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Holidays, weekends and range-based volatility

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ABSTRACT

This study analyses the effect of non-trading periods on the forecasting ability of S&P500 index range-based volatility models. We find that volatility significantly diminishes on the first trading day after holidays and weekends, but not after long weekends. Our findings indicate that models that include autoregressive terms that interact with dummies that allow us to capture changes in volatility levels after interrupting periods provide greater explanatory power than simple autoregressive models. Therefore, the shorter the length of the non-trading periods between two trading days, the higher the overestimation of the volatility if this effect is not considered in volatility forecasting.

1. Introduction

Volatility is one of the key concepts in the financial literature. The estimation of volatility is crucial for analysts, portfolio managers and professional traders. A large number of academic studies have focused on the study of different volatility models for decades, such as GARCH and stochastic volatility models. However, following [Chou, Chou, and Liu \(2015\)](#), both models are inaccurate and inefficient because they are based on the closing prices of the reference period, ignoring what happens to the evolution of the prices throughout the trading session. [Fiess and MacDonald \(2002\)](#) note that the difference between the opening and closing prices serves as a measure of the intraday trend, while the difference between the high and low prices marks the intraday trading range and represents a measure of volatility. [Parkinson \(1980\)](#), [Garman and Klass \(1980\)](#) and [Rogers, Satchell, and Yoon \(1994\)](#), among others, have demonstrated theoretically that range-based volatility estimators are more efficient than return-based volatility estimators. In addition, [Alizadeh, Brandt, and Diebold \(2002\)](#) show theoretically, numerically, and empirically that range-based volatility measures are highly efficient and approximately Gaussian and are robust to microstructure noise. Specifically, several papers have highlighted empirically the superior forecasting performance of range-based volatility models when compared to traditional close-to-close volatility models in the stock market. [Li and Weinbaum \(2001\)](#) find overwhelming support for the extreme value volatility estimators for the S&P 500 and S&P100 stock indices, in terms of bias and efficiency. [Pandey \(2002\)](#) obtains a similar result for the S&P CNX Nifty. [Shu and Zhang \(2006\)](#) investigate, with S&P 500 index data, the relative performance of various historical volatility estimators that incorporate daily trading range and conclude that all perform very well when an asset price follows a continuous geometric Brownian motion. Furthermore, they observed that the range estimators are fairly robust toward microstructure effects. [Li and Hong \(2011\)](#) employ weekly observations of S&P 500 to show the superiority of range-based autoregressive models to forecast the future volatility relative to GARCH models. [Miralles-Marcelo, Miralles-Quirós, and Miralles-Quirós \(2013\)](#) analyse the forecasting ability of the CARR model proposed by [Chou \(2005\)](#) using the S&P 500. Their results show that the

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Parkinson model is better for upward trends and volatilities which are higher and lower than the mean, while the CARR model is better for downward trends and mean volatilities. Wang, Hsu, and Liu (2014) demonstrate that the introduction of range volatility estimators into the conditional variance of GARCH (1,1) improves the out-of-sample volatility forecasts of Nasdaq-100 stock index returns with a daily horizon. A similar result is obtained by Molnár (2016), who conducted an empirical analysis on stocks, stock indices and simulated data that shows that the range-GARCH (1,1) model performs significantly better than the standard GARCH (1,1) model both in terms of in-sample fit and out-of-sample forecasting ability. Petneházi and Gáll (2018) use data from all current constituents of the Dow Jones Industrial Average index to investigate the predictability of several range-based stock volatility estimators by applying recurrent neural networks and find that changes in the values of range-based estimators are more predictable than those of the estimators using only daily closing values. Finally, Miralles-Quirós, Miralles-Quirós, and Nogueira (2019) analyse the out-of-sample performance of different portfolio strategies using the returns and volatility forecasts from a VAR-ADCC-GARCH approach and find a significant improvement in performance when returns standardised by the Garman-Klass volatility estimator are used as endogenous variables in the said approach.

The above mentioned papers analyse the data as regular series, ignoring the role of non-trading periods on market volatility. However, an increasing number of papers have focused on the analysis of the influence of interrupting periods on volatility. On the one hand, there are some papers that address non-trading periods in a novel approach that is based on the use of data from related markets that are open to forecast volatility in a market that is closed. Todorova and Souček (2014) compare different approaches to model the information flow arising during the closed market time for enhancing the predictive power of the standard volatility forecasting HAR model for the Australian market. They perform this analysis by combining the Australian overnight returns with realized volatility estimates of related assets from other markets that are open when the Australian exchange is closed. In a similar way, Jayawardena, Todorova, Li, and Su (2016) adopt this new forecasting approach to predict Australian stock volatility by using squared overnight returns, pre-open volatility of the same asset and realized volatilities of related assets from other markets.

