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ARTICLE

The Dynamics of Perspective in Quantum Physics

An Analysis in the Context of Teacher Education

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Abstract
Debates on the philosophical interpretations of quantum physics have motivated a renewed interest in how secondary and lower undergraduate students interpret quantum phenomena. In an attempt to contribute to this effort, this paper examines the dynamics of perspective in quantum physics in the context of teacher education. The goal of the study is to investigate how students ($N = 36$) from a Master's Degree in Secondary Education Teacher Training in Spain negotiate perspective as they participate in small-group discussions of quantum physics topics. This study focuses on the wave-particle duality, superposition of states, and the calculation of probabilities for two-state systems. The method of research is grounded in sociocultural discourse analysis and focuses on the properties of the utterance as outlined by Bakhtin. Analysis shows that the subjects of the study adopt multiple perspectives when representing the referents of quantum theory. We also find that students' perspective change is usually followed by a change in the referentially semantic content. Finally, it is suggested that some perspectives are more appropriate than others depending on the task at hand and the learning goals previously defined for instruction.

1 Introduction

The teaching of quantum physics has a long tradition as a topic of research in the field of science education. Since the early 1990s, there is an interest in student conceptions and learning processes related to modern physics (Gil & Solbes, 1993; Fischler & Lichtfeldt, 1992). In a recent review of the literature, Krijtenburg-Lewerissa et al. (2017) show that studies on quantum physics teaching are primarily concerned with student difficulties and focus on teaching strategies, multimedia applications, and research tools. In general, these studies show that students often interpret quantum ideas in classical terms (Greca & Freire, 2003; Johnston et al., 1998; Kalkanis et al., 2003; Petri & Niedderer, 1998). According to Johnston et al. (1998), there are at least two difficulties facing educational research in

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quantum physics: (1) the content is structured in a highly mathematical formalism and there is no consensus on how it can be taught less abstractly; (2) questions of how the quantum formalism should be interpreted are still discussed in the scientific literature.

With regard to the former, effective answers to the instruction problem led to conceptual approaches to quantum mechanics that made it possible to teach its basic concepts at earlier stages of physics education (Krijtenburg-Lewerissa et al., 2017). In fact, quantum physics is now part of secondary school curricula in many countries (Stadermann et al., 2019), although traditional teaching methods and textbooks may often introduce quantum ideas in ways that promote misconceptions (Kalkanis et al., 2003). With respect to the interpretation problem, although such an observation was made more than twenty years ago, only recently educational research has addressed issues of interpretation in quantum physics (Baily & Finkelstein, 2010, 2015; Bungum et al., 2018; Cheong & Song, 2014; Garritz, 2013; Henriksen et al., 2018; Lautesse et al., 2015; Mohan, 2020; Niaz & Fernández, 2008). While some studies approach quantum interpretations from a historical perspective (e.g., Garritz, 2013), others emphasize the lack of consensus that still exists in current debates (Baily & Finkelstein, 2009; Cheong & Song, 2014; Henriksen et al., 2018; Lautesse et al., 2015; Mohan, 2020).

Discussions around interpretations of quantum physics have motivated a renewed interest in how upper secondary and lower undergraduate students represent quantum phenomena (Baily & Finkelstein, 2015; Henriksen et al., 2018). Previous studies have shown that instructor's philosophical stance during instruction impacts on student perspectives and that students tend to prefer a realist local perspective when instructors are not explicit in addressing questions of quantum ontology (Baily & Finkelstein, 2009, 2010). These findings are directly related to the problem of reference in quantum physics (Lautesse et al., 2015). Another result from the study of Baily and Finkelstein (2009) is the lack of consistency of student perspectives across different contexts. According to their study, a significant number of students who prefer a quantum perspective in the double-slit experiment, for example, would still choose an intuitive classical perspective when representing the electron inside the atom.

In an attempt to contribute to these efforts, the present study investigates the dynamics of perspectives in quantum physics in the context of science teacher education. The goal of the study is to examine how students from a Master's Degree in Secondary Education Teacher Training, in Spain, negotiate perspective as they participate in small-group discussions of quantum phenomena. Previous studies have investigated small-group discussions of quantum topics in the context of Norwegian upper secondary physics education (Bungum et al., 2018; Henriksen et al., 2018). However, only a few studies investigate personal perspectives in the context of pre-/in-service teacher education (Asikainen & Hirvonen, 2014; Kalkanis et al., 2003). This might be particularly relevant in settings where quantum physics is part of the upper secondary school curriculum, and the majority of science teachers do not have an adequate training in quantum mechanics (Lautesse et al., 2015).

The study focuses on a conceptual approach to quantum physics that includes discussions on the wave-particle duality, superposition of states, and calculations of probabilities for two-state systems. In particular, we address the components of spin to discuss the collapse of the wave function in the context of the Stern-Gerlach experiment. Previous researches have investigated student conceptions on wave-particle duality (Asikainen & Hirvonen, 2014; Henriksen et al., 2018; Mannila et al., 2002; Olsen, 2002). According to the review of the literature provided by Krijtenburg-Lewerissa et al. (2017), the teaching of complex quantum behavior such as quantum states, superposition, and measurement has barely been investigated for upper secondary and lower undergraduate level (e.g., Bungum et al., 2018;

Greca & Freire, 2003). In addition, the concept of spin is usually taught in the context of atomic physics, including atomic spectra, quantization of energy levels, atomic models, and Pauli principle (Krijtenburg-Lewerissa et al., 2017). The present study is guided by the following research questions:

1. How do students represent the referents of quantum physics and how do their perspectives vary during small-group discussions?
2. How does an emphasis on two-state systems impact on student perspectives?

