Entry with Two Correlated Signals: The Case of Industrial Espionage and Its Positive Competitive Effects

Alex Barrachina*, Yair Tauman** and **Amparo Urbano*****

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Abstract

Recent advances in communication and information technologies have increased firms' incentive to acquire information about other firms. This fact may have important implications for market entry since in this context potential entrants can find easier to gather valuable information about, for example, the incumbents' cost structure. However, little theoretical work has been undertaken to analyze them. This paper takes a step forward by extending a one-sided asymmetric information version of Milgrom and Roberts' (1982) limit pricing model allowing the entrant to have an access to an Intelligence System (IS hereafter) of a certain precision that generates a noisy signal on the incumbent's cost structure. Therefore, she decides whether to enter the market, based on two signals: the price charged by the incumbent and the signal sent by the IS. Our main finding is that for intermediate values of the IS precision, the set of pooling equilibria with ex-ante profitable market entry is non-empty. Moreover, the probability of ex-ante non-profitable entry is strictly positive. Since in the classical limit-pricing models the entrant never enters in a pooling equilibrium, this result suggests that the use of the IS may potentially increase competition.

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Keywords: Entry deterrence; Asymmetric information; Limit pricing; Pooling equilibria.

All authors contributed in a peer-to-peer way, but Alex Barrachina is the principal author.

**Dept. of Finance and Accounting and LEE, University Jaume I, Castellón (Spain). Alejandro.Barrachina@gmail.com*

***IDC Herzliya (Israel) and Stony Brook University (USA). Amty21@gmail.com*

****Dept. of Economic Analysis and ERI-CES, Univ of Valencia (Spain). Amparo.Urbano@uv.es*

ORCID number: 0000-0002-2971-9612

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1. Introduction.

Information is a valuable resource for every firm since it will allow to improve the quality of decisions. Particularly important is the information about other firms with which the interaction is more or less direct. Relevant aspects in this sense are, for example, other firms' infrastructures and technologies, manufacture processes and cost structures, product pipelines and strategies, among others. When obtaining some of this information is seen as crucial but its availability through public sources ("open sources") is insufficient, firms sometimes exceed the limits of competitive intelligence activities and acquire it by engaging in industrial espionage[1] (Roche 2016). For example, Roche (2006) reported that "when General Motors learned that a competitor had purchased property to construct a very large factory, but did not know for what purpose, it set up a "spy center" to determine what its competitor was doing" (Roche 2006, p. 61).[2]

In the past few years, industrial espionage has experienced an increasing importance (Bhatti and Alymenko 2017). In a first moment, this could have been related to the fact that thousands of professionals in information-gathering activities were seeking an employment in the private sector after the Cold War (Solberg 2016). But what really made the difference were the more recent advances in communication and information technologies. These advances have increased firms' incentive to conduct these illegal information-gathering activities since they implied not only that a huge amount of firms' information is electronically written and their information systems are connected to the Internet, but also that cyber espionage activities are "far safer and less risky" (Solberg 2016, p. 52). There are many tools modern cyber industrial espionage can employ to collect other firms' confidential information. Trojan horses, adwares and

[1] Ferdinand and Simm (2007) conceptualize knowledge resulted from industrial espionage as part of organizational external learning, calling it illegal 'larcenous learning'. Solberg (2016) considers that this conceptualization is coherent with the historical patterns of industrial espionage.

[2] Other example, maybe the most classic and famous case of industrial espionage, is referred in Solberg (2016). In between 1989 and 1997, a chemical engineer, Tenhong Lee from Taiwan (also known as the glue man), of the company Avery Dennison in the U.S., making glue-based products, stole confidential information which allowed his other employer in Taiwan, Four Pillars Enterprise Co., to become the leading competitor of Avery Dennison in Asia.

cookies are some examples. Moreover, these instruments can allow the unauthorized access and remote control of several devices ("Botnets"), which will facilitate the exfiltration of the desired information (Bederna and Szadeczky 2020).

Nowadays as in the past, companies are weak at detecting and preventing espionage episodes and prefer to hide them considering they are negative publicity. As stated by Solberg (2016, p. 52), "there is always a fear that admission of breach may lead to loss of confidence and lower share price. So the stories seldom become public, if they are not leaked by state intelligence organizations or spread as anecdotes by retired executives at cocktail parties".[3] This implies that tenable knowledge about real espionage cases is scarce (Bhatti and Alymenko 2017). Moreover, when information about some industrial espionage case is obtained for research purposes, a hefty confidentiality agreement is typically signed, meaning that the real names of the involved companies are not revealed and the narrative is one-sided (Solberg 2016)

This is really unfortunate because theoretical work in the field cannot be inspired/contrasted with examples of real life. Nevertheless, some few recent cases have become public. For instance, a paradigmatic one came to light in 2015. It was discovered that the above-mentioned cyber espionage tools had been employed some years before to slowly and methodically exfiltrate confidential information from two U.S. tech companies, Avago and Skywords. The objective of the attackers was to collect relevant data to start their own business in the industry and the exfiltrated information included recipes and product designs, equipment and facilities specifications, project plans and performance data (Securonix 2015).

One of the most crucial business decisions is whether to start operating in a certain industry. In this sense, and as the previous case of cyber industrial espionage shows, in such a situation the incentive to gather relevant and confidential information from incumbents in the target industry before making the entry decision is very high, including data related to incumbents' performance. Actually, incomplete information about incumbents' cost structure has been considered a relevant aspect in the theoretical explanation of market entry behavior. Milgrom and Roberts (1982) initiated

[3] One of the reasons the glue man's case is well known is that Tenhong Lee went to court, where the motivation of his actions was revealed (Solberg 2016).

this strand in the theoretical literature about market entry,[4] but little theoretical work has been undertaken to analyze the implications of a potential entrant's activities to reduce such informational disadvantage.

The goal of this paper is to take a step forward in this direction by incorporating these potential entrant's activities in the context of modern cyber espionage. In particular, the paper extends a one-sided asymmetric information version of Milgrom and Roberts' (1982) model considering that the entrant has an access to an Intelligence System (IS hereafter) of a certain precision (which consists in some of the above-mentioned cyber espionage instruments) and employs it to better detect the cost structure of an incumbent monopolist[5] before deciding whether to enter the market (similarly as in the case study discussed by Securonix 2015). More precisely, the IS generates a noisy signal on the incumbent's cost structure and, therefore, the entrant decides whether to enter the market, based on two signals: the price charged by the incumbent and the signal sent by the IS.

Assuming that the precision of the IS is exogenously given (which is consistent with the fact that the entrant had already access to the spying technology before considering to enter the market), we show that gathering information about the cost structure of the incumbent firm produces two types of results: 1) for intermediate values of the IS precision, the set of pooling equilibria with ex-ante profitable market entry is non-empty, and 2) there exist pooling equilibria in which the probability of ex-ante non-profitable entry is strictly positive.

The origin of the theoretical literature considering incomplete information about an incumbent's cost structure as a crucial aspect in the explanation of market entry behavior was the long-standing question in the field of Industrial Organization about whether an incumbent firm can price so as to deter entry that otherwise would be profitable. Bain (1949) provided an early argument that an incumbent may deter entry by limit pricing. Subsequent analysis, however, suggested that early economists exaggerated the entry-deterring effects of incumbent pricing. As Needham (1976) argued, the incumbent's pre-entry behavior deters entry only if some link exists

[4] This seminal paper and some posterior developments in this literature are reviewed later.

[5] In terms of Ferdinand and Simm (2007), the entrant uses the IS for 'larcenous learning'.

between this behavior and the potential entrant's expected post-entry profit. This would be the case if the incumbent could commit to maintain his pre-entry price in the event of entry, but such an assumption seems implausible.

Later research use game-theoretic models to reconsider whether limit pricing may deter entry. In particular, a set of research proposes an informational link between the incumbent's pre-entry behavior and the entrant's expected post-entry profit. Thus, in a classic paper, Milgrom and Roberts (1982, MR hereafter) assume that the incumbent has private information about his costs of production, thus endogenously generating an interdependence between the pre-entry output rate and the potential entrant's expected post-entry profits and her entry decision. MR show that a separating equilibrium may exist in which the incumbent sets a below monopoly price (limit price) and thereby signals that his costs are low. The potential entrant then infers the incumbent's cost type and enters exactly when entry would be profitable under complete information. Pooling equilibria only exist when entry is not profitable since profitable entry cannot be deterred. Therefore, in the setting considered by MR, there exists no pooling equilibrium in which the potential entrant enters the market with positive probability.

Bagwell and Ramey (1988) extend the MR model to allow the incumbent to have two signals: price and advertising.[6] In their model, the incumbent is privately informed as to whether its costs are high or low, the potential entrant's costs are commonly known, and entry is profitable if and only if the incumbent has high costs. In a refined separating equilibrium, the low cost incumbent engages in "cost-reducing distortion", this meaning that it adopts the same price and advertising selection as it would be, hypothetically, an uncontested monopoly with costs that were even lower. The low cost incumbent thus limits prices and distorts its demand-enhancing advertising upwards. Once again, due to signaling, profitable entry is not deterred. But, once pooling equilibria are considered Bagwell and Ramey (1988) show that for some parameters refined pooling equilibria exist in which the high cost incumbent uses limit pricing and an upward distortion in advertising to deter entry that would be profitable

[6]For other extensions, see Albaek and Overgaard (1992a,b), Bagwell (1992), Bagwell and Ramey (1990, 1991), Linnemer (1998), and Orzach et al. (2002).

under complete information. The MR result is in the benchmark model of Bagwell (2007), where both prices and advertising expenditure are signals of the incumbent monopolist cost. Bagwell (2007) extends the benchmark game to include two dimensions of private information. Specifically, the incumbent is privately informed as to its cost type and its level of patience and selects price and advertising in the pre-entry period. He finds pooling equilibrium (satisfying the intuitive criterion) associated with the behavior of the patient high cost incumbent, which pools with the impatient low cost incumbent.

In this paper we deal with a monopoly who is engaged in R&D activity with the aim to reduce his cost of production. The outcome of the R&D project is the private information of the incumbent. A potential entrant assigns a certain probability that the monopolist fails to reduce his cost. If the project fails and the entrant enter, it will obtain positive profit. Otherwise, if the project succeeds and the entrant enters, it will not be able to cover entry cost. As already mentioned, the paper considers the case in which the entrant has an access to an Intelligence System (IS), which consists in some of the modern cyber espionage tools discussed above, and it is used to collect (noisy) information about the incumbent's cost structure before deciding whether or not to enter the market. The IS sends one out of two signals. Signal h, indicating that the investment was not successful, in which case we refer to the incumbent as having high cost (type H), and signal l, indicating that the investment was successful and the incumbent reduces its cost (type L).

Consistently with the entrant having access to the spying device (e.g. having the ability to plant one or several cyber espionage instruments in the information system of the incumbent firm) before considering to enter the market, it is assumed that the precision of the IS is exogenously given. In this context, the entrant decides whether to enter the market based on a pair of signals: the price that the incumbent charges for its product and the signal sent by the IS. If the entrant enters the market, it competes with the incumbent (whether it is a Cournot or Bertrand competition or any other mode of competition).

The interaction between the entrant and the monopolist is described as a three stage game. In the first stage, the incumbent who knows the outcome of the R&D project, sets a price and the IS sends a signal. Based on this pair of signals the entrant decides whether or not to enter the market in the second stage. If it enters, it will be engaged in a certain mode of competition with the incumbent in the third stage of the game. The game is of incomplete information and, following Harsanyi (1967, 1968), we analyze it as a three player game, where the players are the two types of the incumbent and the entrant. We analyze the sequential equilibria of this game. The case where the IS precision is $1/2$ (not informative) is the limit pricing model of MR, for the case where the entrant's cost is common knowledge.

We distinguish two cases: the first one is the separating equilibrium where the two types of incumbent charge different prices and the second one is the pooling equilibrium case where both types charge the same price.

The analysis provides several interesting findings. Firstly, the entrant's best response entails two different threshold entry prices, one for each IS signal. That is, for each signal there is a threshold price such that the entrant enters if and only if the observed price is higher than the signal-related threshold price. The threshold price associated with signal l (the incumbent is of type L) is higher than the one associated with the other signal (signal h). This result means that the entrant will stay out for a higher range of prices when observing l than when h is realized. Secondly, the analysis supports the separating equilibria in MR and Bagwell and Ramey (1988). Namely, the low-cost incumbent separates itself from the high-cost type, and separation will be achieved through a cost-reducing distortion if the cost difference is not too far apart. In other words, in this case at any separating equilibrium, the low-cost incumbent limits prices; this behavior enables the potential entrant to infer the incumbent's cost so that profitable entry is not deterred. We show that the separating equilibria of our model coincide with that of MR and Bagwell and Ramey (1988) and the IS makes no difference for either the entrant or the incumbent. This is not surprising since the entrant in a separating equilibrium identifies the incumbent's type with or without the use of the IS. The only difference between our separating equilibria and those of the aforementioned papers is in the behavior of the entrant when observing prices off the equilibrium path.

Thirdly, we show that the IS plays an important role in pooling equilibria. As already mentioned, a classical game-theoretical result is that limit pricing cannot deter profitable entry and thus the set of pooling equilibria when the entrant's expected profits are positive is empty. The same result is obtained in our model if the IS precision is sufficiently low to affect the entrant's decision. In the other extreme, if the IS precision is very accurate (close to 1), then contrary to the MR model, pooling equilibrium does not exist, even when entry is not profitable ex-ante. In this case, the entrant identifies with high probability the incumbent's type and enters the market if the IS sends signal h and stays out if the signal is l. The high cost monopolist, who knows that with high probability its type is detected, benefits from a deviation to its monopoly price, upsetting a pooling equilibrium.

However, the results change for intermediate values of the IS precision, namely, when the precision of the IS is bounded away from 1 and from $1/2$. We show that the set of pooling equilibria is non-empty even under ex-ante profitable entry. The entrant's decision is to follow the signal, namely entering if the signal is h and staying out if the signal is l. Thus, when the IS precision is bounded away from 1, the high cost monopolist knows that with significant probability the entrant will obtain the wrong signal and will stay out. Hence, it succeeds with positive probability to "fool" the entrant about his type.