On the other hand, other authors deal with non-trading periods using data from the same market. Hansson and Hördahl (2005), estimate the conditional variance of daily Swedish OMX-index returns from 1984 to 1996 with stochastic volatility models and GARCH models by allowing for weekend and holiday effects in the variance. They find that although the in-sample models that incorporate weekend/holiday effects outperform models that do not allow for these effects, the volatility forecasting ability does not increase when the effects are considered. Tsiakas (2010) analyses the economic gains of trading stocks around non-trading periods in the US stock market from 1962 to 2005 and introduces a stochastic volatility model that distinguishes between weeknights, weekends, holidays and long weekends, and detects higher close-to-close volatility both in post-holidays and post-long weekends in comparison to pre-holidays and pre-long weekends, respectively. In the same line, Wang and Hsiao (2010) examine the S&P 500 and FTSE 100 indices from 1997 to 2008 using an adaptation of the GJR (1,1) model proposed by Glosten, Jagannathan, and Runkle (1993), including the effects of weekends, weekday holiday periods, and half-day trading periods. They observe for both indices that only weekday holiday periods and half-day trading periods have significant positive and negative impacts on the determination of the expected daily volatility. Finally, Lyócsa and Molnár (2017) have investigated the effect of non-trading days on heterogeneous autoregressive volatility models in equity markets. They observe that realized volatility significantly decreases after weekends and suggest incorporating an autoregressive coefficient into the models that interacts with a dummy that identifies whether a trading day follows a weekend.

As we have shown, a growing body of financial literature studies how to model and forecast volatility of time series data containing interrupting periods. This study aims to contribute to this research area by analysing volatility after non-trading periods using range-based volatility measures, taking into account not only weekends, but also holidays and long weekends. As far as we know, no study has analysed the influence of all these interrupting periods on the forecasting ability of range-based volatility models, so it is our purpose herein to shed some light on how these models are affected when these effects are considered. The results obtained indicate significant decreases in the level of volatility, especially after holidays; suggesting the importance of differentiating the type of interrupting period when estimating range-based volatilities. The remainder of the work is organised as follows. Section 2 explains briefly the range-based volatility estimators used in this study. Section 3 describes the data. Section 4 discusses the methodology and presents the main empirical findings. Section 5 concludes.

2. Daily volatility estimators

Rogers et al., 1994, Li and Weinbaum (2001), and Chou et al. (2015), among others, have carried out comprehensive literature reviews on extreme value volatility estimators.¹ In this section, we briefly revise the range-based measures of volatility used in this study, as well as their most important characteristics. In general, the main advantage of these measures regarding the close-to-close standard deviation is that they take into account some kind of intraday information and, as a consequence, even if two consecutive closing prices were the same, these measures could detect high intraday volatility.

The first range-based volatility measure we calculate is the estimator proposed by Parkinson (1980), whose main characteristic is that it only uses the maximum and the minimum price of the trading day. The expression of the Parkinson measure in terms of variance is the following:

¹ See Chou et al. (2015) for an updated and complete literature review of range-based volatility models, including range-based multivariate volatility models and realized ranges and their application in finance, such as value at risk estimation, hedge, spillover effect, portfolio management and microstructure issues.

$$\sigma_{P,t}^2 = \frac{1}{4Ln(2)} \left(Ln \left(\frac{h_t}{l_t} \right) \right)^2 \quad (1)$$

where h_t and l_t correspond to the highest and the lowest daily price on day t , respectively. Following [Chou et al. \(2015\)](#), the efficiency of the [Parkinson \(1980\)](#) estimator intuitively comes from the fact that the intraday price range gives more information regarding the future volatility than two arbitrary points such as the closing prices.

The second measure we estimate is the [Garman and Klass \(1980\)](#) volatility estimator. This measure is considered an extension of that proposed by [Parkinson \(1980\)](#) and is characterised by including the maximum, the minimum, the opening, and the closing prices of the trading session. The expression of the Garman-Klass estimator is as follows:

$$\sigma_{GK,t}^2 = \frac{1}{2} \left(Ln \left(\frac{h_t}{l_t} \right) \right)^2 - (2Ln(2) - 1) \left(Ln \left(\frac{c_t}{o_t} \right) \right)^2 \quad (2)$$

where o_t corresponds to the opening price of the session on day t , and c_t corresponds to the closing price of the session on day t .

Both Parkinson and Garman-Klass are estimators derived under the assumption of a driftless world. [Rogers and Satchell \(1991\)](#) extended [Parkinson \(1980\)](#) by allowing the geometric Brownian motion to have a non-zero drift term, and obtain a measure that is similar to the Garman-Klass measure in terms of efficiency. The expression of Rogers-Satchell is as follows:

$$\sigma_{RS,t}^2 = Ln \left(\frac{h_t}{c_t} \right) Ln \left(\frac{h_t}{o_t} \right) Ln \left(\frac{l_t}{c_t} \right) Ln \left(\frac{l_t}{o_t} \right) \quad (3)$$

This measure presents a problem when the maximum or the minimum prices of the session coincide with the opening or closing prices. Furthermore, this measure, like the two previous ones, does not address jumps during the night, and therefore underestimates volatility.

