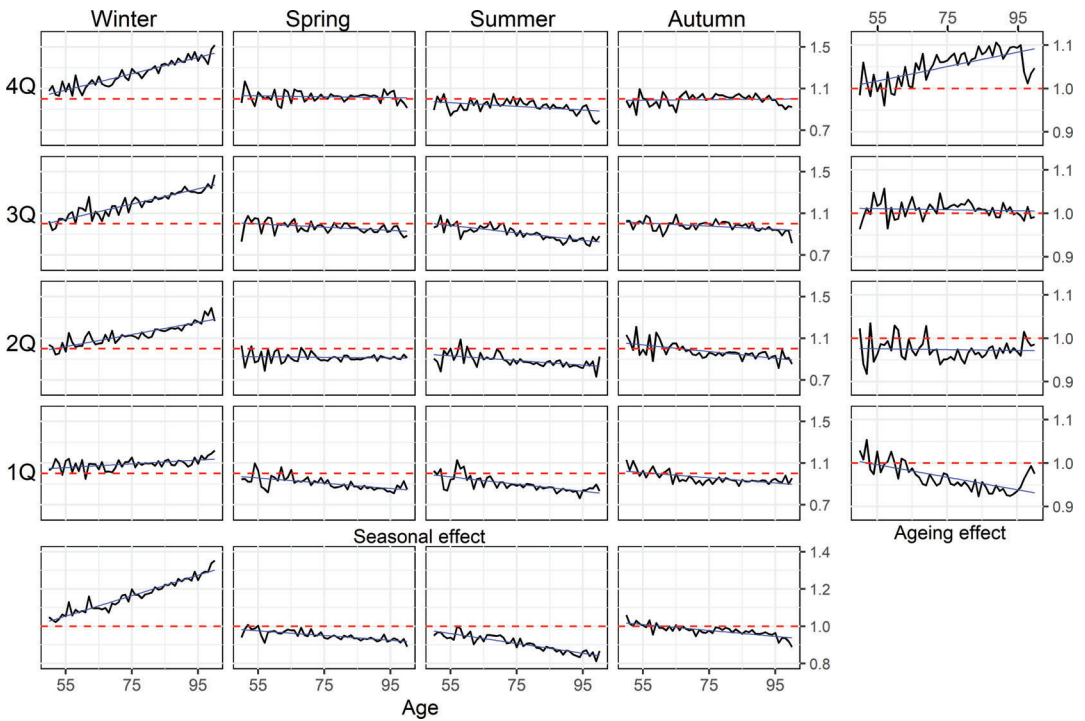
FIGURE 3 Estimates of seasonal–ageing indexes for women aged between 55 and 100. The top 16 panels on the left show, for each age-quarter and season-quarter, the combined seasonal–ageing effects, $\gamma_{rs}^{(x)}$. The four rightmost panels show the ageing (marginal) effects, $\alpha_{rs}^{(x)}$, and the four lowest panels show the seasonal (marginal) effects, $\beta_{rs}^{(x)}$. Dashed red lines have been included as reference of no effect. Age trend blue lines of the indexes have also been included in the plots. Note the difference in scale between the central and marginal panels

during the first quarter of life, irrespective of the season. Second, the intensity of the effects varies among seasons and age-quarters. On the one hand, the hotter the season, the lower the associated coefficient. On the other hand, effects are stronger in the first and the fourth age-quarters. Third, the intensity of the effects increases with age. As people get older, the increasing effect of winter seasons on risks and the decreasing effect of the other seasons intensify. This result is in line with previous research (Rau, 2007; Richards et al., 2020). Equally, the increasing effect on risk of being alive in the fourth quarter of a given age and the decreasing effect of just having an age between x and $x + \frac{1}{4}$ also intensify as age increases. Likewise, we also observe the expected result that having an age between $x + \frac{1}{4}$ and $x + \frac{1}{2}$ reduces the average risk of dying with completed age x, whereas having an age between $x + \frac{1}{2}$ and $x + \frac{3}{4}$ increases it. These last effects are nevertheless quite uniform over the whole age range. Extreme ages are the exceptions. It seems that for ages above 95 the intra-annual ageing effect tends to disappear. However, given the small samples at these ages, this result should be treated with caution. Fourthly, the results also point out the presence of some interaction effects between seasonal and ageing effects. The magnitude of the interactions, however, evolves with age. Further analyses are required to study the relationship with age of the interactions. Finally, looking in more detail at the figures and tables in Sections S7 to S9, we also note another interesting phenomenon. For young males, the indexes are consistently and noticeably higher in summer. This effect is not observed for women.