To compare this result with the result obtained in the MR model, suppose first that prior to the completion of the R&D project, the expected payoff of E from entering the market is positive. Then, no pooling equilibrium exists in the MR model. Moreover, the entrant never enters in a pooling equilibrium when the expected profit of entry is negative. Contrary to the MR model, the entrant in our model enters the market with positive probability (when the IS signal is h) even if her ex-ante expected profit is negative, suggesting positive competitive effects of industrial espionage in contrast to the negative ones that would emerge when a non-spied incumbent operates in more than one market (Pires and Jorge 2012). Moreover, an IS with intermediate values of precision allows for pooling equilibria with ex-ante profitable entry.

In our model the incumbent only signals his costs by the price and the other signal is generated by the IS operated by the entrant, in contrast with Bagwell and Ramey

(1988), where the incumbent signals his costs with both price and advertisement. Bagwell (2007) finds a (intuitive) pooling equilibrium, where the incumbent has two dimensions of private information, his costs and his level of patience. In contrast, our model also offers the existence of pooling equilibria under ex-ante profitable entry with only one dimension of private information by the incumbent but with two IS signals correlated with price, that provide additional (probabilistic) information to the entrant about the incumbent type's and help her to smooth her best response. The entrant's best response is completely smooth in Matthews and Mirman (1983), in a limit pricing model where demand is stochastic, so that prices reveal only statistical information about the incumbent's private information. Their (separating) equilibrium differs from standard signaling equilibria in that it can be unique, it depends on prior beliefs and it is rich in comparative statics.

This paper is also closely related to a relatively recent strand in the theoretical literature, represented by Barrachina et al. (2014) and Barrachina (2019), that analyzes the effects of gathering noisy information (through an IS of the same nature as the one considered in the present paper) in the context of entry deterrence. Barrachina et al. (2014) elaborates on the general game-theoretic framework to analyze espionage games, as suggested by Solan and Yariv (2004), and considers the case in which a potential entrant can gather noisy information about the incumbent's decision regarding capacity expansion. As in the present paper, their results suggest that market competition is likely to increase under the entrant's industrial espionage activities. Alternatively, Barrachina (2019) considers the case in which the owner of the IS is the incumbent and identifies the conditions under which communicating that entrant's strength can be detected is effective as an entry deterrence strategy. Barrachina (2019), like the present paper, consider espionage in the context of asymmetric information, and so do Perea and Swinkels (1999) and Ho (2008). However, in the latters' model the spying activity as carried out by a decision maker who can act strategically.

This theoretical literature analyzing espionage in an economic and industrial context is quite sparse. In a recent paper, Barrachina and Forner-Carreras (2020) also consider a market entry context but focusing on the interaction of one country's noiseless espionage activity with other country's counter-espionage effort. The analysis shows that the optimal counter-espionage effort, concerned about social welfare in the target market, is always positive but decreasing with the level of competition in that market. Counter-espionage activities are also considered by Whitney and Gaisford (1999), Grabiszewski and Minor (2019) and Fan et al. (2019).

The expected increase in the level of market competition showed by Barrachina et al. (2014) and the present paper is likely to improve social welfare. More focused on the effect of industrial information-gathering activities on this social welfare are the theoretical studies by Sakai (1985), Billand et al. (2016) and Kozlovskaya (2018). Like in our paper, Sakai (1985) analyzes two firms and information gathering in order to know the cost structure of the opponent firm. However, unlike us, Sakai (1985) considers that both firms are already competing in the market and they know neither the costs of their opponent nor their own costs.

The remainder of the paper is organized as follows. Section 2 sets out the model. The entrant's strategy is offered in Section 3. Section 4 shows the pooling equilibria and Section 5 analyzes the separating equilibria of the game. Section 6 concludes the paper. Most of the proofs are presented in the Appendix.

2. The Model.

We consider a monopoly M and a potential entrant E. The monopoly M is engaged in R&D activity with the aim to reduce his cost of production from the current cost $C_H(q)$ to $C_L(q)$, where q is the production level. The outcome of the R&D project is the private information of M. The potential entrant, E, assigns a certain probability $\mu > 0$, that M fails to reduce his cost and probability $1-\mu > 0$ that the project was successful. Therefore, the cost function of M is a private information and it can be of two types: L (low cost) and H (high cost) and the potential entrant, E, assigns probability μ that M is of type H. If the project fails and E enters, she obtains positive profit. Otherwise, if the project succeeds and E enters, she will not be able to cover her entry cost and she will end up with negative profit.

The entrant has an access to an Intelligence System (IS) that allows her to gather (noisy) information about the cost structure of M. The IS sends one out of two signals. The signal h, which indicates that the investment was not successful (in which case we refer to M as having the type H), and the signal l, which indicates that the investment was successful (namely, M is of type L). The precision of the IS is α, $1/2 \le \alpha \le 1$. That is, the signal sent by the IS is correct with probability α (for simplicity, whether the cost function is $C_H(q)$ or $C_L(q)$). The case where $\alpha = 1/2$ is equivalent to the case where E does not use an IS. The case $\alpha = 1$ is the one where E knows exactly the outcome of the project. It is assumed that the precision α of the IS is exogenously given.

The interaction between E and M is described as a three stage game $G(\alpha)$. In the first period M chooses a price as a function of his type. The entrant decides whether to enter based on a pair of signals: the price, p, that M charges for his product and the signal s (h or l) sent by the IS. If E enters, she will incur an entry cost K and compete with M (whether it is a Cournot or Bertrand competition or any other mode of competition). The form of competition (Cournot, Bertrand or other) is commonly known and once E enters, the outcome of the competition is assumed to be uniquely determined. It is assumed that the above is commonly known (including the precision α of the IS).

The game $G(\alpha)$ is a game of incomplete information and, using Harsanyi's approach, we analyze it as a three player game, where the players are the two types, H and L, of M and the entrant, E. The case where $\alpha = 1/2$, namely, where the IS has no value (and, therefore, can be ignored), is exactly the limit pricing model MR, when the entrant only has an entry cost type. Therefore, our model is an extension of the MR model where the entrant has an access to an intelligence system and it is only for $1/2 < \alpha < 1$.

Let $Q(p)$ be the demand function and $C_t(q)$ be the cost function of the t-type monopoly. Let D_H and D_L be the duopoly profits of the H-type and the L-type monopolists, respectively. For short we denote by H and L the H-type and the L-type monopolists, respectively. Let $\Pi_H(p)$ be the profit of H and let $\Pi_L(p)$ be the profit of L when they set the price p and when E does not enter. Denote by $D_E(H)$ and $D_E(L)$ the duopoly profits of E when she competes with H and L respectively. Denote by p_H^M and p_L^M the monopoly prices of H and L respectively (and by q_H^M and q_L^M the monopoly quantities). The following assumptions are standard in the literature.

Assumptions

1. $D_E(L)-K\equiv\Delta_E(L)<0$ and $D_E(H)-K\equiv\Delta_E(H)>0$.
2. $\Pi_t(p)$, $t\in\{H,L\}$, is increasing in p whenever $p\le p_t^M$ and is decreasing in p whenever $p\ge p_t^M$.
3. $\Pi_L(p_L^M)-D_L>\Pi_H(p_H^M)-D_H$. Namely, L loses from entry more than H.
4. The cost functions $C_t(x)$, $t\in\{H,L\}$, are differentiable , $C_H'(q)>C_L'(q)$ and $C_H(0)\ge C_L(0)$.
5. $Q(p)$ is differentiable and $Q'(p)<0$ for all $p\ge 0$.
6. All the parameters of the model and the above five assumptions are commonly known.

Let $\hat{p}$ be the price for H and let p_0 be the price for L that yields the duopoly profits for H and L respectively, i.e.,

$$\Pi_H(\hat{p})=D_H \text{ and } \hat{p}<p_H^M$$

and

$$\Pi_L(p_0)=D_L \text{ and } p_0<p_L^M.$$

Lemma 1. (i) $\Pi_L(p)-\Pi_H(p)$ decreases in p.

(ii) $p_H^M>p_L^M$.

(iii) $\hat{p}>p_0$.

Proof: See the Appendix.

We restrict our study to sequential equilibria of $G(\alpha)$. A sequential equilibrium is a combination of strategies and beliefs such that strategies are sequentially rational given the players' beliefs and beliefs are consistent in all information sets.

A strategy for the entrant E is an *entry rule,* $\sigma_E:\{h,l\}\times R\to\{0,1\}$. After observing a first period price $p\in\mathbb{R}_+$ and a signal $s\in\{h,l\}$, E enters if $\sigma_E(s,p)=1$ and does not enter if $\sigma_E(s,p)=0$. A strategy for firm M is a *pricing rule,* $p:\{H,L\}\to R$ that specifies a price p_t, $t=\{H,L\}$.

Given α, for every pair of signals (s,p), $s \in \{h,l\}$ and $p \in \mathbb{R}_+$, let $Prob(H|s,p)$ and $Prob(L|s,p) = 1 - Prob(H|s,p)$ be the conditional probability that E assigns to the event that M is of type H and of type L, respectively.

It is assumed that, conditional on the type of M, the signals are mutually independent. Namely, M chooses price p independently of the choice of the IS. Nevertheless, the signals p and s are correlated. If E observes a very high price, it will be more likely to observe signal h. If however E observes a low price, it will be more likely to observe signal l. The Bayesian posterior belief that E assigns to the types of M is

$$Prob(H|h,p) = \frac{Prob(h,p|H)Prob(H)}{Prob(h,p|H)Prob(H) + Prob(h,p|L)Prob(L)}$$

$$= \frac{Prob(h|H)Prob(p|H)Prob(H)}{Prob(h|H)Prob(p|H)Prob(H) + Prob(h|L)Prob(p|L)Prob(L)}$$

Equivalently,

$$Prob(H|h,p) = \frac{\mu\alpha f(p|H)}{\mu\alpha f(p|H) + (1-\mu)(1-\alpha) f(p|L)} \qquad (1)$$

Similarly,

$$Prob(H|l,p) = \frac{\mu(1-\alpha) f(p|H)}{\mu(1-\alpha) f(p|H) + (1-\mu)\alpha f(p|L)} \qquad (2)$$

where $f(p|t)$ is the (density) probability that E assigns to the event that M of type t, $t \in \{H,L\}$ sends the signal p.

In a pure strategy equilibrium, if H assigns probability 1 to the event that $p = p_H$, then $f(p_H|H) = 1$ and $f(p|H) = 0$ if $p \neq p_H$. In this case, $f(p|H)$ is identified with the probability that H selects p. Similarly, $f(p_L|L) = 1$ and $f(p|L) = 0$, $\forall p \neq p_L$. Hence, for $p \neq p_H$ and $p \neq p_L$, (1) and (2) are not well defined (the numerators and denominators are zero). To apply the sequential equilibrium concept we need to consistently define beliefs for any observed p, therefore off the equilibrium path we approach $f\left(p|t\right)$ by a sequence $\left(f_n\left(p|t\right)\right)_{n=1}^{\infty}$, such that $f_n\left(p|t\right)>0$ and $\lim_{n\to\infty} f_n\left(p|t\right)=f\left(p|t\right)$ for all $p\in\mathbb{R}_+$. Let

$$Prob_n\left(H|h,p\right)\equiv\frac{\mu\alpha f_n\left(p|H\right)}{\mu\alpha f_n\left(p|H\right)+(1-\mu)(1-\alpha)f_n\left(p|L\right)} \tag{3}$$

$$Prob_n\left(H|l,p\right)\equiv\frac{\mu(1-\alpha)f_n\left(p|H\right)}{\mu(1-\alpha)f_n\left(p|H\right)+(1-\mu)\alpha f_n\left(p|L\right)} \tag{4}$$

Now $Prob_n\left(H|h,p\right)$ is well defined for all $p\in\mathbb{R}_+$ and (1) can be modified to be

$$Prob\left(H|h,p\right)\equiv\lim_{n\to\infty}\frac{\mu\alpha f_n\left(p|H\right)}{\mu\alpha f_n\left(p|H\right)+(1-\mu)(1-\alpha)f_n\left(p|L\right)}$$

We modify (2) in the same way. Note that different sequences of $\left(f_n\left(p|t\right)\right)_{n=1}^{\infty}$ generate different conditional probabilities $Prob\left(t|s,p\right)$, $t\in\{H,L\}$, $s\in\{h,l\}$, $p\in\mathbb{R}_+$.

Let $\Pi_E\left(s,p\right)$ be the expected payoff of E given its on and off equilibrium beliefs, namely

$$\Pi_E\left(s,p\right)\equiv Prob\left(H|s,p\right)\Delta_E\left(H\right)+Prob\left(L|s,p\right)\Delta_E\left(L\right) \tag{5}$$

In a sequential equilibrium, if $\Pi_E\left(s,p\right)<0$, E does not enter the market and if $\Pi_E\left(s,p\right)>0$, E enters. To simplify the analysis we assume that E stays out also when $\Pi_E\left(s,p\right)=0$. Namely, E stays out if and only if she observes $\left(s,p\right)$ such that

$$\Pi_E\left(s,p\right)\equiv Prob\left(H|s,p\right)\Delta_E\left(H\right)+Prob\left(L|s,p\right)\Delta_E\left(L\right)\le 0$$

3. Conditions for limit pricing: The entry rule

For firm M to engage in limit pricing, entry should be more likely, in some sense, when prices are higher rather than lower for any observed signal. This is clearly the case if $\sigma_E\left(s,p\right)$ specifies entry if and only if for each signal s the observed price exceeds the entry price. The following assumptions will help insure that $\sigma_E\left(s,p\right)$ is of this form for any p and each $s\in\{h,l\}$.

Assumption 7.

(1) For each $t \in \{H, L\}$ and each n, $f_n(p|t)$ is differentiable in p for all $p \geq 0$.

(2) Let

$$g_n(p) = f_n(p|H) / f_n(p|L)$$

Then $g_n(p)$ is increasing in n for each p, and is increasing in p for each n. Furthermore, for every n, $\lim_{p \to 0} g_n(p) = 0$ and $\lim_{p \to \infty} g_n(p) = \infty$.