Finally, the last volatility measure we have calculated is the Garman-Klass Yang-Zhang Extension. It is a merger of the Garman-Klass measure (1980) and that of Yang-Zhang (2000). This volatility estimator takes into account the jumps during the night, including in its formulation the closing price of the previous day. The expression of the Garman-Klass Yang-Zhang Extension is as follows:

$$\sigma_{GKYZ,t}^2 = \left(Ln \left(\frac{o_t}{c_{t-1}} \right) \right)^2 + \frac{1}{2} \left(Ln \left(\frac{h_t}{l_t} \right) \right)^2 - (2Ln(2) - 1) \left(Ln \left(\frac{c_t}{o_t} \right) \right)^2 \quad (4)$$

where c_{t-1} corresponds to the closing price on day $t - 1$.

3. Data

Data from the Thomson Reuters portal have been used. Opening price, closing price, and the highest and the lowest prices traded during the daily session have been extracted for the index S&P 500 from January 1, 1993 to December 31, 2017. This sample period presents multiple scenarios that encompass periods of growth and economic crises. We have observed that the highest and lowest prices recorded by the Thomson Reuters database only make reference to prices traded throughout the session. Therefore, if the opening or closing prices are higher or lower than the highest and the lowest prices, the daily observations of the maximum and the minimum prices have been corrected in order to avoid contradictions between prices, i.e. if the closing price is greater than the highest price, then this price has been replaced by the closing price.

The original sample has been divided into two subsamples: from January 1, 1993 to December 31, 2015 and from January 1, 2016 to December 31, 2017. The first one has been used for in-sample estimations and the second one for out-of-sample forecast comparisons. Based on daily estimates, we have calculated the sample mean for each variance estimator where E refers to the method used to calculate the daily variance estimator, i.e., $E = P, GK, RS$ and $GKYZ$. Panel A of [Table 1](#) reports summary statistics for all the variance measures used in the analysis. The Parkinson and Roger-Satchell volatility estimators exhibit the highest and the lowest sample means, respectively. The corresponding annualized volatilities (AV) are around 14% and can be considered as normal values over long-periods for the S&P 500. The range of all the daily estimators is high in comparison with their means, and there are important differences between the means and the medians in all the cases. It is important to note the role of serial correlation. The first-order autocorrelation coefficients range from 0.357 to 0.597 and they are significant and positive at the 1% level in all the measures. Following [Lyócsa and Molnár \(2017\)](#), if we assume the same dependence throughout the week, but we observe and ignore a difference in volatility after the interrupting period, we will underestimate the dependence between weekdays and overestimate the dependence after non-trading periods. Specifically, the in-sample period in our study contains 5791 range-based daily estimates in which there are 51 holidays, 1047 weekends, and 153 long weekends. This implies that 21.6% of the trading days are interrupted by more than one day.

Panel B of [Table 1](#) reports the daily sample means after each type of interrupting period. We have considered three cases of non-trading days: one day without trading (hereafter holiday), two consecutive days without trading (weekend), and three or more consecutive days without trading (long weekend). All range-based estimators show low sample means after the three non-trading periods considered, except for Parkinson's. Panel B of [Table 1](#) also provides the difference between the mean after each type of interrupting period and the sample mean. The results obtained indicate notable decreases in the level of volatility, about -25% , especially after holidays. However, in order to determine whether any of these differences between these mean values are statistically

Table 1
Descriptive Statistics.

Panel A	$\hat{\sigma}_E^2$	Min	Max	AV	m	$\rho(1)$
PARK	0.964	0.011	42.884	15.52	0.409	0.597
GK	0.839	0.001	59.181	14.48	0.381	0.537
RS	0.820	0.001	64.073	14.32	0.333	0.357
GKYZ	0.854	0.001	59.887	14.61	0.386	0.550
Panel B	$\hat{\sigma}_{E,H}^2$	% _{H-E}	$\hat{\sigma}_{E,W}^2$	% _{W-E}	$\hat{\sigma}_{E,L}^2$	% _{L-E}
PARK	0.730	-23.4	0.974	1	1.046	8.2
GK	0.560	-27.9	0.765	-7.4	0.781	-5.8
RS	0.516	-30.4	0.716	-10.4	0.707	-11.3
GKYZ	0.600	-25.4	0.782	-7.2	0.817	-3.7