TABLE 1 Estimates of Spanish seasonal-ageing indexes for age 0 (upper panels) and age 65 (lower panels)

Men						Women					
0 Years	**Winter**	**Spring**	**Summer**	**Autumn**	**Ageing**	**0 Years**	**Winter**	**Spring**	**Summer**	**Autumn**	**Ageing**
4Q	0.15530	0.15858	0.15565	0.13898	**0.1570**	**4Q**	0.13749	0.14597	0.13979	0.20156	**0.1614**
3Q	0.21089	0.20268	0.20004	0.23656	**0.2153**	**3Q**	0.24016	0.26325	0.22217	0.23546	**0.2438**
2Q	0.41927	0.39069	0.31628	0.48331	**0.4063**	**2Q**	0.52783	0.38309	0.30115	0.31548	**0.3823**
1Q	3.47679	2.95403	3.22742	3.27352	**3.2215**	**1Q**	3.35819	3.23359	3.06936	3.22546	**3.2125**
Seasonal	**1.0021**	**0.9520**	**1.0014**	**1.0444**		**Seasonal**	**1.0014**	**1.0338**	**0.9573**	**1.0075**	

Men						Women					
65 Years	**Winter**	**Spring**	**Summer**	**Autumn**	**Ageing**	**65 Years**	**Winter**	**Spring**	**Summer**	**Autumn**	**Ageing**
4Q	1.20159	1.02135	0.94637	0.98831	**1.0425**	**4Q**	1.20509	1.00564	0.94698	1.00360	**1.0438**
3Q	1.17588	0.97295	0.92515	0.97749	**1.0098**	**3Q**	1.15908	0.97167	0.94560	0.97652	**1.0099**
2Q	1.11577	0.94231	0.90613	0.95584	**0.9770**	**2Q**	1.12129	0.93236	0.91213	0.95214	**0.9769**
1Q	1.07471	0.92361	0.91200	0.96057	**0.9707**	**1Q**	1.07685	0.91275	0.91878	0.95952	**0.9694**
Seasonal	**1.1403**	**0.9627**	**0.9232**	**0.9738**		**Seasonal**	**1.1403**	**0.9549**	**0.9311**	**0.9737**	

6.3 | On the impacts of the sub-annual variations of mortality risks on the insurance industry

The above findings reveal (at least for Spain) a non-uniform intra-annual behaviour of mortality risks. They vary between integer ages, but also strongly depend on, and interact with, seasonal factors. The life expectancy and the probabilities of dying and surviving at each age depend on the season of birth of the person. Despite this, neither pension systems nor insurances companies account for this. As far as we know, in the actuarial field, insurances companies do not use quarterly life tables; they employ annual tables for reserving and pricing purposes and, at most, use some of the FAAs to quarterly distribute annual figures. This fact means no account is taken of intra-annual variations in mortality and it may result in, for instance, over- or under-estimation of insurance premiums or annuities.

In Section 5 of this paper, we offer an innovative solution for building quarterly tables from SAIs by applying them to the current annual life table in use by the insurance company or the pension system. In what follows, we compare the premiums obtained in a classical insurance product, a year-term life insurance, after applying to the same annual life table the three FAAs and the SAIs detailed in Tables S104–S204 in the supplementary material (see Table 1 as an example of SAIs for two specific ages).

According to the Spanish insurance regulator, the so-called PASEM2019_second_order (BOE, 2020) annual life table is the risk-loaded table (see, e.g. Pavía et al., 2019) to be used as reference for Spanish insurance companies in pricing risk-life insurance products. So, in order to compare the impact of using FAAs and SAIs on pricing, we use this table to construct quarterly life tables. In particular, we build SAI quarterly tables by applying the strategy detailed in Section 5. For instance, taking the values $q_{65}^{PASEM} = 0.00725439$ and $m_{65}^{PASEM} = 0.00728081$ obtained from the PASEM2019 tables and the SAI $\tilde{\gamma}_{1,3}^{(65)} = 0.91200$ from Table 1, we have that $\frac{1}{3}\tilde{m}_{65} = m_{65} \cdot \tilde{\gamma}_{1,3}^{(65)} = 0.006640099$ for a male policyholder with completed age 65 and, from this, that the probability of dying for this policyholder during summer when aged between 65 and 65.25 is $_{0.25}\tilde{q}_{65}^{(3)} = \frac{1}{3}\tilde{m}_{65} / \left(4 + 0.5 \cdot \frac{1}{3}\tilde{m}_{65}\right) = 0.00165864$. Once probabilities are computed, premiums are easily calculated by multiplying probabilities and amounts, assuming a null discount rate.