(3) Let $g(p) = \lim_{n \to \infty} g_n(p)$. Then, $g(p)$ is continuous in p.

Notice that, $f(p|H)/f(p|L)$ is the likelihood ratio and to be increasing in p or, equivalently, to satisfy the Monotone Likelihood Ratio Property in p (Milgrom, 1981) implies that a high price is more likely to come from H rather than from L. Most common densities such as the uniform, normal or exponential satisfy the MLRP. Assumption 7, guarantees continuity and monotonicity of the conditional probability $Prob(t|s, p)$, $t \in \{H, L\}$, $s \in \{h, l\}$, $p \in \mathbb{R}_+$. The next lemma shows the continuity and monotonicity of such conditional probabilities. The proof is relegated to the Appendix.

Lemma 2. (i) For each $s \in \{h, l\}$ and $t \in \{H, L\}$, $Prob(t|s, p)$ is continuous in p and $Prob(H|s, p)$ is non-decreasing in p, $p \geq 0$.

(ii) For every $p \geq 0$, $Prob(H|h, p) > Prob(H|l, p)$.

By the above lemma, $\Pi_E(s, p)$ is continuous and non-decreasing in p (this follows from the fact that $Prob(H|s, p)$ is continuous and non-decreasing in p, $\Delta_E(H) > 0$, $Prob(L|s, p) = 1 - Prob(H|s, p)$ and $\Delta_E(L) < 0$). Moreover,

Lemma 3. Let $J_s = \{p \geq 0 | \Pi_E(s, p) \leq 0\}$. Then, J_s and $\mathbb{R}_+ \setminus J_s$ are both non-empty sets. In other words, $\Pi_E(s, p) < 0$ for sufficiently small p, and $\Pi_E(s, p) > 0$ for p sufficiently large.

Proof: See the Appendix.

Recall that by Assumption 1, $D_E(L) - K \equiv \Delta_E(L) < 0$, $D_E(H) - K \equiv \Delta_E(H) > 0$. Then,

Proposition 1. Suppose that Assumption 1 holds. Then, any beliefs of E which satisfy Assumption 7, implies that $\Pi_E(s,p)$ is continuous and non-decreasing in p and uniquely determines p_h and p_l. In every sequential equilibrium with these beliefs, $p_h < p_l$ and E enters the market if and only if she observes signal (h,p) with $p > p_h$ or signal (l,p) with $p > p_l$.

Proof: See the Appendix.

Since we have that have $p_l > p_h$, then the best response entry rule of E when she observes the pair of signals (s,p) is given by Figure 1 below.

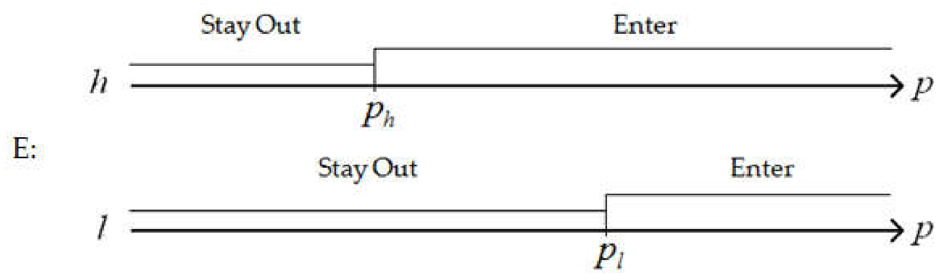

Figure 1

The correlation between signals and prices gives rise to the ordering of the threshold prices associated to signals and thus the threshold price associated to signal l is higher than the one related signal h. This means, for instance, that the entrant will stay out for a higher range of prices when observing l than when observing h, which is quite intuitive since the signal sent by the IS is informative (although noisy) and E will be more inclined to enter the market when receiving signal h than when receiving signal l.

Our next goal is to characterize the sequential equilibrium of $G(\alpha)$ given the above decision rule of E.

4. Conditions for entry deterrence: Pooling equilibria

We analyze first the existence of sequential pooling equilibria, which is our main contribution. We claim here that the set of pooling equilibria with ex-ante profitable market entry is non-empty and the probability of ex-ante non-profitable entry is strictly positive (more precisely, when the IS signal is h) if the cost function of H is not significantly higher than that of L and the IS precision belong to some intermediate levels. This result is accomplished in our model, where the incumbent only has a dimension of private information, because the entrant receives one of the two IS signals correlated with price, providing her with additional (probabilistic) information about the incumbent's type. This result also offers support for the predictions of an earlier literature, wherein the contribution by Bain (1949) describes the condition under which limit pricing may deter entry.

The conditional-to-signal entry rule.

By pooling equilibrium we refer to triples of the form (σ_E, p_H, p_L) where σ_E is the strategy of E and $p_H = p_L \equiv p^*$.

We calculate first the entrant's expected payoff conditional to receiving a signal from the IS and observing price p^*. Recall that by Assumption 1, $D_E(L) - K \equiv \Delta_E(L) < 0$ and $D_E(H) - K \equiv \Delta_E(H) > 0$, where $D_E(H)$ and $D_E(L)$ denote the duopoly profits of E when she competes with H and L respectively.

Given signal l of the IS, the expected payoff of E conditional to receiving such a signal is

$$\Pi_E(l|\alpha) \equiv Prob(H|l)\Delta_E(H) + Prob(L|l)\Delta_E(L)$$

Equivalently,

$$\Pi_E(l|\alpha) = \frac{\mu(1-\alpha)}{\mu(1-\alpha)+(1-\mu)\alpha}\Delta_E(H) + \frac{(1-\mu)\alpha}{\mu(1-\alpha)+(1-\mu)\alpha}\Delta_E(L)$$

Hence, if the IS sends signal l, E does not enter the market when observing price p^* if and only if $\Pi_E(l|\alpha) \le 0$.

Let

$$\bar{\alpha}_l = \frac{\mu\Delta_E(H)}{\mu\Delta_E(H) - (1-\mu)\Delta_E(L)} \tag{6}$$

Therefore, suppose that the entrant's conditional expected profits to receiving signal l is non-positive, then she does not enter when observing price p^* if and only if the IS precision is sufficiently high, i.e., $\alpha \ge \bar{\alpha}_l$.

Since in our model $1/2 < \alpha < 1$, and $1/2 < \overline{\alpha}_l < 1$ then, to make the analysis fruitful, we would like to know when $\overline{\alpha}_l < 1/2$. From (6) and recalling that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)$ is the entrant's expected profit without the IS then, $\overline{\alpha}_l < 1/2$ if and only if

$$\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0 \tag{7}$$

Therefore, E does not enter when receiving signal l if and only if the entrant's unconditional expected payoffs are negative (i.e., (7) is satisfied), this meaning that the precision of the IS is high enough.

Suppose next that the IS sends signal h. Then the expected payoff of E conditional to receiving h is

$$\Pi_E(h|\alpha) \equiv Prob(H|h)\Delta_E(H)+Prob(L|h)\Delta_E(L)$$

Equivalently,

$$\Pi_E(h|\alpha)=\frac{\mu\alpha}{\mu\alpha+(1-\mu)(1-\alpha)}\Delta_E(H)+\frac{(1-\mu)(1-\alpha)}{\mu\alpha+(1-\mu)(1-\alpha)}\Delta_E(L)$$

Hence, if the IS sends signal h, E does not enter the market when observing price p^* if and only if $\Pi_E(h|\alpha)\le 0$.

Let

$$\overline{\alpha}_h=\frac{-(1-\mu)\Delta_E(L)}{\mu\Delta_E(H)-(1-\mu)\Delta_E(L)} \tag{8}$$

Note that $\Pi_E(h|\alpha)\le 0$ if and only if the IS precision is sufficiently low, i.e., $\alpha \le \overline{\alpha}_h$.

As above, $0<\overline{\alpha}_h<1$ and from (8), $\overline{\alpha}_h > 1/2$ if and only if (7) is satisfied, i.e., the entrant's expected profit without signals is negative.

Corollary 1. Suppose that $1/2<\alpha<1$ and

$$\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$$

Then, E stays out if and only if she observes signal l or receives signal h and the IS precision is low enough, i.e., $\alpha \le \overline{\alpha}_h$.

Alternatively, when the entrant's expected profit without signals is negative, she enters the market if and only if receives signal is h and the IS precision is sufficiently high, i. e., $\alpha > \bar{\alpha}_h$.

Obviously, when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$, then $\bar{\alpha}_h < 1/2 < \bar{\alpha}_l < 1$. Hence $\alpha > \bar{\alpha}_h$ $\forall\alpha$, $1/2<\alpha<1$. Namely, if the IS sends signal h, E enters the market when observing price p^* irrespective the precision α of the IS. Also, $\bar{\alpha}_l > 1/2$ and then we may have $1/2<\alpha<\bar{\alpha}_l$ or $\bar{\alpha}_l \le \alpha < 1$. In the former case, E enters the market when observing price p^* irrespective of the signal sent by the IS, and in the latter E enters the market when observing price p^* if the IS sends signal h and does not enter if the IS sends signal l. Therefore,

<u>Corollary 2.</u> Suppose that $1/2<\alpha<1$ and

$$\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$$

Then, the entrant stays out if and only if she observes signal l and the IS precision is high enough, i.e., $\alpha \ge \bar{\alpha}_l$.

Alternatively, when the entrant's expected profit without signals is positive, then she enters the market when observing price p^* if and only if she observes signal h or an imprecise signal l (i.e., $\alpha < \bar{\alpha}_l$).

In the classical threshold price model, pooling equilibria entails a pooling price smaller than or equal to such a threshold. Here, however, by the entry rule, there are two threshold prices p_h and p_l, such that E enters the market if and only if she observes signal (h,p) with $p>p_h$ or signal (l,p) with $p>p_l$. Moreover, a key factor for entry is the sign of the entrant's expected payoff conditional to receiving signal $s\in\{h,l\}$. By Corollary 1, this sign depends on her expected profit without signals and the IS precision.

Proposition 2 below characterizes the set of pooling equilibria of game $G(\alpha)$. The proof is long and tedious because the incentive compatibility conditions have to take into account the entrant's positive and negative expected payoffs, the different bounds on α, the differences of the incumbent's cost technology and the different orderings of the entrant's threshold prices p_h and p_l with respect to the monopoly and duopoly prices. Therefore, most of it is relegated to the Appendix. Nevertheless, let us motivate first the approach, show some general results and set up some notation.

We offer here some useful properties and the incentive compatibility conditions of the monopolist's types that any pooling equilibrium of $G(\alpha)$ must satisfy.

Properties of pooling equilibria.

Let $A_l = \{\alpha | \Pi_E(l|\alpha) \le 0\}$ and $A_h = \{\alpha | \Pi_E(h|\alpha) \le 0\}$. Suppose first that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$. In this case $\bar{\alpha}_l < 1/2 < \bar{\alpha}_h < 1$. Hence, $\alpha > \bar{\alpha}_l$ and $\alpha \in A_l \ \forall \alpha$, $1/2 < \alpha < 1$. Namely, if the IS sends signal l, E does not enter the market when observing price p^* irrespective the precision α of the IS. This case is split in two main subcases: a) $1/2 < \alpha \le \bar{\alpha}_h$ and b) $\bar{\alpha}_h < \alpha < 1$.

(a) We start with the subcase $1/2 < \alpha \le \bar{\alpha}_h$. Here, $\alpha \in A_l \cap A_h$. Namely, E does not enter the market when observing price p^* irrespective of the signal sent by the IS. Hence, belief consistency (see Proposition 1) implies that $p^* \le p_h$. The following lemma, proven in the Appendix, establishes a useful result.

<u>Lemma 4.</u> Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $1/2 < \alpha \le \bar{\alpha}_h$. Then in every pooling equilibrium $p_H^M > p_L^M \ge p_h$.

With the above lemma we offer next the incentive compatibility conditions of the two types of incumbent, for low values of the IS precision.

The Incentive Compatibility Condition of H (ICC_H) when $1/2 < \alpha \le \bar{\alpha}_h$ ***.***

Since by belief consistency $p^* \le p_h$ and by Lemma 4 $p_H^M > p_h$, it suffices that the ICC_H considers only deviations to $p > p_h$. In this case E, when observing such a p, may enter the market with some probability and H might be better off choosing p_H^M. Conditional to the monopolist being of type H and for any $p \in (p_h, p_l]$, E receives signal h with probability α and enters, and signal l with probability $(1-\alpha)$ and does not enter. If $p > p_l$, then E enters for sure.

Let $\widehat{p}_H(\alpha)$ be the only p whose profits are equal to the expected profits of H when E enters with probability α, i.e.,

$$\Pi_H(p) = \alpha D_H + (1-\alpha)\Pi_H(p_H^M) = D_H + (1-\alpha)(\Pi_H(p_H^M) - D_H) > D_H$$

And let $\tilde{p}_H(\alpha)$ be the (unique) solution in p of the following equation

$$\Pi_H(p) = \alpha\Pi_H(p_H^M) + (1-\alpha)D_H > D_H$$

In other words, $\tilde{p}_H(\alpha)$ is the unique p whose profits are equal to the expected profits of H when E enters with probability $(1-\alpha)$. Notice that $\tilde{p}_H(\alpha) > \widehat{p}_H(\alpha) > \hat{p}$.

Both $\widehat{p}_H(\alpha)$ and $\tilde{p}_H(\alpha)$ play a key role in the relevant ICC_H when $1/2 < \alpha \leq \bar{\alpha}_h$ summarized by the following lemma.

<u>Lemma 5.</u> If $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $1/2 < \alpha \leq \bar{\alpha}_h$, then the ICC_H requires that at any pooling equilibrium p^*

(1) $p_H^M > p^* \geq \widehat{p}_H(\alpha)$ if $p_h < p_H^M \leq p_l$;

(2) $p_H^M > p^* \geq \hat{p}$ if $p_H^M > p_l$, with $\tilde{p}_H(\alpha) \geq p_l$

<u>Proof:</u> See the Appendix.