Panel A reports summary statistics for all the variables used in the analyses. PARK, GK, RS, and GKYZ denote the measures proposed by Parkinson (1980), Garman and Klass (1980), Roger and Satchell (1991), and Yang and Zhang (2000), respectively. $\hat{\sigma}_E^2$ is the sample mean for each variance estimator; Min and Max are the minimum and the maximum of the daily variance estimator; AV indicates the annualized volatility; m is the median; and $\rho(1)$ is the first-order autocorrelation coefficient. Panel B presents the means for each E variance estimator after each type of interrupting period. $\hat{\sigma}_{E,H}^2$ indicates the mean of the first trading day after a holiday; $\hat{\sigma}_{E,W}^2$ indicates the mean of the first trading day after a weekend; $\hat{\sigma}_{E,L}^2$ indicates the mean of the first trading day after a long weekend, and % indicates the percentage difference compared to the sample mean of Panel A. All variance estimators are scaled up by $10E+4$. The sample period goes from January 1, 1993 to December 31, 2015 in which there are 51 holidays, 1,047 weekends, and 153 long weekends.

significant, the following dummy variable regression model has been estimated for each variance estimator:

$$\hat{\sigma}_{E,t}^2 = \alpha + \beta_H [I(t, t-1) = 2] + \beta_W [I(t, t-1) = 3] + \beta_L [I(t, t-1) > 3] + e_{E,t} \quad (5)$$

where the dummy variable $[I(t, t-1) = 2]$ takes the value 1 if the difference between two consecutive trading days is 2 (holiday) and zero otherwise; $[I(t, t-1) = 3]$ takes the value 1 if the difference between two consecutive trading days is 3 (weekend) and zero otherwise; and $[I(t, t-1) > 3]$ takes the value 1 if the difference between two consecutive trading days is higher than 3 (long weekend) and zero otherwise.

The series are analysed by means of the least squares method taking into account both heteroskedasticity and autocorrelation problems in the error term by using the Newey-West estimator for the variance-covariance matrix. A negative and significant coefficient in whatever dummy would indicate that the volatility after the corresponding interrupting period is significantly lower than the volatility on a trading day that is not preceded by one or more holidays. Table 2 presents the results of the estimation for each E variance estimator considering the three types of non-trading periods. All the dummy coefficients that are significant are negative. Coefficients of the dummy variables that capture the difference in the level of the variances after holidays and weekends (β_H and β_W) are negative and significantly different from zero in GK, RS and GKYZ at the usual significance levels. It is interesting to note that the variance after long weekends does not diminish in any estimator. Therefore, in general, both outcomes of Table 1 and 2 imply that in order to study the fall in volatility after a non-trading period, it is recommendable to differentiate the type of interrupting period: holiday, weekend or long weekend.

Table 2
Volatility after non-trading periods.

Parameter	PARK		GK		RS		GKYZ	
	Estimation	t-statistic	Estimation	t-statistic	Estimation	t-statistic	Estimation	t-statistic
α	0.9610	13.6747 ^c	0.8611	13.2261 ^c	0.8510	13.0934 ^c	0.8746	13.2319 ^c
β_H	-0.2307	-1.3948	-0.3007	-2.3802 ^b	-0.3352	-2.6837 ^c	-0.2750	-1.8876 ^a
β_W	0.0133	0.2235	-0.0963	-2.2816 ^b	-0.1352	-2.4737 ^b	-0.0927	-2.2138 ^b
β_L	0.0849	0.6146	-0.0800	-0.8518	-0.1437	-1.3543	-0.0575	-0.5834
R^2	0.0001		0.0005		0.0008		0.0005	
AIC	-13.9708		-14.1899		-13.9916		-14.1665	
SC	-13.9662		-14.1853		-13.9870		-14.1619	
HQC	-13.9692		-14.1883		-13.9900		-14.1649	

This table presents the estimates of the dummy regression variable model proposed in Eq. (5) for each E variance estimator with the Newey and West correction. The estimations have been multiplied by $10E+5$ to improve readability. PARK, GK, RS, and GKYZ denote the measures proposed by Parkinson (1980), Garman and Klass (1980), Roger and Satchell (1991), and Yang and Zhang (2000), respectively. β_H , β_W , and β_L are the coefficients of the dummy variables that identify the first trading day after a holiday, a weekend, and a long weekend. Superscripts a, b, and c indicate statistical significance at the 10%, 5%, and 1% significance level, respectively. R^2 denotes the coefficient of determination; AIC stands for the Akaike information criterion; SC is the Schwarz criterion; and HQC denotes the Hannan-Quinn criterion. The sample period goes from January 1, 1993 to December 31, 2015 in which there are 5791 observations.