The estimates of the quarterly probabilities of death by season of birth associated with the PASEM2019_second_order life table are presented graphically in Figure 4. This figure synthesises for men (left panel) and women (right panel) their four related quarterly life tables, one for each season of birth. More details can be found in Section S10 of the supplementary material in Figures S36 and S37, where the range of ages has been broken down into two: from 0 to 49 and from 50 to 100 years old. As can be observed, quarterly probabilities depend on the season of birth of the holder. Figures S38–S43 show the same information, but ordered in a different fashion (a hypothetical situation, with no meaning in the real world) to better appreciate the impacts of seasonal and ageing effects.

With regard to assessment of the impact of intra-annual fluctuations of mortality on insurance products, Table 2 presents the premiums to be paid for a man (woman) at age 65 for a quarter-term life insurance of 100,000€ when the dying probabilities come from either any of the SAI quarterly tables previously calculated (see Figure 4) or the PASEM table after using any of three classical fractional age assumptions. The FAA premiums are presented as baseline. The analysis of Table 2 reveals a couple of interesting results beyond the expected outcome that the average premium to be paid with SAI tables is equal to the annual premium to be paid with any of the FAAs. On the one hand, we observe, at least with this table and for this age, almost no differences among the quarterly FAA premiums. However, we find differences in SAI premiums as large as 27.3%
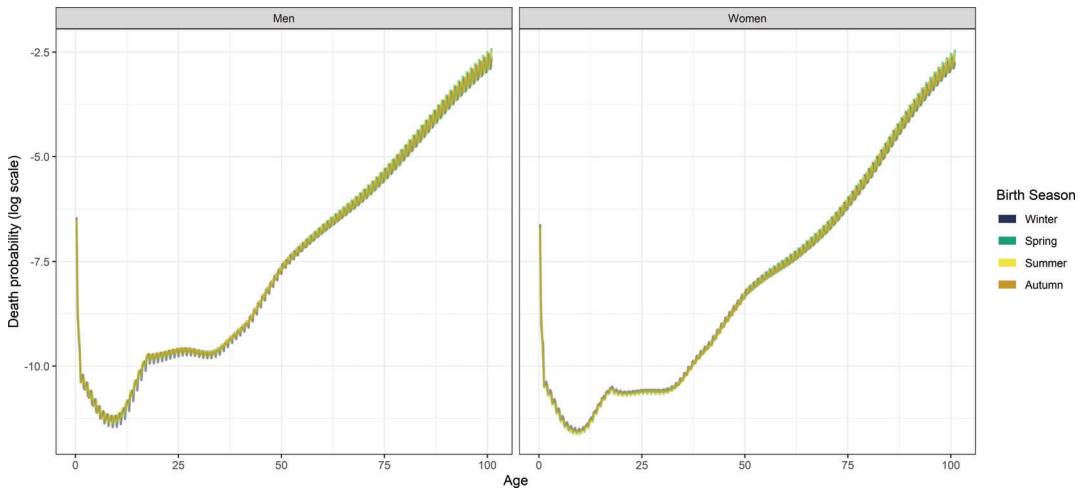
FIGURE 4 Estimates of quarterly probabilities of death by season of birth to be applied to life insurance risk products in Spain. Men (left panel) and women (right panel). The sequence of probabilities presented (in log scale) in the panels defines the quarterly life tables. The probabilities have been obtained by mapping death rates into death probabilities after applying, using the mathematical expressions introduced in Section 5, the smoothed estimates of SAIs ($\tilde{\gamma}_{rs}^{(x)}$) available in Tables S104–S204 of the supplementary material to the PASEM2019_second_ order life table. In Section S10 of the supplementary material, interested readers can consult Table S205 for the numbers depicted in the figures as well as those derived from a uniform-risk annual table

TABLE 2 Premium to be paid for a quarter-term life insurance of 100,000€ for a man (woman) at age 65

| Ageing | Fractional age assumption | | | Season of birth of holder – SAI adjusted table | | | |
	UDD	constant μ_x	Balducci	Winter	Spring	Summer	Autumn
1Q	181.36 €	181.86 €	182.35 €	195.43 €	167.97 €	165.86 €	174.69 €
	(80.57 €)	(80.67 €)	(80.76 €)	(86.86 €)	(73.63 €)	(74.12 €)	(77.40 €)
2Q	181.36 €	181.52 €	181.69 €	171.04 €	164.52 €	173.54 €	202.53 €
	(80.57 €)	(80.6 €)	(80.63 €)	(75.15 €)	(73.53 €)	(76.75 €)	(90.38 €)
3Q	181.36 €	181.19 €	181.03 €	167.64 €	177.17 €	213.08 €	176.27 €
	(80.57 €)	(80.54 €)	(80.50 €)	(76.16 €)	(78.66 €)	(93.35 €)	(78.25 €)
4Q	181.36 €	180.87 €	180.37 €	178.77 €	217.36 €	184.71 €	171.16 €
	(80.57 €)	(80.47 €)	(80.37 €)	(80.76 €)	(96.98 €)	(80.92 €)	(76.2 €)
Total	**725.44 €**	**725.44 €**	**725.44 €**	**712.87 €**	**727.03 €**	**737.19 €**	**724.65 €**
	(322.27 €)	**(322.27 €)**	**(322.27 €)**	**(318.93 €)**	**(322.79 €)**	**(325.14 €)**	**(322.23 €)**