The incentive compatibility conditions for H say that in order p^* to be a pooling equilibrium price it should give profits higher than or equal to those from E entering with probability α when $p_h < p_H^M \leq p_l$ (part (1) of Lemma 5), and higher than or equal to those of the duopoly profits, when $p_H^M > p_l$, provided that $\tilde{p}_H(\alpha) \geq p_l$ (part (2) of Lemma 5).

The role of $\tilde{p}_H(\alpha)$ is clear. For any value of the IS precision $\alpha \in (1/2, \bar{\alpha}_h]$, second period expected profits after a first period pooling are the monopoly profits, and those following a deviation to p_l are a linear combination of the duopoly and the monopoly expected profits, weighted by probability α. Therefore, a pooling $p^* \geq \hat{p}$ avoids a deviation to p_l whenever, for example, first period profits from $p^* = \hat{p}$, plus the difference between second period expected profits from the pooling and those from the deviation are higher than or equal to the profits from p_l. This implies that $\tilde{p}_H(\alpha) \geq p_l$.

The Incentive Compatibility Condition of L (ICC_L ***) when*** $1/2 < \alpha \leq \bar{\alpha}_h$.

The specification of the relevant ICC_L when $1/2 < \alpha \leq \bar{\alpha}_h$ must consider, again, that by Lemma 4, $p_L^M \geq p_h$. Hence, when $p_L^M = p_h$, the only possible pooling equilibrium price satisfying belief consistency is $p^* = p_L^M$, and L would have no incentive to deviate. When $p_L^M > p_h$, L may consider deviations to p_L^M. Now, note that conditional to the monopolist being of type L and for any $p \in (p_h, p_l]$, E receives signal l with probability α and does not enter, and signal h with probability $(1-\alpha)$ and enters. Also, for any $p > p_l$, E enters for sure.

Define $\tilde{p}_L(\alpha)$ as the unique price solving

$$\Pi_L(p) = \alpha\Pi_L(p_L^M) + (1-\alpha)D_L = D_L + \alpha(\Pi_L(p_L^M) - D_L) > D_L$$

In other words, $\tilde{p}_L(\alpha)$ is the only p whose profits are equal to the expected profits of L when E enters with probability $(1-\alpha)$. And let $\hat{p}_L(\alpha)$ be the unique price solving

$$\Pi_L(p) = \alpha D_L + (1-\alpha)\Pi_L(p_L^M) > D_L$$

i.e., $\hat{p}_L(\alpha)$ is the unique p whose profits are equal to the expected profits of L when E enters with probability α. Notice that $\tilde{p}_L(\alpha) > \hat{p}_L(\alpha) > p_0$.

Both $\tilde{p}_L(\alpha)$ and $\hat{p}_L(\alpha)$ play a key role in the relevant ICC_L when $1/2 < \alpha \leq \bar{\alpha}_h$ specified in the following lemma.

Lemma 6. If $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $1/2 < \alpha \leq \bar{\alpha}_h$, then the ICC_L requires that at any pooling equilibrium p^*

(1) $p_L^M > p^* \geq \tilde{p}_L(\alpha)$ if $p_h < p_L^M \leq p_l$;

(2) $p_L^M > p^* \geq p_0$ if $p_L^M > p_l$, with $\hat{p}_L(\alpha) \geq p_l$.

Proof: See the Appendix.

Similarly to ICC_H in Lemma 5, the incentive compatibility conditions for L say that in order p^* to be a pooling equilibrium price it should give profits higher than or equal to those from E entering with probability $(1-\alpha)$ when $p_h < p_L^M \le p_l$ (part (1) of Lemma 6), and higher than or equal to those of the duopoly profits, when $p_L^M > p_l$, provided that $\widehat{p}_L(\alpha) \ge p_l$ (part (1) of Lemma 6).

Notice that part (2) in Lemmata 5 and 6 imply that p_h and p_l have to be sufficiently close to each other.

(b) We analyze next the incentive compatibility conditions of the two types of incumbent, for high values of the IS precision. Consider now that $\overline{\alpha}_h < \alpha < 1$. In this case $\alpha \in A_l \setminus \overline{A}_h$. Namely, E enters the market when observing price p^* if the IS sends signal h and does not enter if the IS sends signal l. Hence, by belief consistency (see Proposition 1), $p_h < p^* \le p_l$. Let us find some useful properties of the pooling equilibria in this case. First, a useful result is established by the following lemma, which proof is in the Appendix.

Lemma 7. If $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $\overline{\alpha}_h < \alpha < 1$, then at any p^*, $p_H^M > p_L^M \ge p_l$.

The above lemma allows us to offer next the incentive compatibility conditions of the two types of incumbent when the IS precision is relatively high.

The Incentive Compatibility Condition of H (ICC_H) when $\overline{\alpha}_h < \alpha < 1$.

By Lemma 7, $p_H^M > p_l$. That, together with the fact that $\overline{\alpha}_h < \alpha < 1$ and hence E enters if she observes signal h, are behind the relevant ICC_H summarized in the following lemma (more details are given in the proof of this lemma in the Appendix).

Lemma 8. If $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $\overline{\alpha}_h < \alpha < 1$, then the ICC_H requires that at any pooling equilibrium p^*, $p^* \ge \tilde{p}_H(\alpha) > \hat{p}$ and $\hat{p} \ge p_h$.

The intuition of this lemma is as follows. At any pooling $p^* \in (p_h, p_l]$, by Proposition 1, the entrant will enter with positive probability when receiving signal h, and second period expected profits for H are

$$\alpha D_H + (1-\alpha)\Pi_H(p_H^M) = \Pi_H(p_H^M) - \alpha\left(\Pi_H(p_H^M) - D_H\right)$$

where $\alpha = Prob(s = h / H)$ or entry probability when the monopoly is of type H.

In other words, for any value of the IS precision $\alpha \in (\bar{\alpha}_h, 1)$, second period expected profits after a first period pooling are the monopoly profits minus the probability of entry times the difference between the monopoly and duopoly profits. Since by Lemma 7, $p_H^M > p_l$, then any pooling $p^* \in (p_h, p_l]$ requires that the inequality

$$\Pi_H(p^*) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H$$

is satisfied, which is equivalent to

$$\Pi_H(p^*) \geq D_H + \alpha\left(\Pi_H(p_H^M) - D_H\right)$$

Thus, the profits from the first period pooling have to compensate the second period expected loss from entry, which, as shown above, for H-type monopoly implies that $p^* \geq \tilde{p}_H(\alpha)$.

Also, the pooling profits have to be higher than the duopoly profits. This is accomplished whenever $p^* \geq \tilde{p}_H(\alpha) > \hat{p}$ with $\hat{p} \geq p_h$, where $\hat{p}$ is the price for H that yields the duopoly profits.

The Incentive Compatibility Condition of L (ICC_L) when $\bar{\alpha}_h < \alpha < 1$.

By Lemma 7, $p_L^M \geq p_l$. Hence, when $p_L^M = p_l$, the only possible pooling equilibrium price satisfying belief consistency is $p^* = p_L^M$, and L would have no incentive to deviate. Consequently, the relevant ICC_L are the ones specified in the following lemma (proven in the Appendix)

Lemma 9. If $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $\bar{\alpha}_h < \alpha < 1$, then the ICC_L requires that at any pooling equilibrium satisfies $p_L^M > p^* \geq \hat{p}_L(\alpha)$ and $p_0 \geq p_h$.

The intuition of Lemma 9 is similar to the one of Lemma 8 but for the monopoly of type L. In particular, second period expected profits for the L-type after the pooling p^*, for any value of the IS precision $\alpha \in (\bar{\alpha}_h, 1)$, are

$$\alpha\Pi_L(p_L^M) + (1-\alpha)D_L = \Pi_L(p_L^M) - (1-\alpha)\left(\Pi_L(p_L^M) - D_L\right)$$

where $(1-\alpha) = Prob(s = h / L)$ or the probability of market entry when the monopoly is of type L. Then, since by Lemma 7 $p_L^M \geq p_l$, any pooling $p^* \in (p_h, p_l]$ requires

$$\Pi_L\left(p^*\right)+\alpha\Pi_L\left(p_L^M\right)+\left(1-\alpha\right)D_L\geq\Pi_L\left(p_L^M\right)+D_L$$

to be satisfied, which is equivalent to

$$\Pi_L\left(p^*\right)\geq\alpha D_L+\left(1-\alpha\right)\Pi_L\left(p_L^M\right)=D_L+\left(1-\alpha\right)\left(\Pi_L\left(p_L^M\right)-D_L\right)$$

Thus, again, the profits from the first period pooling have to compensate the second period expected loss from entry. The above entails $p^*\geq\widehat{p}_L\left(\alpha\right)$ for type L. Additionally, the pooling profits have to be higher than the duopoly profits. This is accomplished whenever $p_L^M>p^*\geq\widehat{p}_L\left(\alpha\right)$ with $p_0\geq p_h$, where p_0 is the price for L that yields the duopoly profits.

Consider now the case in which the entrant's expected profit without signals is positive, i.e., $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)>0$. By Corollary 2, the relevant threshold for her to assess the precision of the IS and hence her entry decision is $\overline{\alpha}_l$. It is easy to show that the properties and the incentive compatibility conditions that must be satisfied in every pooling equilibrium here are equivalent to the ones analyzed above for the case in which $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)<0$. More details are offered in the Appendix.

We offer next the pooling equilibria of $G\left(\alpha\right)$. Basically, given the entrant's entry rule, the two incentive compatibility conditions have to be compatible and sequentially rational at equilibrium, and the entry rule has to be consistent with equilibrium prices. Let $\delta=\left(\Pi_H\left(p_L^M\right)-D_H\right)/\left(\Pi_H\left(p_H^M\right)-D_H\right)$ be a threshold to bound by above the IS precision.

Proposition 2. Consider the game $G\left(\alpha\right)$, where $1/2<\alpha<1$. Let $SPEP$ be the set of all sequential pooling equilibrium prices and SPE the set of all sequential pooling equilibria of $G\left(\alpha\right)$.

(1) Suppose that expected profits (not conditioned to the IS signals) from entry are negative, i.e., $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)<0$. Then

 (i) If $p_L^M<\hat{p}$ (the cost technology is quite far apart), then $SPE=\varnothing$,

 (ii) If $p_L^M=\hat{p}$ and $\alpha\leq\overline{\alpha}_h$, then $SPEP=\left\{p_L^M\right\}$. If $\alpha>\overline{\alpha}_h$, then $SPE=\varnothing$,

(iii) If $p_L^M > \hat{p}$ (intermediate cost technology) then

(iii.1) For $\alpha \le \bar{\alpha}_h$, in every equilibrium in SPE, E stays out irrespective of the signal s and $SPEP = \left[\hat{p}, p_L^M \right]$.

(iii.2) If $\bar{\alpha}_h < \delta$, then for all α, $\bar{\alpha}_h < \alpha \le \delta$, E enters if and only if $s = h$, $SPE \ne \varnothing$ and $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha) \right), p_L^M \right]$.

(iii.3) For $\alpha > \delta$, $SPE = \varnothing$.

(2) Suppose that expected profits (not conditioned to the IS signals) from entry are positive, i.e. $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$. Then,

(i) If $p_L^M \le \hat{p}$ (the cost technology is quite far apart), then $SPE = \varnothing$,

(ii) If $p_L^M > \hat{p}$ (intermediate cost technology) then

(ii.1) For $\alpha < \bar{\alpha}_l$, $SPE = \varnothing$.

(ii.2) If $\bar{\alpha}_l \le \delta$, then for all α, $\bar{\alpha}_l \le \alpha \le \delta$, E enters if an only if $s = h$, $SPE \ne \varnothing$ and $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha) \right), p_L^M \right]$.

(ii.3) For $\alpha > \delta$, $SPE = \varnothing$.

(3) Suppose that $\delta < \max\left(\bar{\alpha}_l, \bar{\alpha}_h \right) = \bar{\alpha}_l$, then $SPE = \varnothing$. Suppose that $\delta < \max\left(\bar{\alpha}_l, \bar{\alpha}_h \right) = \bar{\alpha}_h$, then $SPE = \varnothing$ whenever[7] $\alpha > \bar{\alpha}_h$.

Proof: See the Appendix.

Proposition 2 asserts that sequential pooling equilibrium does not exist if either $p_L^M < \hat{p}$ or if $\alpha > \delta$. The first condition, $p_L^M < \hat{p}$, implies that the cost function of H is significantly higher than that of L. Even the duopoly price $\hat{p}$, when H competes with E, is above the monopoly price of L. In this case, it is too costly for H to mimic L and to "fool" E about his type. The other condition, $\alpha > \delta$, means that the IS is sufficiently accurate so that when E observes signal h, she knows that the true type of M is H with high probability, and she is better off entering the market. In this case, H, who knows that his type is detected with high probability, has no reason to pool and he is better off charging the monopoly price p_H^M, upsetting the pooling equilibrium.

[7] It is easy to verify that if $\delta = \bar{\alpha}_h$, then $SPE = \varnothing$ whenever $\alpha > \bar{\alpha}_h$.

When the cost technology is not too far apart and for intermediate values of α (i.e., $\bar{\alpha}_h < \alpha \leq \delta$ or $\bar{\alpha}_l \leq \alpha \leq \delta$), the set of pooling equilibria is non-empty even under profitable entry and the decision of E is to enter the market if and only if the signal sent by the IS is h. In this case, M of type H knows that α is sufficiently low so with significant probability, $(1-\alpha)$, E will obtain the wrong signal l and will stay out.

The meaning of set $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right), p_L^M\right]$ is clear. As already explained, $p^* \geq \tilde{p}_H(\alpha)$ gives H the first period profits from pooling that compensate the second period expected loss from entry. Similarly, $p^* \geq \hat{p}_L(\alpha)$ plays the same role for L. Therefore any pooling $p^* \in \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right), p_L^M\right]$ satisfies the *ICC's* of the two types of monopoly and sequential rationality with the entry rule.

However, it is also required that the precision α is not too low since, otherwise, E will not trust the signal and she will enter whether the signal is h or l. But then, the two type monopolists will be better off with their monopoly prices, upsetting a pooling equilibrium.