4. Methodology and results

4.1. Methodology

This paper follows the idea proposed by Lyócsa and Molnár (2017) to address non-trading periods but applied to range-based volatility measures. As we have previously mentioned, Lyócsa and Molnár (2017) study the realized volatility in the S&P 500 both on consecutive trading days and after weekends, and observe that realized volatility significantly decreases after weekends. Furthermore, they document that realized volatility series present a high first-order autocorrelation coefficient. Combining these empirical facts, they suggest that if we ignore the difference in volatility after weekends and assume the same dependence throughout the week, we will underestimate the dependence in volatility between weekdays and overestimate the dependence in volatility between Friday and Monday. To cope with this problem, they suggest incorporating an autoregressive coefficient into the heterogeneous autoregressive volatility model that interacts with a dummy that identifies whether or not a trading day follows a weekend. However, given the outcomes obtained in Table 1 and 2, unlike Lyócsa and Molnár (2017), we have defined three dummy variables that identify whether a trading day follows a holiday, a weekend or a long weekend. More specifically, for each range-based volatility estimator, we have estimated two volatility models. The first one is an autoregressive model (AM) in which each E volatility estimator is a linear function of p past lags:

$$\sigma_{E,t}^2 = \alpha + \sum_{i=1}^p \beta_i \sigma_{E,t-i}^2 + \varepsilon_{E,t} \tag{6}$$

The second model (AMD) takes into account p autoregressive components and three first-order autoregressive coefficients that interact with the three dummies described in section 3 in order to capture volatility changes that can appear after different non-trading periods:

$$\sigma_{E,t}^2 = \alpha + \sum_{i=1}^p \beta_i \sigma_{E,t-i}^2 + \gamma_H \sigma_{E,t-1}^2 [I(t, t-1) = 2] + \gamma_W \sigma_{E,t-1}^2 [I(t, t-1) = 3] + \gamma_L \sigma_{E,t-1}^2 [I(t, t-1) > 3] + \xi_{E,t} \tag{7}$$

Note that this model allows autoregressive coefficients to differ between days following holidays, weekends, long weekends, and days following a single trading day if the difference is 1.

Then, only for the best AM and AMD estimated models, we have calculated the forecast error by comparing the forecasted volatility to actual volatility. Comparison of the forecast techniques takes place from January 1, 2016 to December 31, 2017.

4.2. Results

Tables 3 and 4 present the in-sample estimates of AM and AMD volatility models. These regression models have also been estimated using both ordinary least squares and the Newey and West correction that accounts for heteroskedasticity and serial correlation problems. The number of lags has been determined using Akaike info criterion (AIC), the Schwarz criterion (SC), and the Hannan-Quinn criterion (HQC). The chosen AM and AMD models have been specified by introducing a five-order autoregressive process.

Table 3
Autoregressive Models.

Parameter	PARK		GK		RS		GKYZ	
	Estimation	t-statistic	Estimation	t-statistic	Estimation	t-statistic	Estimation	t-statistic
α	1.59E-05	3.2229 ^c	1.51E-05	3.5072 ^c	2.05E-05	3.8259 ^c	1.50E-05	3.4981 ^c
$\beta_{.1}$	0.3144	5.8661 ^c	0.2575	6.6442 ^c	0.1451	3.0159 ^c	0.2659	7.0788 ^c
$\beta_{.2}$	0.1313	2.7834 ^c	0.1362	3.4003 ^c	0.1566	4.1616 ^c	0.1393	3.5039 ^c
$\beta_{.3}$	0.1183	2.2306 ^b	0.1032	2.2754 ^b	0.0546	1.0076	0.1019	2.1347 ^b
$\beta_{.4}$	0.1462	2.7131 ^c	0.2086	2.0735 ^b	0.2438	2.0921 ^b	0.2086	2.0930 ^b
$\beta_{.5}$	0.1248	1.7780 ^a	0.1150	2.0844 ^b	0.1497	3.2919 ^b	0.1085	1.9514 ^a
R^2	0.4584		0.4144		0.2816		0.4249	
$Adj-R^2$	0.4579		0.4139		0.2810		0.4244	
AIC	-14.5824		-14.7230		-14.3201		-14.7177	
SC	-14.5755		-14.7161		-14.3132		-14.7108	
HQC	-14.5800		-14.7206		-14.3177		-14.7153	

This table presents the estimates of the autoregressive model with the Newey and West correction proposed in Eq. (6) for each E variance estimator. PARK, GK, RS, and GKYZ denote the measures proposed by Parkinson (1980), Garman and Klass (1980), Roger and Satchell (1991) and Yang and Zhang (2000), respectively. Superscripts a, b, and c indicate statistical significance at the 10%, 5%, and 1% significance level, respectively. R^2 denotes the coefficient of determination; $Adj-R^2$ indicates the adjusted coefficient of determination; AIC stands for the Akaike information criterion; SC is the Schwarz criterion; and HQC denotes the Hannan-Quinn criterion. The sample period goes from January 1, 1993 to December 31, 2015 in which there are 5,791 observations.

Table 4
Autoregressive Models considering non-trading periods.