(27.1%) for women (men), depending on the season of birth of the holder. On the other hand, we observe that the quarterly premium to be paid depends on both the exact age of the holder and the season in which the holder is exposed to at that age. The exact order of seasonal-ageing combinations, which depends on the season of birth of the holder, entails a different aggregate annual premium for each type of holder. Likewise, as expected, there is more variability among SAI quarterly premiums than among SAI annual premiums.

A careful analysis of the results reported in Table 2 reveals that adopting a framework based on quarterly tables would have relevant consequences in the real world for insurance companies,

impacting on their competitiveness and balance sheets via, for instance, risk management and risk selection. For example, in terms of pricing and underwriting, in a scenario with two companies in which one of the companies, say company C1, adopts the approach proposed in this research and the other (C2) does not, the company C1 would have a significant competitive advantage by following the next two-step strategy: (i) maintaining the prices in line with a scenario where premiums are calculated using the annual table and (ii) selecting the risks based on the quarterly tables. In this scenario, and using the numbers of Table 2 for 65-year-old men, we can see that C1 would earn, on average, 6.68€ per policyholder whereas C2 would lose, on average, 3.33€ per policyholder; with these numbers obtained after supposing that (a) the PASEM2019_second_order life table actually captures the risk of dying of 65-year-old men in Spain and (b) each policyholder initially chooses C1 and C2 with equal probability. In particular, using the structure of population of 2008 and assuming that all the 65 year-old Spanish men bought an annual-term life insurance of 100,000€, we would see that the aggregate risk-surpluses for C1 and C2 would be 383,137€ and −335,822€, respectively. The portfolio of C1 would be composed of half of the men born in autumn and winter whereas the portfolio of C2 would be composed of all the men born in spring and summer and half of the men born in autumn and winter. In a similar fashion, one could also envisage examples with other strategies and scenarios with implications for reserving (although these latter would be a bit more complex). In short, a better measurement of risks would have consequences in terms of increasing competition and/or competitiveness.

The computed indexes are not only valid for Spain but also, in our view, the SAIs obtained in this research could be used in other areas or countries with similar climatic (and socio-economic) conditions, such as Italy. Furthermore, they could even be used more widely since, according to Richards et al. (2020), the seasonal mortality fluctuations show a 'high degree of commonality across countries with different climates and different health systems' (p. 864). As an example, we have applied the procedure to the SIM/SIF 2018 table, which is the equivalent of the Spanish PASEM in Italy, and calculated the same premiums as those presented in Table 2. Interested readers can find these results in Table S206 of Section S11 of the supplementary material, from which similar conclusions can be drawn. To end this section, it should be noted that, although due to the Test-Achats case the EU regulation states that gender cannot be utilised as a variable to discriminate premiums and benefits under EU insurance contracts, we have followed the recommendation of Aseervatham et al. (2016) and Chen et al. (2018), who warn about the impact of this practice on some economic indicators, and have computed and presented in Table 2 and S206 the premiums separated by gender, given the dissimilarity in the patterns of mortality shown by men and women.

7 | DISCUSSION AND CONCLUDING REMARKS

Intra-annual demographic patterns have been seen in births, deaths and migration flows, depending on, among other issues, climatic, social, warfare and health factors. Despite this, they are not explicitly integrated in general population mortality risk analyses, which are routinely approached from an annual basis. The most that insurance companies do, and starting from an annual life table, is use fractional age assumptions (FAAs) in the actuarial calculations that involve fractional ages and/or fractional periods, despite the limitations of FAAs having been proven in a series of papers (Hoem, 1984; Jones & Mereu, 2000, 2002). To solve this issue, in this paper, we propose a new estimator for estimating death rates that, considering the exact moment

of occurrence (in terms of both exact age and exact day) of the demographic events, simultaneously accounts for both the ageing and the calendar fluctuations of the risks of mortality between any two consecutive integer ages. We also propose to model these intra-annual variations with respect to a scenario of risk uniformity, which makes it possible to build innovative specific SAIs. Specific new indexes that, once applied to annual life tables, provide relevant sub-annual life tables that can be used, for instance, to properly compute actuarial present values of annuities with payments more frequent than annual or net single premiums for insurance benefits payable at the moment of death. SAIs could be used by managers of pension systems for refining the individual benefits of the pension schemes and by insurance companies for improving the distribution of risks in their portfolios or for implementing tighter pricing policies.