Proposition 2 also asserts that for $p_L^M > \hat{p}$ (intermediate cost technology) if $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$ (in which case $\bar{\alpha}_h < \bar{\alpha}_l$), and if α is relatively small ($\alpha < \bar{\alpha}_l$), then $SPE = \varnothing$. In other words, pooling equilibrium does not exist since E enters the market, independently of the signal *s*, and both types of M are better off deviating to their monopoly price. In contrast, and for the same technology, when $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and if α is relatively small ($\alpha \leq \bar{\alpha}_h$), then $SPE \neq \varnothing$, more precisely $SPEP = \left[\hat{p}, p_L^M\right]$, and E stays out irrespective of the signal s. This is clearly so because the IS is not accurate enough for the entrant to trust signal h when she receives it but the pooling prevents the entrant to guess the true type of M, and M of type H mimics type L.

Nevertheless, the incumbent can deter profitable entry with significant probability, for intermediate values of the IS precision. Namely, for $\bar{\alpha}_l \leq \alpha \leq \delta$, $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$, entry will be deterred if the signal sent by the IS is l. This probability is $\alpha > 1/2$ when M is of type L and $(1-\alpha)$ when M is of type H.

Remark 1. Note that when $\alpha = \delta$, then $\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) = \tilde{p}_H(\alpha) = p_L^M$ and $SPEP = \left\{p_L^M\right\}$. Also note that the relationship between δ and $\bar{\alpha}_s$, where $s \in \{h, l\}$, in game $G(\alpha)$ is not obvious and in general it is quite complex.

Remark 2. This relationship between δ and $\bar{\alpha}_s$, and therefore the existence of pooling equilibria in which there is a positive likelihood of market entry, is highly sensitive not only to the entrant's beliefs about the success of the incumbent's R&D project (determined by μ), but also to the characteristics of the market demand, the firms' cost structures and the mode of competition in case the entrant enters de market. This is shown in the following example.

Example: We study in this example the statement in Remark 2 under Cournot and Bertrand competition in a market with linear demand and linear cost functions. In particular, suppose that $p = a - Q$ is the total demand function and suppose that the cost functions are given by

$$C_L(q) = C_E(q) = c_L q,\ C_H(q) = c_H q, \text{ where } c_L < c_H < \hat{c},\ \hat{c} = (a + c_L)/2.$$

In this linear model,

$$p_L^M = \frac{a + c_L}{2},\ p_H^M = \frac{a + c_H}{2} \text{ and } \Pi_L\left(p_L^M\right) = \left(\frac{a - c_L}{2}\right)^2,\ \Pi_H\left(p_H^M\right) = \left(\frac{a - c_H}{2}\right)^2$$

We characterize next the existence of pooling equilibria in which there is a positive likelihood of market entry in this linear version of $G(\alpha)$ under Cournot and Bertrand competition. More specific details can be found in the Appendix.

<u>*Cournot competition*</u>

Let us focus on the particular case in which $c_H < \tilde{c}_H$, where $\tilde{c}_H = \left(5a + \left(3\sqrt{14} - 4\right)c_L\right) / \left(1 + 3\sqrt{14}\right) < \hat{c}$. Let $\tilde{K}_1$ be the solution to $\delta = \bar{\alpha}_l$, and $\tilde{K}_2$ be the solution to $\delta = \bar{\alpha}_h$, where $\tilde{K}_1 > (a - c_L)^2/9$ and $\tilde{K}_2 < (a - 2c_L + c_H)^2/9$ since $c_L < c_H < \hat{c}$. Moreover,

$$\tilde{K}_1 < \mu(a - 2c_L + c_H)^2/9 + (1 - \mu)(a - c_L)^2/9 < \tilde{K}_2$$

since $c_H < \tilde{c}_H$. Therefore, there are two interesting cases in terms of the entry cost K in which there is a positive likelihood of market entry under pooling equilibrium.

Case 1: $\tilde{K}_1 \le K < \mu\left(a-2c_L+c_H\right)^2/9+\left(1-\mu\right)\left(a-c_L\right)^2/9$ *(intermediate-low entry cost).* Note that in this case $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)>0$ and $\delta \ge \bar{\alpha}_l > 1/2$. Therefore, there exists some α, such that $\bar{\alpha}_l \le \alpha \le \delta$, for which E enters if and only if $s=h$, (this is, with probability α) and $SPEP=\left[\max\left(\tilde{p}_H\left(\alpha\right),\hat{p}_L\left(\alpha\right)\right),p_L^M\right]$. These pooling equilibria do not exist when the entry cost is too low, more precisely when $0<K<\tilde{K}_1$, since in this case $\delta<\bar{\alpha}_l$.

Case 2: $\mu\left(a-2c_L+c_H\right)^2/9+\left(1-\mu\right)\left(a-c_L\right)^2/9<K<\tilde{K}_2$ *(intermediate-high entry cost).* Note that in this case $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)<0$ and $\delta>\bar{\alpha}_h>1/2$. Therefore, there exists some α, such that $\bar{\alpha}_h<\alpha\le\delta$, for which E enters if and only if $s=h$, (this is, with probability α) and $SPEP=\left[\max\left(\tilde{p}_H\left(\alpha\right),\hat{p}_L\left(\alpha\right)\right),p_L^M\right]$. These pooling equilibria do not exist when the entry cost is too high, more precisely when $\tilde{K}_2\le K<\left(a-2c_L+c_H\right)^2/9$, since in that case $\delta\le\bar{\alpha}_h$.

<u>Bertrand competition</u>

Let us focus on the particular case in which $c_H<\bar{c}_H$, where $\bar{c}_H=\left(a+\sqrt{2}c_L\right)/\left(1+\sqrt{2}\right)<\hat{c}$. Let $\bar{K}_1$ be the solution to $\delta=\bar{\alpha}_l$, and $\bar{K}_2$ be the solution to $\delta=\bar{\alpha}_h$, where $\bar{K}_1>0$ and $\bar{K}_2<\left(c_H-c_L\right)\left(a-c_H\right)$ since $c_L<c_H<\hat{c}$. Moreover, $\bar{K}_1<\mu\left(c_H-c_L\right)\left(a-c_H\right)<\bar{K}_2$ since $c_H<\bar{c}_H$. Therefore, there are two interesting cases in terms of the entry cost K in which there is a positive likelihood of market entry under pooling equilibrium.

Case 1: $\bar{K}_1\le K<\mu\left(c_H-c_L\right)\left(a-c_H\right)$ *(intermediate-low entry cost).* Note that in this case $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)>0$ and $\delta\ge\bar{\alpha}_l>1/2$. Therefore, there exists some α, such that $\bar{\alpha}_l\le\alpha\le\delta$, for which E enters if and only if $s=h$, (this is, with probability α) and $SPEP=\left[\max\left(\tilde{p}_H\left(\alpha\right),\hat{p}_L\left(\alpha\right)\right),p_L^M\right]$. These pooling equilibria do not exist when the entry cost is too low, more precisely when $0<K<\bar{K}_1$, since in this case $\delta<\bar{\alpha}_l$.

Case 2: $\mu\left(c_H-c_L\right)\left(a-c_H\right)<K<\bar{K}_2$ *(intermediate-high entry cost).* Note that in this case $\mu\Delta_E\left(H\right)+\left(1-\mu\right)\Delta_E\left(L\right)<0$ and $\delta>\bar{\alpha}_h>1/2$. Therefore, there exists some α, such that $\bar{\alpha}_h < \alpha \le \delta$, for which E enters if and only if $s = h$, (this is, with probability α) and $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right), p_L^M\right]$. These pooling equilibria do not exist when the entry cost is too high, more precisely when $\bar{K}_2 \le K < (c_H - c_L)(a - c_H)$, since in that case $\delta \le \bar{\alpha}_h$.

This example shows the statement in Remark 2. For instance, in this example pooling equilibria in which there is a positive likelihood of market entry do not exist if $\tilde{c}_H \le c_H < \hat{c}$ when the mode of competition is *à la* Cournot, and if $\bar{c}_H \le c_H < \hat{c}$ when the mode of competition is *à la* Bertrand. As this shows, the mode of competition directly determines the characteristics of the market demand and the firms' cost structures for which there exist pooling equilibria in which espionage is likely to increase market competition.

Comparison with the case in which the IS is not informative ($\alpha = 1/2$).

A natural benchmark of comparison is when the IS precision is $\alpha = 1/2$ or, equivalently, a modification of the MR set up, when the entrant only has a cost type and does not operate an IS on M. This game is denoted by G_{MR}. Recall that $\hat{p}$ is the price for H and that p_0 is the price for L that yields the duopoly profits for H and L respectively. In this game, the entrant's strategy $\sigma_E(p)$, is a threshold strategy,

$$\sigma_E(p) = \begin{cases} "Stay\,out", & p \le \bar{p} \\ "Enter", & p > \bar{p} \end{cases}$$

where the threshold $\bar{p}$ is the choice of E, given her beliefs $Prob(H|p)$ and $Prob(L|p)$, for any p. Trivially, for $\alpha = 1/2$, $p_l = p_h = \bar{p}$. Therefore, for any $\alpha > 1/2$

$$\Pi_E(l, p_l) = \Pi_E(h, p_h) = \Pi_E(\bar{p}) > \Pi_E(l, p_h),$$

that implies $\Pi_E(l, p_l) > \Pi_E(l, p_h)$ and $\Pi_E(h, p_h) > \Pi_E(l, p_h)$ and hence $\bar{p} = p_h < p_l$.

Therefore, when $\alpha = 1/2$ the game $G(\alpha)$ collapses to G_{MR} and the entrant's expected profit is now $\mu\Delta_E(H) + (1-\mu)\Delta_E(L)$. We offer the pooling equilibria of G_{MR},

<u>Proposition 3</u>. Consider the game G_{MR}. Let $SPEP_{MR}$ be the set of all sequential pooling equilibrium prices and SPE_{MR} the set of all sequential pooling equilibria of G_{MR}. Then,

(1) When $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)< 0$

(i) $SPEP_{MR}=\{p_H=p_L=p^*=\overline{p}\}$, and

(ii) $\hat{p}\leq p^*\leq p_L^M$.

(2) When $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)> 0$, the set of all sequential pooling equilibria in G_{MR} is the empty set: $SPE_{MR}=\varnothing$.

Proposition 3 shows that in G_{MR} there exists no pooling equilibrium in which the potential entrant enters the market with positive probability. Moreover, when the entrant's expected profit is positive, i.e., $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)$>0, the entrant will enter when observes price p^*. Hence, both types H and L of monopoly should select their monopoly prices p_H^M and p_L^M, respectively, destroying the pooling equilibria. Therefore, profitable entry is never deterred. This result is maintained even when the incumbent monopolist does not know the entry costs of the entrant (see MR) and in the benchmark model of Bagwell and Ramey (1988), where both prices and advertising are signal for the incumbent monopolist. Bagwell (2007) extend the benchmark game to include two dimension of private information. Specifically, the incumbent is privately informed as to its cost type *and its level of patience* and selects price and advertising in the pre-entry period. He finds (intuitive) pooling equilibrium associated with the behavior of the patient high cost incumbent, who pools with the impatient low cost incumbent.

In contrast, our model also offers the existence of pooling equilibria under ex-ante profitable entry with only a dimension of private information by the incumbent but with two IS signals correlated with price, that provide additional (probabilistic) information about the incumbent type's. In fact, by Proposition 2, when the cost technology is intermediate, the entrant's expected profits without the IS are positive, and the IS is of intermediate accuracy, pooling equilibria with a positive likelihood of market entry exist (more precisely, when the entrant receives signal h). Moreover, entry may also happens when, under the same technology, the entrant's expected profits without the IS are negative, and the IS is accurate enough for the entrant to trust signal h when she receives it.

However, by comparing the pooling equilibria of game $G(\alpha)$ with those of G_{MR} it is clear that the use of a relatively not accurate but informative IS has no impact on either entry or entry deterrence. Thus, for intermediate cost technology, positive expected profits without the IS of the entrant, and a not informative or relatively small α, $SPE = \varnothing$ in both games. In other words, pooling equilibrium does not exist since E will enter the market and both types of M are better off deviating to their monopoly price. Similarly, if, for the same technology, the expected profits without the IS of the entrant are negative, and if the IS is not informative or it is informative but relatively inaccurate, then $SPE \neq \varnothing$ in both games but E stays out for sure.

As discussed before, the fact that the precision of the IS is common knowledge implies that existence of pooling equilibria requires such precision to be not as high such that the type H of M, knowing he is detected with high probability, is better off deviating and upsetting the pooling equilibrium.

5. Separating equilibrium

The analysis in the section captures the central theme of the classic limit pricing paper by MR, in that limit pricing occurs and yet profitable entry is never deterred. Bagwell and Ramey (1988) present a related model but assume that the probability of entry jumps from zero to one once the belief rises above a critical value. Although in our model signals help the entrant smooth her best response, the difference between our sequential separating equilibria and those of MR and Bagwell and Ramey (1988) is only in the behavior of the entrant for prices off the equilibrium path.

A separating equilibrium consists of a pair of prices (p_H, p_L) with $p_H \neq p_L$, and an entry rule, $\sigma_E(s,p)$ which is sequentially rational given consistent beliefs $Prob(H|h,p)$ and $Prob(H|l,p)$ for any pair (s,p). In this equilibrium E identifies with probability 1 the type of M. Hence, E enters the market when observing the price p_H irrespective of the signal of the IS, and E stays out when observing p_L, again irrespective of s. Therefore, by the entrant's strategy $p_H > p_l$ and $p_L \leq p_h$.

The following proposition characterizes the sequential separating equilibrium prices of $G(\alpha)$. The proof is also quite long and since the results are not too different from those of the existing literature, we only offer a sketch in the Appendix.[8]

Proposition 4. Consider the game $G(\alpha)$ for $\frac{1}{2} < \alpha < 1$, and let SSE be the set of all sequential separating equilibrium points of $G(\alpha)$. Let SSE_t be the set of all equilibrium prices of the t-type monopolist in SSE. Then, given consistent beliefs $Prob(H|h,p)$ and $Prob(H|l,p)$, for any p and s,

(1) $SSE_L = \{p_L | p_0 \le p_L \le \min(p_L^M, \hat{p})\}$ and $SSE_H = \{p_H^M\}$.