Parameter	PARK		GK		RS		GKYZ	
	Estimation	t-Statistic	Estimation	t-Statistic	Estimation	t-Statistic	Estimation	t-Statistic
α	1.52E-05	3.1638 ^c	1.38E-05	2.7187 ^c	1.92E-05	3.0570 ^c	1.36E-05	2.7649 ^c
β_{-1}	0.3044	5.9890 ^c	0.3124	5.2758 ^c	0.1955	5.9975 ^c	0.3181	5.2563 ^c
β_{-2}	0.1178	2.5377 ^c	0.1529	4.5276 ^c	0.1725	5.0603 ^c	0.1538	4.6501 ^c
β_{-3}	0.1269	2.5374 ^c	0.0928	1.7272 ^a	0.0485	0.7930	0.0922	1.6524 ^a
β_{-4}	0.1492	2.7064 ^c	0.2008	2.1527 ^b	0.2398	2.1400 ^b	0.2001	2.1676 ^b
β_{-5}	0.1320	1.7686 ^b	0.1248	2.2571 ^b	0.1555	3.8794 ^c	0.1186	2.1367 ^b
γ_H	-0.9286	-2.7941 ^c	-1.0114	-2.9341 ^c	-0.8819	-2.1107 ^b	-0.9574	-2.6611 ^c
γ_W	0.0672	1.0357	-0.2153	-2.1270 ^b	-0.2017	-1.5793	-0.1981	-2.0983 ^b
γ_L	0.3282	1.8411 ^a	0.0181	0.0910	-0.1520	-0.5339	0.0464	0.2262
R^2	0.4637		0.4277		0.2920		0.4365	
$Adj-R^2$	0.4630		0.4269		0.2910		0.4358	
AIC	-14.5913		-14.7450		-14.3335		-14.7372	
SC	-14.5809		-14.7347		-14.3232		-14.7269	
HQC	-14.5877		-14.7414		-14.3299		-14.7336	

This table presents the estimates of the autoregressive model with the Newey and West correction proposed in Eq. (7) for each E volatility estimator. PARK, GK, RS, and GKYZ denote the measures proposed by Parkinson (1980), Garman and Klass (1980), Roger and Satchell (1991) and Yang and Zhang (2000), respectively. Superscripts a, b, and c indicate statistical significance at the 10%, 5%, and 1% significance level, respectively. R^2 denotes the coefficient of determination; $Adj-R^2$ indicates the adjusted coefficient of determination; AIC stands for the Akaike information criterion; SC is the Schwarz criterion; and HQC denotes the Hannan-Quinn criterion. The sample period goes from January 1, 1993 to December 31, 2015 in which there are 51 holidays, 1047 weekends, and 153 long weekends.

The results for the estimation of Eq. (6) are presented in Table 3. The constant and all the autoregressive coefficients of each E volatility estimator are statistically significant at the conventional levels, confirming the stylised fact known as persistence in volatility (all β parameters are positive and significant). The coefficients of the lagged variables in the AM models decreased until lag 3 to increase again in lag 4. In general, the estimated models exhibit a high predictive power, with an adjusted R^2 that ranges from 28% to 45%.

Table 4 presents the regression results for Eq. (7). The adjusted R^2 in AMD models slightly increases and the AIC, SC, and HQC criteria decrease again in relation to the AM models presented in Table 3. The pattern observed for the β parameters is similar to that observed in Table 3 for all the models. However, the new autoregressive coefficients for observations that follow a holiday (γ_H) are significant and negative in all the models. Furthermore, the coefficients for trading days that follow a weekend (γ_W) are significant and negative for GK and GKYZ measures but they are, in absolute terms, lower than those corresponding to post-holidays. As usual, volatility diminishes after holidays and weekends but, surprisingly, not after long weekends. In this case, volatility rises (PARK measure) or does not change (the rest of the estimators). These findings are in line with the reduction of volatility levels observed in means after non-trading days (see Table 1), and suggest the importance of differentiating the type of interrupting period when estimating range-based volatilities.

In order to give more robustness to our findings, we have estimated several methodological approaches. Firstly, Kambouroudis and McMillan (2015) show that for the majority of the markets, long in-sample periods are not necessary in order to produce the most accurate forecasts supporting the practitioners'/investors' view. To provide additional strength to our results, we have re-estimated all the models, eliminating 1000 observations (approximately four years of data). The magnitude and significance of the coefficients estimated with the shorter in-sample period is almost identical to those reported in Tables 3 and 4, and, therefore, they are omitted for the sake of brevity. Secondly, given that the data-generating process is unknown, we have calculated the simple average of the four range-based estimators and performed a dummy regression model taking the average as the dependent variable. We have observed significant and negative decreases in the volatility after weekends. Thirdly, as modelling variance can lead to results sensitive to outliers, to make our results more robust we have estimated the model applying Weighted Least Squares where the weights are reciprocal to the dependent variable (see Clement and Preve, 2019). The results obtained are in accordance with those presented in Table 4, confirming that volatility decreases only after holidays and weekends.²

4.3. Forecast evaluation

Out-of-sample forecast comparisons are made using additional data from January 1, 2016 to December 31, 2017. The volatility forecasting accuracy has been evaluated through the analysis of different loss criteria such as the Mean Squared Error (MSE), the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE). The model with the lowest values for each loss criterion will be the best predictor of volatility. Furthermore, we have calculated the decomposition of the Mean Squared Forecast Error into bias, variance and covariance proportions. In this case, the best model is that with the lowest bias and variance proportions

² The results of these analyses are not included for the sake of brevity, but they are available from the authors upon request.