We illustrate these new tools using a real database from Spain made up of more than 186 million microdata events, from which we estimate quarterly life tables. Our computations reveal clear deviations from uniformity of intra-annual mortality risks. They vary not only between integer ages, but also strongly depend on, and interact with, climatological, social and calendar issues. The life expectancy and the probabilities of dying and surviving at each age strongly depend on the season of birth of the people. Despite this, as far as we know, neither pension systems nor insurances companies account for this. Nevertheless, this could be quickly remedied. As we show, the estimated SAIs can be used to easily derive quarterly life tables from annual tables; and not only for Spain. Indeed, given the high levels of similarities reported in the literature among the seasonal mortality fluctuations across countries (e.g. Rau, 2007; Richards et al., 2020), they could be employed (at least on first impressions) in other areas or countries. In our view, given that total population and deceased people are the main components to compute the $_s^r m_x^a$ rates, a first test to decide whether the SAIs of a population could be used in another population should consist in studying whether the monthly distributions of birth dates and deaths are similar in both populations. More research is required, however, to assess the validity of this tentative rule and on the relationships between the total monthly distributions and the corresponding distributions broken down by age, both inter- and intra-populations.

To appreciate the magnitudes of the impact of the intra-annual ageing and seasonal effects on mortality, we can analyse the sequences of quarterly death probabilities represented graphically in Figures S38–S43 of Section S10 of the supplementary material as if they really corresponded to correct orders of death probabilities. (Note that these ordered sequences do not exist in the real world.) These figures show the estimates of the quarterly probabilities as if the persons exposed-to-risk were always living during age x either in the same season (Figures S38–S40) or in the same quarter of age (Figures S41–S43), conditioned to having had a normal life before (and after) age x. This way of presenting the data makes it easier to evaluate the impact of seasonal and ageing effects on the probabilities of death at each age and can be used to answer counterfactual questions, such as, what would be the impact of living in either a hypothetical world in which the season when the person reached age x remained the same for a whole year or in another hypothetical world in which the person kept the exact age that he/she had at the beginning of the calendar-year for a full year. Under these conditions, for example, the aggregation of life expectancies during a year of a group of 101 men each one with a different age between 0 and 100 years old living the whole year in summer would be 2.38 years higher than the corresponding aggregation for a similar group of men living the whole year in winter. This contrasts with the difference in life expectancy of a man born in summer versus one born in winter, which is 0.30 years. The life expectancies linked to all these hypothetical worlds, computed from the SAI quarterly death probabilities derived from the PASEM tables, are offered in Table S207 of Section S12 of the supplementary material. In the same table, the reader can also find the actual life expectancies of

the actual quarterly life tables associated with (hypothetical) cohorts of people born in the same instant but in different seasons.

Note that with our data we cannot compute the hypothetical life table corresponding to a person always living, say, in winter, because the data do not capture the accumulative eroding effect on health that it would entail. In our population, between winters, people always have three other less health-demanding seasons in which to recover. In light of our findings, a person, say, aged 60 always living in winters would be expected to have, compared to a person of our population with the same age, a worse health condition and, consequently, a higher probability of dying at that age.

In summary, demographic events do not behave uniformly over the calendar year and this should be considered, jointly with the non-uniform behaviour of mortality between integer ages, for a proper management of risks. As we have shown in this research, the intra-annual variations on mortality rates have a measurable impact on pricing (and reserving), therefore for a proper management of pension systems and life insurance, in our view, the proposals of this paper should be taken into account and adopted, for example, by public administrators of pension systems and private insurances companies.

We should point out, however, that both our methodological proposals and our data are not free of limitations. As has been stated in different sections throughout the paper, due to the exponential dividing effect of increasing frequencies, our approach to estimating sub-annual rates and probabilities requires large populations to yield proper results. The smaller the population and the finer the granularity, the greater the volatility of the estimates. This issue highlights the dangers of estimating sub-annual probabilities directly at regional and local levels and from insurance companies' portfolios. We propose the use of SAIs estimated from a larger population to overcome this limitation. However, this gives rise to (at least) two new issues that should be considered further. One is related to the uncertainty of estimating the SAIs and their assumption of stationarity. Another is related to the deviations and heterogeneities that the use of SAIs obtained from a different population could entail, as both local and socio-economic factors could impact differently on our target and reference populations. On the one hand, there is significant local variability in life expectancy, causes of death and dying probabilities. On the other hand, life-insured populations are reported to be more educated and richer than general populations. We therefore run the risk of insufflating to our target population alien intra-annual fluctuations. This could be solved having more detailed data about causes of death and socio-economic conditions; variables that we lack in our data set.