(2) Let $p_L \in SSE_L$. If $p_L < p_L^M$, then $p_L = p_h$. If $p_L = p_L^M$, then $p_L^M \le p_h$.

Note that when the cost functions are not too far apart, i.e. $\hat{p} < p_L^M$, all the separating equilibria limit price: $p_0 \le p_L < p_L^M$, while when the cost function of H is significantly higher than that of L in the sense that $p_L^M \le \hat{p}$, then $p_L \le p_L^M$. Therefore limit pricing is more likely in sequential separating equilibria when the cost technology is not too far apart, because in this case L needs a reduction of his monopoly price p_L^M in order to separate from H. When the cost function of H is significantly higher than that of L in the sense that $p_L^M \le \hat{p}$, limit pricing will only occur when the monopoly price p_L^M is not too low in the sense that either $p_h < p_l < p_L^M$ or $p_h < p_L^M < p_l$.

One crucial question is whether some of these separating equilibria exist in the same parameter region as some of the pooling equilibria with a positive likelihood of market entry (characterized in Proposition 2 in the previous section) and dominate them. Note that, as stated above, in a separating equilibrium, E identifies with probability 1 the type of M and the H-type monopoly, knowing that E will enter the market when observing price p_H, is better off choosing his monopoly price. Therefore, $SSE_H = \{p_H^M\}$ as stated by Proposition 4. It is straightforward to see that $p_H = p_H^M > p_l$

[8] The complete proof is available from the authors if required.

implies that in every possible separating equilibrium $p_h < \hat{p}$ and $p_l < \tilde{p}_H(\alpha)$ must hold. Otherwise, H might have incentives to deviate to p_h or to p_l, respectively. The important implication of this last constraint, $p_l < \tilde{p}_H(\alpha)$, is that pooling equilibrium prices for intermediate values of α ($\bar{\alpha}_h < \alpha \le \delta$ if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, and $\bar{\alpha}_l \le \alpha \le \delta$ if $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$) ensure that no separating equilibrium exists in the same parameter region, which leads to the following important result.

Corollary 3. No pooling equilibrium price of the form $\tilde{p}_H(\alpha) < p^* = p_l \le p_L^M$ can be dominated by any separating equilibrium and, therefore, existence of pooling equilibrium with a positive likelihood of market entry is ensured.

Let us next characterize the separating equilibria in G_{MR} in the following proposition in order to compare them with those of game $G(\alpha)$.

Proposition 5. Consider the game G_{MR} and let SSE^{MR} be the set of all sequential separating equilibrium points of G_{MR}. Then, given consistent beliefs $Prob(H|h,p)$ and $Prob(H|l,p)$, for any p,

(1) $SSE_L^{MR} = \{p_L | p_0 \le p_L \le \min(p_L^M, \hat{p})\}$ and $SSE_H^{MR} = \{p_H^M\}$.

(2) Let $p_L \in SSE_L^{MR}$. If $p_L < p_L^M$, then $p_L = \bar{p}$. If $p_L = p_L^M$, then $p_L^M \le \bar{p}$.

Remark 3. By Lemma 1, $\hat{p} > p_0$ and SSE_{MR} is non-empty.

Proposition 5 together with Proposition 4 allow to compare the separating equilibria of both G_{MR} and $G(\alpha)$. This comparison is summarized in the following corollary.

Corollary 4.

(1) The set SSE coincides with SSE_{MR}, the set of all sequential separating equilibrium points of G_{MR}.

(2) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Let p_h and $\bar{p}$ be the equilibrium cutoff price for entry when in $G(\alpha)$ (when $s = h$) and in G_{MR} respectively. Then, $\bar{p} = p_h$.

(3) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Then the equilibrium strategy of E in $G(\alpha)$ coincides with the equilibrium strategy of E in G_{MR} for all $p_L \notin (p_h, p_l]$. If $p_L \in (p_h, p_l]$, then E in $G(\alpha)$ enters the market with positive probability (which is α if M is of type H and $(1-\alpha)$ if M is of type L) and enters for sure in G_{MR}.

Part (3) of the corollary asserts that E is less inclined to enter the market in $G(\alpha)$. For all prices below $\overline{p} = p_h$, E stays out of the market in both games G_{MR} and $G(\alpha)$. For prices above p_l, E enters for sure in both games. But for prices p, $p_h < p \le p_l$, E enters the market in game $G(\alpha)$ if and only if the signal sent by the IS is h. In contrast, in this region, E enters the market for sure in game G_{MR}. The difference between $G(\alpha)$ and G_{MR} with regard to sequential separating equilibria is only in the behavior of E off the equilibrium path. Therefore, for prices off the equilibrium path the monopolist is better off with an entrant with access to an IS of commonly known precision.

6. Conclusions.

Recent advances in communication and information technologies have increased firms' incentive to acquire information about other firms exceeding the limits of competitive intelligence activities, and therefore, engaging in industrial espionage. This may have important implications for market entry given that potential market entrants can find easier in this context to gather valuable information about incumbents in the target market. Actually, from a theoretical point of view, incomplete information about incumbents' cost structure has been considered a relevant aspect in the explanation of market entry behavior. However, little theoretical work has been undertaken to analyze situations in which a potential entrant attempts to reduce such informational disadvantage.

This paper took a step forward in this direction by incorporating these potential entrant's activities in the context of modern cyber espionage. More precisely, it extended a one-sided asymmetric information version of Milgrom and Roberts' (1982) model considering that a potential market entrant, E, does not observe the outcome of the R&D project carried out by an incumbent monopolist with the aim to reduce his cost of production. The potential entrant develops an Intelligence System (IS) of precision α that allows her to gather noisy information about the cost structure of M. Based on this information and the price that M charges for his product, E decides whether to enter the market. We assumed that α is exogenously given and commonly known by both firms.

Our main contribution is to offer pooling equilibria even under ex-ante profitable entry and positive likelihood of market entry even in pooling equilibria under ex-ante non-profitable entry, with only a dimension of private information by the incumbent. This existence takes place for intermediate values of the IS precision, for which E enters the market if the IS tells her the cost of M is high. From this point of view, spying on incumbent firms increases market competition with high probability. Obviously, if the precision α of the IS is sufficiently low to affect the entrant's decision of staying out, then pooling equilibrium does not exist as it does not when α is very accurate.

Finally, we showed that the separating equilibria of our model are not affected by the spying activity of E. This is not very surprising since in a separating equilibrium E identifies the type of M with or without the use of the IS.

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Appendix

Proof of Lemma 1.

(i) $\Pi_L(p)-\Pi_H(p)=C_H(Q(p))-C_L(Q(p))$

$$\frac{\partial}{\partial p}\left[\Pi_L(p)-\Pi_H(p)\right]=Q'(p)\left[C'_H(Q(p))-C'_L(Q(p))\right]$$

By Assumptions 4 and 5 the right hand side of the above expression is negative.

(ii)

$$p_L^M q_L^M - C_L\left(q_L^M\right) \geq p_H^M q_H^M - C_L\left(q_H^M\right)$$
$$p_H^M q_H^M - C_H\left(q_H^M\right) \geq p_L^M q_L^M - C_H\left(q_L^M\right)$$

Adding the two inequalities we have

$$C_H\left(q_L^M\right)-C_L\left(q_L^M\right)\geq C_H\left(q_H^M\right)-C_L\left(q_H^M\right)$$

By Assumption 4 we have that $q_L^M \geq q_H^M$ and hence $p_L^M \leq p_H^M$.

Let us show that $p_L^M < p_H^M$. If not, then $p_L^M = p_H^M$. Since the First Order Condition (FOC) for M of type t is

$$\frac{\partial \Pi_t}{\partial p}\left(Q(p)\right)=0 \leftrightarrow C_t'\left(Q(p)\right)=p+\frac{Q(p)}{Q'(p)}$$

the solution does not depend on t, namely $C_L'\left(Q\left(p_L^M\right)\right)=C_H'\left(Q\left(p_L^M\right)\right)$. But this contradicts Assumption 4.

(iii) By Assumption 3,

$$\Pi_L\left(p_L^M\right)-D_L>\Pi_H\left(p_H^M\right)-D_H$$

Note that $D_L=\Pi_L\left(p_o\right)$ and $D_H=\Pi_H\left(\hat{p}\right)$. Hence this inequality can be written as

$$\Pi_L\left(p_L^M\right)-\Pi_L\left(p_0\right)>\Pi_H\left(p_H^M\right)-\Pi_H\left(\hat{p}\right)$$

Thus,

$$\Pi_L\left(p_L^M\right)-\Pi_H\left(p_L^M\right)+\underbrace{\Pi_H\left(p_L^M\right)-\Pi_H\left(p_H^M\right)}_{<0}>\Pi_L\left(p_0\right)-\Pi_H\left(\hat{p}\right)$$

Hence,

$$\Pi_L\left(p_L^M\right)-\Pi_H\left(p_L^M\right)>\Pi_L\left(p_0\right)-\Pi_H\left(\hat{p}\right) \tag{A1}$$

Since $p_0 \leq p_L^M$, we have by section (i) of Lemma 1

$$\Pi_L\left(p_0\right)-\Pi_H\left(p_0\right)>\Pi_L\left(p_L^M\right)-\Pi_H\left(p_L^M\right)$$

This together with (A1) imply that

$$\Pi_H\left(\hat{p}\right)>\Pi_H\left(p_0\right)$$

But $p_0 < p_H^M$ and $\hat{p} < p_H^M$ and by Assumption 2 $p_0 < \hat{p}$.

■

Proof of Lemma 2.

(i) By (3) in the main text,

$$Prob_n\left(H|h,p\right)=\frac{\mu\alpha\dfrac{f_n\left(p|H\right)}{f_n\left(p|L\right)}}{\mu\alpha\dfrac{f_n\left(p|H\right)}{f_n\left(p|L\right)}+\left(1-\mu\right)\left(1-\alpha\right)}$$

Hence,

$$Prob\left(H|h,p\right)=\frac{\mu\alpha g\left(p\right)}{\mu\alpha g\left(p\right)+\left(1-\mu\right)\left(1-\alpha\right)} \quad \text{(A2)}$$

and by Assumption 7, $Prob\left(H|h,p\right)$ is continuous in p.

The proof that $Prob\left(H|l,p\right)$ is continuous is similarly derived by (4) in the main text.

Since $Prob\left(L|s,p\right)=1-Prob\left(H|s,p\right)$, then $Prob\left(L|s,p\right)$ is also continuous.

Next note that $g\left(p\right)$ is non-decreasing in p since $g_n\left(p\right)$ is increasing in p for all n. Therefore Theorem 1 in Milgrom (1981) implies that if $p_1>p_2$, the posteriors $Prob\left(H|s,p_1\right)$ dominates $Prob\left(H|s,p_2\right)$, $s=\{h,l\}$, in the sense of first order stochastic dominance. In fact, it is easy to verify by (A2) that $\frac{\partial}{\partial p}Prob\left(H|h,p\right)\geq 0$ if and only if $g'\left(p\right)\geq 0$ and thus $Prob\left(H|h,p\right)$ is non-decreasing in p. The proof that $Prob\left(H|l,p\right)$ is non-decreasing is similar.

(ii)

Let

$$x_n\left(p\right)=\frac{Prob_n\left(H|h,p\right)}{Prob_n\left(H|l,p\right)}$$

By (3) and (4) in the main text,

$$x_n(p)-1=\frac{\frac{\alpha}{1-\alpha}\left[\mu(1-\alpha)f_n(p|H)+(1-\mu)\alpha f_n(p|L)\right]}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)}-1$$
$$=\frac{(1-\mu)\frac{\alpha^2}{1-\alpha}f_n(p|L)-(1-\mu)(1-\alpha)f_n(p|L)}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)}$$
$$=\frac{(1-\mu)f_n(p|L)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)\right]}$$
$$=\frac{(1-\mu)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha g_n(p)+(1-\mu)(1-\alpha)\right]}$$

Hence,

$$\lim_{n\to\infty}\left[x_n(p)-1\right]=\frac{(1-\mu)(2\alpha-1)}{(1-\alpha)\left[\mu\alpha g(p)+(1-\mu)(1-\alpha)\right]}>0$$

Hence $\lim_{n\to\infty} x_n(p)>1$ and, consequently, for every $p\geq 0$,

$$Prob(H|h,p)>Prob(H|l,p)$$

■

Proof of Lemma 3. By (5) in the main text,

$$\Pi_E(s,p)=Prob(H|s,p)\Delta_E(H)+Prob(L|s,p)\Delta_E(L)$$
$$=Prob(L|s,p)\left[\frac{Prob(H|s,p)}{Prob(L|s,p)}\Delta_E(H)+\Delta_E(L)\right] \quad \text{(A3)}$$

Let $s=h$. For every p,

$$\frac{Prob(H|h,p)}{Prob(L|h,p)}=\lim_{n\to\infty}\frac{\mu\alpha f_n(p|H)}{(1-\mu)(1-\alpha)f_n(p|L)}=\frac{\mu\alpha}{(1-\mu)(1-\alpha)}g(p)$$

We claim that $g(p)\to 0$ as $p\to 0$. This follows by Dini's theorem, as $g_n(p)$ is increasing in n, $g_n(p)$ is continuous in p and $g(p)$ is also continuous. Hence, for every $\gamma>0$, $\lim_{n\to\infty} g_n(p)=g(p)$ uniformly on $[0,\gamma]$. Since for every n, $g_n(p)\to 0$ as $p\to 0$, we have $g(p)\to 0$ as $p\to 0$. Consequently,

$$\lim_{p}\frac{Prob(H|h,p)}{Prob(L|h,p)}=0\text{, as } p\to 0 \quad \text{(A4)}$$

Inequality (A4) holds also when h is replaced by l (the proof is similar).