Table 5
Out-of-sample forecast performance of competing models.

Panel A: AM Models	MSE	MAE	MAPE	BIAS	VARIANCE	COVARIANCE
PARK	0.0158	2.5384	2.7396	0.0801	0.1697	0.7502
GK	0.0152	2.3863	2.5627	0.0678	0.1877	0.7445
RS	0.0302	2.9906	7.2654	0.0595	0.2536	0.6869
GKYZ	0.0151	2.4296	2.0289	0.0701	0.1683	0.7617
Panel B: AMD Models	MSE	MAE	MAPE	BIAS	VARIANCE	COVARIANCE
PARK	0.0166	2.5395	2.6694	0.0744	0.1225	0.8030
GK	0.0152	2.3391	2.3890	0.0586	0.1386	0.8027
RS	0.0301	2.9356	7.0035	0.0546	0.2160	0.7294
GKYZ	0.0151	2.4052	1.9114	0.0618	0.1207	0.8174
Panel C: DM test	MSE	p-value	MAE	p-value	MAPE	p-value
PARK	-1.0110	0.3120	-0.0405	0.9677	6.6677	0.0000
GK	-0.0636	0.9493	2.2435	0.0249	7.5998	0.0000
RS	0.0635	0.9494	2.7011	0.0069	2.3909	0.0168
GKYZ	-0.0984	0.9216	1.1504	0.2500	6.0293	0.0000

Panels A and B present the statistics of the out-of-sample forecast comparisons for each E volatility estimator. PARK, GK, RS, and GKYZ denote the measures proposed by Parkinson (1980), Garman and Klass (1980), Roger and Satchell (1991) and Yang and Zhang (2000), respectively. *MSE* is the Mean Squared Error and figures have been multiplied by $10E+7$ to improve readability; *MAE* is the Mean Absolute Error and figures have been multiplied by $10E+5$ to improve readability; *MAPE* is the Mean Absolute Percentage Error; *BIAS*, *VARIANCE* and *COVARIANCE* are expressed in percentage terms and have been calculated from the decomposition of the Mean Squared Forecast Error into Bias, Variance and Covariance proportions. Panel C provides the statistic and p-value of the Diebold and Mariano tests (DM test) to compare forecasts between models AM and AMD using different loss criteria (*MSE*, *MAE* and *MAPE*, respectively). The out-of-sample period goes from January 1, 2016 to December 31, 2017.

and with the majority of the error concentrated in the covariance proportion. In addition, Table 5 provides the Diebold and Mariano (1995) test (DM test) for equal predictive accuracy between the AM and AMD models for each volatility estimator. Given the actual series of volatility and the two competing predictions, DM tests the null hypothesis of equal accuracy, applying different loss criteria. Specifically, the statistic tests that the mean difference between each loss criterion for the two predictions is zero.

Table 5 reports the out-of-sample forecast evaluation results. The MSE coefficient diminishes in the AMD models in all the cases except for the Parkinson model, in which it slightly increases. The values of MAE and MAPE are the same or lower in the AMD model when compared with the AM model. DM tests using MSE as the loss function do not show differences between the competing models. However, using MAE or MAPE criteria, DM tests suggest that the AMD models provide superior forecasts of volatility than the AM models. Furthermore, the decomposition of the Mean Squared Forecast Error in all the AMD models tends to concentrate most of the inequalities between the forecasted and the actual series in the covariance proportions, and, consequently, tends to reduce the bias and the variance proportions, suggesting that AMD models outperform AM models in predicting volatility. Therefore, Table 5 supports the conclusion that the range-based autoregressive volatility models that incorporate the effect of non-trading periods provide more accurate forecasts than the corresponding models that do not incorporate these effects under every evaluation criterion for all the volatility estimators. As a consequence, out-of-sample analysis confirms that the impact of the type of interrupting period is relevant to volatility forecasting.

5. Conclusions

This study investigates the effect of non-trading periods on the measurement of the volatility of the S&P 500 index by using range-based autoregressive volatility models during the period 1993 to 2015. It shows the importance of considering the interrupting periods in several volatility estimators. Volatility significantly diminishes on the first trading day after a holiday or a weekend but, surprisingly, this decrease in volatility is not observed on the first trading day after a long weekend. On these days, volatility rises or remains at the same level. Out-of-sample findings indicate that volatility models that introduce autoregressive coefficients of volatility estimators that interact with dummies in order to capture volatility changes that appear after holidays and weekends perform better than simple autoregressive models in forecasting volatility. Therefore, the shorter the length of the non-trading periods between two non-consecutive trading days, the higher the overestimation of the volatility if this effect is not considered in volatility forecasting. The forecasting improvement detected in this study may be relevant for subsequent research on portfolio allocation analysis or when predicting Value-at-Risk.