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DATA AVAILABILITY STATEMENT

Data

The (micro)data used in this research are (i) birth statistics, (ii) stocks of population, (iii) microdata of deaths and (iv) emigrant and immigrant microdata. Birth statistics, which include gender and date of birth, are available free in the section Vital Statistics of Spanish National Institute of Statistics (henceforth, INE) from the following link: https://urldefense.com/v3/__https://www.ine.es/dyngs/INEbase/es/operacion.htm?c=Estadisti ca_C&cid=1254736177007&menu=resultados&idp=1254735573002*__;Iw!!N11eV2iwtfs!taLtG

Tm8S3bjAqCN1zHkrvLNc4ejp2CsSebTeVigpVkBt53nDDAPQ8ej29ZBSmzCM2AC-ZSEzEUt1hi O8g$ Stocks of population come from the Population Now Cast estimates and were obtained by payment in advance. Death microdata include nationality, gender and dates of birth and of death of each deceased and were also obtained by payment in advance. These data were provided under a special agreement contract, according to which under no circumstances were the data to be distributed to third parties. Emigrant and immigrant microdata, which are available free, come from the Statistics of Residential Variation and are available from the following link: https://urldefense.com/v3/__https://www.ine.es/dyngs/INEbase/es/operacion.htm?c=Estadisti ca_C&cid=1254736177013&menu=resultados&idp=1254734710990*__;Iw!!N11eV2iwtfs!taLtG Tm8S3bjAqCN1zHkrvLNc4ejp2CsSebTeVigpVkBt53nDDAPQ8ej29ZBSmzCM2AC-ZSEzEWK3 E2x9w$

Code

We have all the R-scripts used to perform the computations, with comments in Spanish, we can send them to you if you required it. In any case, if the paper is accepted, we will follow your instructions, we can translate the comments and make all our best efforts to make the code easily compressible for other researches.

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REFERENCES

Alderson, M.R. (1985) Season and mortality. *Health Trends*, 17, 87–96.

Aseervatham, V., Lex, C. & Spindler, M. (2016) How do unisex rating regulations affect gender differences in insurance premiums. *The Geneva Papers on Risk and Insurance - Issues and Practice*, 41, 128–160.

Barz, C. & Müller, A. (2012) A tilting algorithm for the estimation of fractional age survival probabilities. *Lifetime Data Analysis*, 18, 234–246.

Basu, R. & Samet, J.M. (2002) Relation between elevated ambient temperature and mortality: a review of the epidemiologic evidence. *Epidemiologic Reviews*, 24, 190–202.

Basulto, J. & García, J. (2009) *Historia de la Probabilidad y la Estadística (IV)*. Huelva-Sevilla: Universidad de Huelva.

Benjamin, B. & Pollard, J. (1986) *The analysis of mortality and other actuarial statistics*. London: Heinemann.

BOE (2020) Resolución de 17 de diciembre de 2020, de la Dirección General de Seguros y Fondos de Pensiones, relativa a las tablas de mortalidad y supervivencia a utilizar por las entidades aseguradoras y reaseguradoras, y por la que se aprueba la guía técnica relativa a los criterios de supervisión en relación con las tablas biométricas, y sobre determinadas recomendaciones para fomentar la elaboración de estadísticas biométricas sectoriales. *Boletín Oficial Del Estado*, 338, 121566–121602.

Boja, C.E., Herțeliu, C., Dârdală, M. & Ileanu, B.V. (2018) Day of the week submission effect for accepted papers in physica A, plos one, nature and cell. *Scientometrics*, 117, 887–918.

Börger, M., Fleischer, D. & Kuksin, N. (2014) Modeling the mortality trend under modern solvency regimes. *ASTIN Bulletin*, 44, 1–38.

Brasche, O. (1870) Beitrag zur Methode der Sterblichkeitsberechnung und zur Mortalitätsstatistik Russland's. Würzburg, A. Struber's Buchhandlung.

Bull, G.M. & Morton, J. (1978) Environment, temperature and death rates. *Age and Ageing*, 7, 210–224.

Cairns, A., Blake, D., Dowd, K., Coughlan, G.D. & Khalaf-Allah, M. (2011) Bayesian stochastic mortality modelling for two populations. *ASTIN Bulletin*, 41, 29–59.

Cairns, A., Blake, D., Dowd, K. & Kessler, A. (2016) Phantoms never die: living with unreliable population data. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 179, 975–1005.

Cairns, A., Kallestrup-Lamb, M., Rosenskjold, C., Blake, D. & Dowd, K. (2019) Modelling socio-economic differences in mortality using a new affluence index. *ASTIN Bulletin*, 49, 555–590.