Next, let us show that $Prob(L|h,p)>0$ for small p.

$$Prob_n(L|h,p)=\frac{(1-\mu)(1-\alpha)f_n(p|L)}{\mu\alpha f_n(p|H)+(1-\mu)(1-\alpha)f_n(p|L)} =\frac{1}{1+\frac{\mu\alpha g_n(p)}{(1-\mu)(1-\alpha)}} \tag{A5}$$

Again, since $g_n(p)\to g(p)$ as $n\to\infty$ uniformly in any interval $[0,\gamma]$, $\gamma>0$, and since $g(p)\to 0$ as $p\to 0$,

$$Prob(L|h,p)=\lim_n Prob_n(L|h,p)\to 1\text{, as } p\to 0$$

In particular, $Prob(L|h,p)>0$ for p sufficiently small. In a similar way, we can prove that $Prob(L|l,p)>0$ for p sufficiently small.

Now, (A3), (A4), and the fact that $\Delta_E(L)<0$ and $Prob(L|s,p)>0$ for small p, imply that for sufficiently small p, $\Pi_E(s,p)<0$ and $J_s\neq\varnothing$.

Let us show that for p sufficiently large, $\Pi_E(s,p)>0$. We use the following claim.

Claim 1. $\lim_p Prob(L|s,p)=0$ as $p\to\infty$.

Proof: Let $n=1$ and $s=h$. By Assumption 7.2, $\lim\frac{f_1(p|H)}{f_1(p|L)}=\infty$. By (A5),

$$Prob_1(L|h,p)\to 0 \text{ as } p\to\infty$$

Hence, for every $\varepsilon>0$, there exists P s.t. for all $p>P$,

$$Prob_1(L|h,p)<\varepsilon$$

By (3) in the main text,

$$Prob_n(H|h,p)=\frac{\mu\alpha}{\mu\alpha+(1-\mu)(1-\alpha)\frac{f_n(p|L)}{f_n(p|H)}}$$

By Assumption 7.2, $Prob_n(H|h,p)$ is increasing in n and, hence, $Prob_n(L|h,p)$ is decreasing in n for every p. Thus, for all $p>P$,

$$Prob_n(L|h,p)<Prob_1(L|h,p)<\varepsilon$$

Hence, for every $\varepsilon > 0$ and for all $p > P$,

$$Prob(L|h,p) = \lim_{n\to\infty} Prob_n(L|h,p) \leq \varepsilon$$

implying that

$$\lim_{p\to\infty} Prob(L|h,p) = 0$$

The proof that $\underset{p}{Prob}(L|l,p) = 0$, as $p \to \infty$ is similarly derived.

■

Claim 1 together with (5), in the main text, imply that for p sufficiently large, $\Pi_E(s,p) > 0$, and the proof of Lemma 3 is completed.

■

Proof of Proposition 1. By part (i) of Lemma 2 and by (5), $\Pi_E(s,p)$ is continuous and non-decreasing in p (this follows from the fact that $Prob(H|s,p)$ is continuous and non-decreasing in p, $\Delta_E(H) > 0$, $Prob(L|s,p) = 1 - Prob(H|s,p)$ and $\Delta_E(L) < 0$).

By Lemma 3, $\Pi_E(s,p) < 0$ for small p and $\Pi_E(s,p) > 0$ for sufficiently large p.

Let

$$p_h = \max\{p \geq 0 | \Pi_E(h,p) \leq 0\}$$

$$p_l = \max\{p \geq 0 | \Pi_E(l,p) \leq 0\}$$

By the continuity of $\Pi_E(s,p)$ in p,

$$\Pi_E(h,p_h) = \Pi_E(l,p_l) = 0 \tag{A6}$$

and E enters the market if and only if she observes either (h,p) s.t. $p > p_h$ or (l,p) s.t. $p > p_l$.

By part (ii) of Lemma 2 it is easy to verify that

$$\Pi_E(h,p) > \Pi_E(l,p) \tag{A7}$$

By (A6) and (A7)

$$\Pi_E(l,p_l) = \Pi_E(h,p_h) > \Pi_E(l,p_h)$$

and since $\Pi_E(s,p)$ is non-decreasing in p, we have $p_l > p_h$.

■

Proof of Lemma 4. Suppose first that $p_H^M \leq p_h$. Then also $p_L^M < p_h$ and, according to Proposition 1, E stays out whether she observes p_L^M or p_H^M. Hence, both types of M will set their (different) monopoly prices, a contradiction.

Suppose next that $p_L^M < p_h$ and $p_H^M > p_h$. Since $\alpha \leq \bar{\alpha}_h$, E stays out and $p^* \leq p_h$, and then $p^* = p_L^M$. In this case, H is better off by deviating to p_h since

$$\Pi_H\left(p_L^M\right)+\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_h\right)+\Pi_H\left(p_H^M\right)$$

implies that $p_L^M \geq p_h$, a contradiction.

■

The Incentive Compatibility Condition when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ ***and*** $1/2<\alpha \leq \bar{\alpha}_h$ **.**

Proof of Lemma 5 (ICC_H ***).*** Considering the discussion about the relevant ICC_H in the main text, for p^* to be a pooling equilibrium price of H, the following should hold,

$$\Pi_H\left(p^*\right)+\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_H^M\right)+\begin{cases}\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right), & \text{if } p_h < p_H^M \leq p_l \\ D_H, & \text{if } p_H^M > p_l\end{cases}$$

The first inequality implies that

$$\Pi_H\left(p^*\right) \geq \alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right)$$

Recalling that $D_H=\Pi_H(\hat{p})$ and the definition of $\hat{p}_H(\alpha)$ in the main text, $\Pi_H\left(p^*\right) \geq \Pi_H\left(\hat{p}_H(\alpha)\right)$ implies that $p^* \geq \hat{p}_H(\alpha)$.

The second inequality requires that $\Pi_H\left(p^*\right) \geq D_H$, or that $p^* \geq \hat{p}$.

Additionally, another potential deviation is the following. When $p_H^M > p_l$ H may deviate to p_l, where E enters with probability α. To avoid this deviation,

$$\Pi_H\left(p^*\right)+\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_l\right)+\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right)$$

Since $p_H^M > p_l$ and, by above, $p^* \geq \hat{p}$, take without loss of generality $p^* = \hat{p}$. Then

$$D_H+\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_l\right)+\alpha D_H+(1-\alpha)\Pi_H\left(p_H^M\right) \tag{A8}$$

Recalling the definition of $\tilde{p}_H(\alpha)$ in the main text, inequality (A8) is satisfied whenever $\tilde{p}_H(\alpha) \geq p_l$.

■

Proof of Lemma 6 (ICC_L ***).*** Considering the discussion about the relevant ICC_L in the main text, to avoid deviations and, therefore, for p^* be a pooling equilibrium price of L, the following should hold,

$$\Pi_L\left(p^*\right)+\Pi_L\left(p_L^M\right)\geq \Pi_L\left(p_L^M\right)+\begin{cases}\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L, & \text{if } p_h < p_L^M \leq p_l \\ D_L, & \text{if } p_L^M > p_l\end{cases}$$

Recall that $D_L = \Pi_L\left(p_0\right)$ and the definition of $\tilde{p}_L\left(\alpha\right)$ in the main text. Then, the first inequality above is satisfied whenever $p_L^M > p^* \geq \tilde{p}_L\left(\alpha\right)$.

For the second inequality it suffices that $p^* \geq p_0$. To deter deviation to p_l when $p_L^M > p_l$, take $p^* = p_0$, then

$$D_L + \Pi_L\left(p_L^M\right)\geq \Pi_L\left(p_l\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L \tag{A9}$$

Therefore, a pooling $p^* \geq p_0$ avoids a deviation to p_l whenever, for example, first period profits from $p^* = p_0$, plus the difference between second period expected profits from the pooling and those from the deviation are higher than or equal to the profits from p_l. This, taking into account the definition of $\widehat{p}_L\left(\alpha\right)$ in the main text, implies that (A9) is satisfied whenever $\widehat{p}_L\left(\alpha\right)\geq p_l$.

■

Proof of Lemma 7. Proof: Suppose first that $p_H^M \leq p_h$. Then also $p_L^M < p_h$ and both, L and H are better off deviating to their monopoly prices. Hence $p_H^M > p_h$.

Now suppose that $p_L^M = p_h$, then for any $p^* \in \left(p_h, p_l\right]$,

$$\Pi_L\left(p^*\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L \geq \Pi_L\left(p_L^M\right)+\Pi_L\left(p_L^M\right)$$

which is a contradiction. Consider that $p_l > p_L^M > p_h$ and $p_H^M \leq p_l$, then for any p^*,

$$\Pi_L\left(p^*\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L \geq \Pi_L\left(p_L^M\right)+\alpha\Pi_L\left(p_L^M\right)+(1-\alpha)D_L$$

The above is impossible unless $p^* = p_L^M$. Therefore, L will deviate to p_L^M. Similarly for H, who will deviate to p_H^M.

Finally consider that $p_h < p_L^M < p_l$ and $p_H^M > p_l$, then L will deviate to p_L^M, and H will be better off deviating to p_l. Therefore, $p_L^M \geq p_l$, and Lemma 7 follows.

■

The Incentive Compatibility Condition when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ ***and*** $\bar{\alpha}_h<\alpha<1$**.**

Proof of lemma 8 (ICC_H ***).*** Since $\bar{\alpha}_h<\alpha<1$ and hence E enters if she observes signal h, and by $p_H^M>p_l$ (according to Lemma 7), then, in order that H does not deviate to p_H^M at a pooling $p^*\in(p_h,p_l]$,

$$\Pi_H(p^*)+\alpha D_H+(1-\alpha)\Pi_H(p_H^M)\geq\Pi_H(p_H^M)+D_H$$

or

$$\Pi_H(p^*)\geq\alpha\Pi_H(p_H^M)+(1-\alpha)D_H=\Pi_H(\tilde{p}_H(\alpha))$$

Where $\tilde{p}_H(\alpha)$ is defined in the main text. Then, $p^*\geq\tilde{p}_H(\alpha)$.

Notice that H might have incentives to deviate to p_h. To deter this deviation, let us assume that $p^*=\tilde{p}_H(\alpha)$. Then

$$\Pi_H(\tilde{p}_H(\alpha))+\alpha D_H+(1-\alpha)\Pi_H(p_H^M)\geq\Pi_H(p_h)+\Pi_H(p_H^M)$$

By the definition of $\tilde{p}_H(\alpha)$ (in the main text), the above inequality is equivalent to $D_H\geq\Pi_H(p_h)$ or $\hat{p}\geq p_h$. Then, it follows the result of Lemma 8.

■

Proof of lemma 9 (ICC_L ***).*** To deter deviations by L to p_L^M when $p_L^M>p_l$, a pooling price has to satisfy,

$$\Pi_L(p^*)+\alpha\Pi_L(p_L^M)+(1-\alpha)D_L\geq\Pi_L(p_L^M)+D_L$$

Or

$$\Pi_L(p^*)\geq(1-\alpha)\Pi_L(p_L^M)+\alpha D_L=\Pi_L(\widehat{p}_L(\alpha))$$

where $\widehat{p}_L(\alpha)$ is defined in the main text. Then, $p^*\geq\widehat{p}_L(\alpha)$.

To avoid a deviation to p_h, consider that $p^*=\widehat{p}_L(\alpha)$ .Then,

$$\Pi_L(\widehat{p}_L(\alpha))+\alpha\Pi_L(p_L^M)+(1-\alpha)D_L\geq\Pi_L(p_h)+\Pi_L(p_L^M)$$

By the definition of $\widehat{p}_L(\alpha)$ (in the main text), the above inequality implies $D_L\geq\Pi_L(p_h)$ or $p_0\geq p_h$ and Lemma 9 follows.

■

The Incentive Compatibility Conditions when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$.

By the discussion before Corollary 2 in the main text, $\bar{\alpha}_h<1/2<\bar{\alpha}_l<1$. Hence $\alpha>\bar{\alpha}_h$ and $\alpha\notin A_h$, $\forall\alpha$, $1/2<\alpha<1$. Namely, if the IS sends signal h, E enters the market when observing price p^* irrespective the precision α of the IS. This case is also split in two subcases: 1) $1/2<\alpha<\bar{\alpha}_l$ and 2) $\bar{\alpha}_l\le\alpha<1$.

(1) Consider that $1/2<\alpha<\bar{\alpha}_l$. In this case $\alpha\notin A_l\cup A_h$. Namely, E enters the market when observing price p^* irrespective of the signal sent by the IS and, therefore, both H and L should select the prices p_H^M and p_L^M, respectively. This, together with $p_L^M<p_H^M$, leads to the following corollary.

<u>Corollary A1.</u> No pooling equilibrium exists when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$ and $1/2<\alpha<\bar{\alpha}_l$.

(2) Suppose now that $\bar{\alpha}_l\le\alpha<1$. In this case $\alpha\in A_l\setminus\bar{A}_h$. Namely, similarly to the case (b) in the main text, E enters the market when observing price p^* if the IS sends the signal h and does not enter if the IS sends signal l. Hence,

<u>Corollary A2.</u> The properties of the pooling equilibria and the incentive compatibility conditions of H and L when $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)>0$ and $\bar{\alpha}_l\le\alpha<1$ are the same as in the case in which $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $\bar{\alpha}_h<\alpha<1$, but the relevant threshold for α is $\bar{\alpha}_l$, not $\bar{\alpha}_h$.

Proof of Proposition 2. Let us consider the four cases analyzed in the main text.

(a) Recall that in this case $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$ and $1/2<\alpha\le\bar{\alpha}_h$.

Consider first that $p_L^M<\hat{p}$. The next lemma proves part 1.(i) of Proposition 1 when $1/2<\alpha\le\bar{\alpha}_h$.

<u>Lemma A1.</u> If $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $1/2<\alpha\le\bar{\alpha}_h$ and $p_L^M<\hat{p}$, then $SPE=\varnothing$.

<u>Proof:</u> As stated in the main text, in every possible pooling equilibrium satisfying belief consistency $p^*\le p_h$. This together with Lemma 4 implies that $p_L^M\ge p_h\ge p^*$ which is

incompatible with the ICC_H, since either $p^* \geq \widehat{p}_H(\alpha) > \hat{p}$ (see Lemma 5(1)) or $p^* \geq \hat{p}$ (see Lemma 5(2)). Hence $p_L^M \geq p^* = p_h \geq \widehat{p}_H(\alpha) > \hat{p} > p_L^M$, a contradiction.