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References

- Alizadeh, S., Brandt, M. W., & Diebold, F. X. (2002). Range-based estimation of stochastic volatility models. *The Journal of Finance*, 57(3), 1047–1091.
- Chou, R. (2005). Forecasting financial volatilities with extreme values: The conditional autoregressive range CARR model. *Journal of Money, Credit and Banking*, 37, 561–582.
- Chou, R. Y., Chou, H., & Liu, N. (2015). Range volatility: A review of models and empirical studies. In C. F. Lee, & J. Lee (Eds.), *Handbook of financial econometrics and statistics*. New York, NY: Springer.
- Clements, A., Preve, D. (2019). A Practical Guide to Harnessing the HAR Volatility Model. Available at SSRN 3369484.
- Diebold, F. X., & Mariano, R. S. (1995). Comparing Predictive Accuracy. *Journal of Business and Economic Statistics*, 13, 253–265.
- Fiess, N. M., & MacDonald, R. (2002). Towards the fundamentals of technical analysis: Analysing the information content of High. *Low and Close prices. Economic Modelling*, 19(3), 353–374.
- Garman, M., & Klass, M. (1980). On the estimation of security price volatilities from historical data. *Journal of Business*, 53, 67–78.
- Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48(5), 1779–1801.
- Hansson, B., & Hördahl, P. (2005). Forecasting variance using stochastic volatility and GARCH. *The European Journal of Finance*, 11(1), 33–57.
- Jayawardena, N. I., Todorova, N., Li, B., & Su, J. J. (2016). Forecasting stock volatility using after-hour information: Evidence from the Australian Stock Exchange. *Economic Modelling*, 52, 592–608.
- Kambouroudis, D. S., & McMillan, D. G. (2015). Is there an ideal in-sample length for forecasting volatility? *Journal of International Financial Markets, Institutions and Money*, 37, 114–137.
- Li, H., & Hong, Y. (2011). Financial volatility forecasting with range-based autoregressive volatility model. *Finance Research Letters*, 8(2), 69–76.
- Li, K., & Weinbaum, D. (2001). *The empirical performance of alternative extreme value volatility estimators*. New York University-Salomon Center-Leonard N. Stern School of Business.
- Lyócsa, Š., & Molnár, P. (2017). The effect of non-trading days on volatility forecasts in equity markets. *Finance Research Letters*, 23, 39–49.
- Miralles-Marcelo, J. L., Miralles-Quirós, J. L., & Miralles-Quirós, M. M. (2013). Improving the CARR model using extreme range estimators. *Applied Financial Economics*, 23(21), 1635–1647.
- Miralles-Quirós, J. L., Miralles-Quirós, M. M., & Nogueira, J. M. (2019). Diversification and the benefits of using returns standardized by range-based volatility estimators. *International Journal of Finance and Economics*, 24, 671–684.
- Molnár, P. (2016). High-low range in GARCH models of stock return volatility. *Applied Economics*, 48(51), 4977–4991.
- Pandey, A. (2002). Extreme-value volatility estimators and their empirical performance in Indian capital markets. NSE Research Paper, (14).
- Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *Journal of Business*, 53, 61–65.
- Petneházi, G., Gáll, J. (2018). Exploring the predictability of range-based volatility estimators using RNNs. arXiv preprint arXiv:1803.07152.
- Rogers, L. C. G., & Satchell, S. E. (1991). Estimating variance from high, low and closing prices. *The Annals of Applied Probability*, 504–512.
- Rogers, L. C., Satchell, S. E., & Yoon, Y. (1994). Estimating the volatility of stock prices: A comparison of methods that use high and low prices. *Applied Financial Economics*, 4(3), 241–247.
- Shu, J., & Zhang, J. E. (2006). Testing range estimators of historical volatility. *Journal of Futures Markets*, 26(3), 297–313.
- Todorova, N., & Souček, M. (2014). Overnight information flow and realized volatility forecasting. *Finance Research Letters*, 11(4), 420–428.
- Tsiakas, I. (2010). The economic gains of trading stocks around holidays. *Journal of Financial Research*, 33(1), 1–26.
- Wang, Y. H., & Hsiao, Y. J. (2010). The impact of non-trading periods on the measurement of volatility. *Review of Pacific Basin Financial Markets and Policies*, 13(04), 607–620.
- Wang, J. N., Hsu, Y. T., & Liu, H. C. (2014). How useful are the various volatility estimators for Improving GARCH-based volatility forecasts? Evidence from the Nasdaq-100 Stock Index. *International Journal of Economics and Financial Issues*, 4(3), 651–656.
- Yang, D., & Zhang, Q. (2000). Drift-independent volatility estimation based on high, low, open, and close prices. *The Journal of Business*, 73(3), 477–492.