Chen, A., Guillen, M. & Vigna, E. (2018) Solvency requirement in a unisex mortality model. *ASTIN Bulletin*, 38, 1219–1243.

Debón, A., Montes, F. & Sala, R. (2005) A comparison of parametric models for mortality graduation. Application to mortality data for the Valencia Region (Spain). *SORT Statistics and Operations Research Transactions*, 29, 269–288.

Deschênes, O. & Greenstone, M. (2011) Climate change, mortality, and adaptation: evidence from annual fluctuations in weather in the US. *American Economic Journal: Applied Economics*, 3, 152–185.

Díaz, J., García, R., López, C., Linares, C., Tobías, A. & Prieto, L. (2005) Mortality impact of extreme winter temperatures. *International Journal of Biometeorology*, 49, 179–183.

Dong, Y., Huang, F., Yu, H. & Haberman, S. (2020) Multi-population mortality forecasting using tensor decomposition. *Scandinavian Actuarial Journal*, 2020, 754–775.

Ellison, P.T., Valeggia, C. & Sherry, D. (2005) Human birth seasonality. In: Brockman, D.K. & van Schaik, C.P. (Eds.) *Seasonality in primates: studies of living and extinct human and non-human primates*. Cambridge: Cambridge University Press, pp. 379–399.

Enchev, V., Klinow, T. & Cairns, A. (2016) Multi-population mortality models: fitting, forecasting and comparisons. *Scandinavian Actuarial Journal*, 2016, 319–342.

Fernández-Durán, J.J. & Gregorio-Dominguez, M.M. (2015) Seasonal mortality for fractional ages in short term life insurance. *Scandinavian Actuarial Journal*, 2015, 266–277.

Foster, G.S., Hummel, R.L. & Adamchak, D.J. (1998) Patterns of conception, natality, and mortality from midwestern cemeteries: a sociological analysis of historical data. *The Sociological Quarterly*, 39, 473–489.

Frostig, E. (2002) Comparison between future lifetime distribution and its approximation. *North American Actuarial Journal*, 6, 11–17.

Frostig, E. (2003) Properties of the power family of fractional age approximations. *Insurance: Mathematics and Economics*, 33, 163–171.

Gompertz, B. (1825) On the nature of the function of the law of human mortality on a new mode of determining the value of life contingencies. *Transactions of the Royal Society*, 115, 513–585.

Grant, W.B., Bhattoa, H.P. & Boucher, B.J. (2017) Seasonal variations of U.S. mortality rates: roles of solar ultraviolet-B doses, vitamin D, gene ex pression, and infection. *Journal of Steroid Biochemistry and Molecular Biology*, 173, 5–12.

Graunt, J. (1662) *Natural and political observations made upon the bills of mortality*. London: Roycroft.

Gray, C. & Bilsborrow, R. (2013) Environmental influences on human migration in rural ecuador. *Demography*, 50, 1217–1241.

Gustafson, P. (2002) Tourism and seasonal retirement migration. *Annals of Tourism Research*, 29, 899–918.

Guy, W.A. (1858) On the annual fluctuations in the number of deaths from various diseases, compared with like fluctuations in crime and in other events within and beyond the control of human will. *Journal of the Statistical Society*, 21, 52–86.

Haberman, S. & Renshaw, A. (2012) Parametric mortality improvement rate modelling and projecting. *Insurance: Mathematics and Economics*, 50, 309–333.

Healy, J.D. (2003) Excess winter mortality in Europe: a cross country analysis identifying key risk factors. *Journal of Epidemiology and Community Health*, 57, 784–789.

Herteliu, C., Ileanu, B.V., Ausloos, M. & Rotundo, G. (2015) Effect of religious rules on time of conception in Romania from 1905 to 2001. *Human Reproduction*, 30, 2202–2214.

Hoem, J.M. (1984) A Flaw in actuarial exposed-to-risk theory. *Scandinavian Actuarial Journal*, 1984, 187–194.

Hossain, S. (2011) Quadratic fractional age assumption revisited. *Lifetime Data Analysis*, 17, 321–332.

Jawad, M., Hone, T., Vamos, E.P., Roderick, P., Sullivan, R. & Millet, C. (2020) Estimating indirect mortality impacts of armed conflict in civilian populations: panel regression analyses of 193 countries. *BMC Medicine*, 18, 1990–2017.

Jones, B.L. & Mereu, J.A. (2000) A family of fractional age assumptions. *Insurance: Mathematics and Economics*, 27, 261–276.

Jones, B.L. & Mereu, J.A. (2002) A critique of fractional age assumptions. *Insurance: Mathematics and Economics*, 30, 363–370.