■

The next lemma proves the first part of case 1.(ii) of Proposition 1.

Lemma A2. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $1/2<\alpha\leq\bar{\alpha}_h$ and $p_L^M = \hat{p}$. Then if $p^* \in SPEP$, $p^* = \{p_L^M\}$.

Proof: First we prove that $p_L^M \in SPEP$. By Lemma 4 $p_L^M \geq p_h$. Suppose that $p_h = p_L^M < p_l < p_H^M$. On the one hand, as stated in the main text, the only possible pooling equilibrium price satisfying belief consistency in this case is $p^* = p_L^M$. On the other hand, by Lemma 5(2) $p_H^M > p^* \geq \hat{p}$ and H has no incentive to deviate to p_H^M in this case since $p_L^M = \hat{p}$.

To avoid deviations by H to p_l, it suffices that $\tilde{p}_H(\alpha) \geq p_l$ (see Lemma 5(2)).

For configurations $p_L^M > p_h$, belief consistency requires $p^* = p_h$ which is incompatible with the ICC_H, where either $p^* \geq \widehat{p}_H(\alpha) > \hat{p}$ if $p_h < p_H^M \leq p_l$, or $p^* \geq \hat{p}$ if $p_H^M > p_l$ (see Lemma 5(1) and 5(2) respectively), since $p_L^M = \hat{p}$. Hence, $SPE = \varnothing$ in these cases.

■

Now, we deal with the case $p_L^M > \hat{p}$, and prove part 1.(iii.1) of Proposition 1. Lemma A3 below proves this result. We show that $SPEP = \left\{p^* \middle| \hat{p} \leq p^* \leq p_L^M\right\}$ for all α, $1/2<\alpha\leq\bar{\alpha}_h$.

Lemma A3. Suppose that $\mu\Delta_E(H)+(1-\mu)\Delta_E(L)<0$, $1/2<\alpha\leq\bar{\alpha}_h$ and $p_L^M > \hat{p}$. Then $SPEP = \left[\hat{p}, p_L^M\right]$.

Proof: First, note that the set

$$\left\{p^* \middle| \hat{p} \leq p^* \leq p_L^M\right\} \subseteq SPEP$$

To see that, consider any configuration such that $p_L^M = p_h$ and $p_H^M > p_l$. As stated in the main text, the only possible pooling equilibrium price satisfying belief consistency in this case is $p^* = p_L^M$. Note that by the ICC_H, $p_H^M > p^* \geq \hat{p}$ (see Lemma 5.(2)), H has no incentive to deviate to p_H^M in this case since $p_L^M > \hat{p}$.

To avoid deviations by H to p_l, by Lemma 5(2) any $p^* > \hat{p}$ deters these deviations whenever $\tilde{p}_H(\alpha) \geq p_l$. Since $p^* = p_L^M > \hat{p}$, then no deviation takes place.

Next consider configurations such that $\hat{p} \leq p_h$ and $p_l < p_L^M < p_H^M$. Clearly in this case (by belief consistency) $p^* = p_h$. By the ICC_H, $p^* \geq \hat{p}$ in order to H not to deviate to p_H^M (see Lemma 5(2)) and by the ICC_L, $p^* \geq p_0$ in order to L not to deviate to p_L^M (see Lemma 6(2)). Both incentive compatibility conditions are compatible since $\hat{p} > p_0$ (see Lemma 1).

To avoid deviations by both H and L to p_l, it suffices that, by Lemma 5(2) $\tilde{p}_H(\alpha) \geq p_l$, and by Lemma 6(2) $\widehat{p}_L(\alpha) \geq p_l$, respectively.

Hence, $\hat{p} \leq p^* = p_h < p_l < p_L^M$ for $\min\left(\tilde{p}_H(\alpha), \widehat{p}_L(\alpha)\right) \geq p_l$ is an incentive compatible pooling equilibrium that satisfies belief consistency.

Therefore, $\left\{p^* \middle| \hat{p} \leq p^* \leq p_L^M\right\} \subseteq SPEP$ as claimed.

Next, we prove that $SPEP \subseteq \left[\hat{p}, p_L^M\right]$. On the one hand, for any configuration $\hat{p} \leq p_h < p_L^M$, if a pooling exists, by belief consistency $p^* = p_h$ and therefore $p^* \in \left[\hat{p}, p_L^M\right]$. On the other hand, notice that whenever $\hat{p} > p_h$ no equilibrium exists since by belief consistency $p^* = p_h$ and this is not compatible with the ICC_H, $p^* \geq \widehat{p}_H(\alpha) > \hat{p}$ (see Lemma 5(1)) or $p^* \geq \hat{p}$ (see Lemma 5(2)).

Therefore, if $p^* \in SPEP$, then $p^* \in \left[\hat{p}, p_L^M\right]$ as claimed.

■

(b) Recall that in this second case $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $\bar{\alpha}_h < \alpha < 1$.

The next lemma completes the proofs of parts 1.(i) and 1.(ii) of Proposition 1.

Lemma A4. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\bar{\alpha}_h < \alpha < 1$ and $p_L^M \leq \hat{p}$, then $SPE = \varnothing$.

Proof: As stated in the main text, in every possible pooling equilibrium satisfying belief consistency in this case $p_h < p^* \leq p_l$. This together with Lemma 7 implies that $p_L^M \geq p^*$, which is incompatible with the ICC_H, $p^* \geq \tilde{p}_H(\alpha) > \hat{p}$ (see Lemma 8).

■

Now suppose that $\hat{p} < p_L^M$. Let us prove next that if $\bar{\alpha}_h < \alpha \leq \delta$, then

$$SPEP = \left[\max\left(\tilde{p}_H(\alpha), \widehat{p}_L(\alpha)\right), p_L^M\right],$$

i.e., part 1.(iii.2) of Proposition 1.

Lemma A5. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\bar{\alpha}_h < \alpha < 1$ and $\hat{p} < p_L^M$. Then

$$SPEP = \left\{p^* \middle| \max\left(\tilde{p}_H(\alpha), \widehat{p}_L(\alpha)\right) \leq p^* \leq p_L^M\right\}$$

and this set is non-empty if $\delta > \bar{\alpha}_h$ and for all α, $\bar{\alpha}_h < \alpha \leq \delta$.

Proof: Consider first the case $p_L^M = p_l$. As stated in the main text, the only possible pooling equilibrium price satisfying belief consistency in this case is $p^* = p_L^M$. Note that the ICC_H requires that $p^* \geq \tilde{p}_H(\alpha) > \hat{p}$ (see Lemma 8), which in this case is equivalent to $p_L^M \geq \tilde{p}_H(\alpha) > \hat{p}$. Note that $p_L^M \geq \tilde{p}_H(\alpha)$ implies

$$\Pi_H\left(p_L^M\right) + \alpha D_H + (1-\alpha)\Pi_H\left(p_H^M\right) \geq \Pi_H\left(p_H^M\right) + D_H$$

or that

$$\alpha \leq \frac{\Pi_H\left(p_L^M\right) - D_H}{\Pi_H\left(p_H^M\right) - D_H} = \frac{\Pi_H\left(p_L^M\right) - \Pi_H(\hat{p})}{\Pi_H\left(p_H^M\right) - \Pi_H(\hat{p})} \equiv \delta$$

Clearly, $0 < \delta < 1$.

To avoid deviations by H and L to p_h, by Lemmata 8 and 9 it suffices that $p_0 \geq p_h$ (see Lemma 1(iii)). Hence $p^* = p_L^M > \hat{p}$, for $p_0 \geq p_h$ and $\bar{\alpha}_h < \alpha \leq \delta$, is an incentive compatible pooling equilibrium that satisfies belief consistency.

Notice now that if $p_L^M > p_l$, then, as stated in the main text, the unique pooling equilibrium guaranteeing belief consistency is $p^* = p_l$. The ICC_H requires that $p^* \geq \tilde{p}_H(\alpha) > \hat{p}$ (see Lemma 8), and the ICC_L that $p^* \geq \widehat{p}_L(\alpha)$ (see Lemma 9). Therefore $\max\left(\tilde{p}_H(\alpha), \widehat{p}_L(\alpha)\right) \leq p_l = p^* < p_L^M$ should hold, where $\tilde{p}_H(\alpha) < p_L^M$ implies $\alpha < \delta$.

To deter deviations by both H and L to p_h, again applies the ICC_H given by Lemma 8 and the ICC_L given by Lemma 9. Hence $\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) \leq p_l = p^* < p_L^M$, for $p_0 \geq p_h$ and $\bar{\alpha}_h < \alpha < \delta$, is an incentive compatible pooling equilibrium that satisfies belief consistency.

Hence set $SPEP = \left[\max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right), p_L^M\right]$ as claimed.

■

Notice that the above implies part 1.(iii.3) of Proposition 1, i.e., for $\alpha > \delta$, $SPE = \emptyset$.

(c) In this case $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$ and $1/2 < \alpha < \bar{\alpha}_l$. By Corollary A1, $SPE = \emptyset$ in this case (part 2.(ii.1) of Proposition 1).

(d) In this case $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$ and $\bar{\alpha}_l \leq \alpha < 1$. By Corollary A2, this case is equivalent to the case (b) above, but the relevant threshold for α is $\bar{\alpha}_l$, not $\bar{\alpha}_h$. Hence, if $\hat{p} < p_L^M$, then

$$SPEP = \left\{p^* \middle| \max\left(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)\right) \leq p^* \leq p_L^M\right\},$$

and this set is non-empty if $\delta \geq \bar{\alpha}_l$ and for all α, $\bar{\alpha}_l \leq \alpha \leq \delta$. This proves part 2.(ii.2) of Proposition 1. As above, notice that 2.(ii.3) of the Proposition, i.e., for $\alpha > \delta$, $SPE = \emptyset$ is satisfied.

As in the proofs of parts 1.(i) and 1.(ii) of the Proposition (see Lemmata A1 and A2 above), if $p_L^M \leq \hat{p}$, then $SPE = \emptyset$, which proves part 2.(i).

Finally, note that parts 2.(ii.1) and 2.(ii.3) of the Proposition imply the first part of point (3), while parts 1.(iii.1) and 1.(iii.3) imply the second part.

■

Example.

Cournot competition

The equilibrium under Cournot competition in between L and E is characterized by $p_0 = (a + 2c_L)/3$ and $D_L = D_E(L) = (a - c_L)^2/9$; and the competition in between H and E is characterized by $\hat{p} = (a + c_L + c_H)/3$, $D_H = (a - 2c_H + c_L)^2/9$ and $D_E(H) = (a - 2c_L + c_H)^2/9$. Note that $p_L^M = (a + c_L)/2 > \hat{p} = (a + c_L + c_H)/3$ since $c_H < \hat{c}$.

Consequently, under Cournot competition, Assumption 1, according to which $D_E(L)-K<0$ and $D_E(H)-K>0$, is satisfied for all K such that $(a-c_L)^2/9<K<(a-2c_L+c_H)^2/9$. Moreover, Assumption 3, according to which $\Pi_L(p_L^M)-D_L>\Pi_H(p_H^M)-D_H$ is satisfied given that $c_L<c_H<\hat{c}$.

Bertrand competition

The equilibrium under Bertrand competition in between L and E is characterized by $p_0=c_L$ and $D_L=D_E(L)=0$; and the competition in between H and E is characterized by $\hat{p}=c_H$, $D_H=0$ and $D_E(H)=(c_H-c_L)(a-c_H)$. Note that $p_L^M=(a+c_L)/2>\hat{p}=c_H$ since $c_H<\hat{c}$.

Consequently, under Bertrand competition, Assumption 1, according to which $D_E(L)-K<0$ and $D_E(H)-K>0$, is satisfied for all K such that $0<K<(c_H-c_L)(a-c_H)$. Moreover, Assumption 3, according to which $\Pi_L(p_L^M)-D_L>\Pi_H(p_H^M)-D_H$ is satisfied given that $c_L<c_H<\hat{c}$.

A sketch of the proof of Proposition 4. As stated in the main text, in this equilibrium E identifies with probability 1 the type of M. Hence, E enters the market when observing price p_H irrespective of the signal of the IS, and E stays out when observing p_L, again irrespective of s. Therefore, by the entrant's strategy $p_H>p_l$ and $p_L\le p_h$. See Figure 1 in the main text.

The H-type monopoly, knowing that entry will occur is better off choosing the price p_H^M. Thus $SSE_H=\{p_H^M\}$ and E enters for sure when she observes price p_H^M. In particular, $p_H^M>p_l$.

We need to prove that $SSE_L=\{p_L \mid p_0\le p_L\le \min(p_L^M,\hat{p})\}$ for all α, $1/2<\alpha<1$. Hence, there are two relevant cases, when $p_L^M\le\hat{p}$ and when $\hat{p}<p_L^M$.

The proof follows two steps. The first step consists in showing that $p_L\in[p_0,\min(p_L^M,\hat{p})]$ in both cases $p_L^M\le\hat{p}$ and $\hat{p}<p_L^M$.

It is straightforward to see that in the first case, when $p_L^M \leq \hat{p}$, $p_L = p_L^M$ can be supported as a separating equilibrium price (no type of M has incentives to deviate), for instance when

$$p_L = p_L^M \leq p_h \leq \hat{p} < p_l \leq \tilde{p}_H(\alpha) < p_H^M$$

And $p_L \in \left[p_0, p_L^M\right)$ can also be supported as a separating equilibrium price, for instance when

$$p_L = p_h < p_l < p_L^M \leq \hat{p} < \tilde{p}_H(\alpha) < p_H^M .$$

Similarly, in the second case $\hat{p} < p_L^M$ it is straightforward to see that $p_L \in \left[p_0, \hat{p}\right]$ can be supported as a separating equilibrium price, for instance when

$$p_L = p_h \leq \hat{p} < p_l < \min\left(p_L^M, \tilde{p}_H(\alpha)\right) < p_H^M$$

The second step of the proof is to show that if $p_L \notin \left[p_0, \min\left(p_L^M, \hat{p}\right)\right]$, then $p_L \notin SSE_L$, which is easy to show considering the relevant positions of p_h and p_l.

■