Kelly, J. (1987) Improving the comparability of international migration statistics: contributions by the conference of European statisticians from 1971 to date. *International Migration Review*, 21, 1017–1037.

Killick, R., Fearnhead, P. & Eckley, I.A. (2012) Optimal detection of changepoints with a linear computational cost. *Journal of American Statistical Association*, 107, 1590–1598.

Lam, D. & Miron, J. (1994) Global patterns of seasonal variation in reproductive outcomes? The European Study Group on Infertility and human fertility. *Annals of the New York Academy of Sciences*, 709, 9–28.

Lam, D. & Miron, J. (1996) The effects of temperature on human fertility. *Demography*, 33, 291–305.

Ledberg, A. (2020) A large decrease in the magnitude of seasonal fluctuations in mortality among elderly explains part of the increase in longevity in Sweden during 20th century. *BMC Public Health*, 20, 1674.

Lee, R.D. & Carter, L.R. (1992) Modelling and forecasting US mortality. *Journal of American Statistical Association*, 87, 659–671.

Lexis, W. (1880) La Représentation Graphique de la Mortalité au Moyen des Points Mortuaires. *Annales De Démographie Internationale*, 4, 297–324.

Li, J. & Liu, J. (2019) A logistic two-population mortality projection model for modelling mortality at advanced ages for both sexes. *Scandinavian Actuarial Journal*, 2019, 97–112.

Lledó, J., Pavía, J.M. & Morillas, F. (2017) Assessing implicit hypotheses in life table construction. *Scandinavian Actuarial Journal*, 2017, 495–518.

Lledó, J., Pavía, J.M. & Morillas, F. (2019) Incorporating big microdata in life table construction: a hypothesis-free estimator. *Insurance: Mathematics and Economics*, 88, 138–150.

Makeham, W.M. (1860) On the law of mortality and the construction of annuity tables. *Journal of the Institute of Actuaries and Assurance Magazine*, 8, 301–310.

Nature. (1874) The relations between human mortality and the seasons of the year. *Nature*, 10, 210.

Newbold, P., Carlson, W. & Thorne, B. (2019) *Statistics for business and economics*. Global Edition, London, UK: Pearson.

Parks, R.M., Bennett, J.E., Foreman, K.J., Toumi, R. & Ezzati, M. (2018) National and regional seasonal dynamics of all-cause and cause-specific mortality in the USA from 1980 to 2016. *Elife*, 7, e35500.

Pascariu, M. (2018) Modelling and Forecasting Mortality. Print & Sign: SDU.

Pavía, J.M., Morillas, F. & Lledó, J. (2012) Introducing migratory flows in life table construction. *SORT-Statistics and Operations Research Transactions*, 36, 103–114.

Pavía, J.M., Morillas, J.F. & Bosch-Rodríguez, J.C. (2019) Using parametric bootstrap to introduce and manage uncertainty: replicated loaded insurance life tables. *North American Actuarial Journal*, 23, 434–446.

R Core Team. (2020) *R: a language and environment for statistical computing*. Vienna: R Foundation for Statistical Computing. [online]. Available from: http://www.R-project.org/.

Rau, R. (2007) *Seasonality in human mortality: a demographic approach*. Berlin, Germany: Springer.

Rau, R., Bohk-Ewald, C., Muszyńska, M.M. & Vaupel, J.W. (2018) *Visualizing mortality dynamics in the lexis diagram*. Cham, Switzerland: Springer International Publishing.

Richards, S.J., Ramonat, S.J., Vesper, G.T. & Kleinow, T. (2020) Modelling seasonal mortality with individual data. *Scandinavian Actuarial Journal*, 2020, 864–878.

Ruiz-Perez, I., Rodriguez-Barranco, M., Rojas-Garcia, A. & Mendoza-Garcia, O. (2017) Economic crisis and suicides in Spain. Socio-demographic and regional variability. *The European Journal of Health Economics*, 18, 313–320.

Simó-Noguera, C.X., Lledó, J. & Pavía, J.M. (2020) Lent impact on the seasonality of conceptions during the 20th century in Spain. *European Journal of Population*, 36, 875–893.

Trenberth, K.E. (1983) What are the seasons? *Bulletin of the American Meteorological Society*, 64, 1276–1282.

Vandesrchirk, C. (2001) The lexis diagram, a misnomer. *Demographic Research*, 4, 97–124.

Wilmoth, J.R., Andreev, K.F., Jdanov, D.A. & Glei, D.A. (2020) Methods Protocol for the Human Mortality Database. Human Mortality Database, University of California Berkeley and Max Planck Institute for Demographic Research [online]. Available from: http://www.mortality.org/.

Zeuner, G. (1869) *Abhandlungen aus der Mathematischen Statistik*. Leipzig: Verlag von Arthur Felix.

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