2 Quantum Debates and Student Perspectives in Quantum Physics

In a historical account of the controversy about the interpretation of quantum mechanics, Freire (2003) identified three different periods. The first one that goes until the 1940s is described as the epoch of almost unchallenged monocracy of the Copenhagen School (Jammer, 1974). Despite the first criticisms by Einstein and Schrödinger, the physicists' community adhered to Bohr's complementarity, regarding those criticisms merely as philosophical quarrels. The second period, which is a transition one, is marked by the appearance of hidden variables and relative state formulation. These challenges to the orthodox view gradually led to a change of attitude among physicists toward the problem of interpretation in quantum physics. The third period, which includes the present time, is characterized by the physicists' community recognizing and legitimating the existence of a scientific controversy about the interpretation of quantum mechanics. The institutionalization of the controversy is manifested in the creation of new journals and conferences aimed at promoting the debates (Freire, 2003).

Obviously, not all participants of this debate admit the existence of a controversy. van Kampen (2008), for example, sees the dissemination of various interpretations of quantum physics as a scandal. He argued that discussions around the interpretation of quantum physics resemble the attitude of seventeenth century Cartesians, who rejected Newton's attraction because they could not accept a force that was not transmitted by a medium. This instrumentalist stance on the philosophy of quantum physics echoes other voices such as those raised by Fuchs and Peres (2000), who claimed that the quantum theory needs no interpretation. Interestingly enough, the authors themselves admit that the attitude that precludes interpretations of the quantum formalism is also an interpretation: "Our purpose here is to explain the internal consistency of an 'interpretation without interpretation'" (Fuchs & Peres, 2000, p. 70).

The institutionalization of the controversy is illustrated by the 2003 special issue of Science & Education (Vol. 12, issue 5–6). In this issue, Bunge (2003) offered a position paper in which he proposes, among other things, the thesis that the Copenhagen interpretation is false and should be replaced with a realist (but not classicist) interpretation. This position contrasts with the attitude of physicists such as Laloë (2001) who argued that recognizing a variety of interpretations should not be a source of misunderstandings. In his words: "It should also be emphasized very clearly that, until now, no new fact whatsoever (or no new reasoning) has appeared that has made the Copenhagen interpretation obsolete in any sense" (Laloë, 2001, p. 66). In Bunge's realist interpretation, the referents of the quantum theory are neither particles nor waves, but objects of a new kind — something he called *quantons* (Bunge, 1968). In addition, it is assumed that their basic laws are probabilistic and some of their properties are blunt rather than sharp.

In this same 2003 special issue of Science & Education, Lévy-Leblond (2003) emphasized the novelty of quantons and suggested the use of the discrete/continuous dichotomy in order to understand their nature. Quantons are discrete in the sense that they come in units and can be counted. On the other hand, they are continuous in the sense that they can be subjected to interference and superposition. Thus, Lévy-Leblond proposed two analytical dimensions: number and extension. Classical particles are discrete in both aspects; they can be counted and are localized at points in space. Classical fields are continuous in both aspects; they cannot be counted and are spread out in space. Quantons differ from classical entities in that they are discrete in number and continuous in extension. This form of description avoids any contradiction because discreteness and continuity does not refer to the same dimension.

The problem of interpretation in quantum mechanics has motivated new studies on student perspectives. Studies have shown that some of the student perspectives are consistent with philosophical interpretations of the quantum theory (Baily & Finkelstein, 2015; Henriksen et al., 2018). This suggests a more nuanced account of student perspectives since it goes beyond the general opposition between classical physics and quantum mechanics (e.g., Kalkanis et al., 2003). In the case of our study, we do not think that students hold a very deep commitment to a particular interpretation of quantum mechanics. And the fact that student perspectives vary within and across domains (Baily & Finkelstein, 2010) suggests that students do not hold internally consistent conceptions as well (those that are deeply rooted and difficult to change). Thus, instead of referring to conception or interpretation, we use in the present study the more general notion of perspective, or point of view, which is expected to differ from one context to another. In particular, we use the notion of "referential perspective," which is defined as a particular way of identifying the referent (Wertsch, 1991). This is consistent with the meaning of reference as used in the philosophical literature, which regards the relationship between expressions and the objects they refer to (Lautesse et al., 2015).

3 Recent Studies on Student Perspectives: a Brief Overview

In the field of science education research, the teaching of wave-particle duality is still under discussion (Krijtenburg-Lewerissa et al., 2017). While some studies use it as a starting point to introduce quantum mechanics (Scarani & Suarez, 1998), others argue for the need to abandon the wave-particle terminology altogether (Fischler & Lichtfeldt, 1992). We believe that studies on the teaching of wave-particle duality are particularly relevant because this topic is usually taught at early stages of quantum physics instruction (Cheong & Song, 2014). On the other hand, the teaching of more advanced topics such as superposition has received much less attention in educational research (Krijtenburg-Lewerissa et al., 2017). In what follows, we present a brief overview of recent studies addressing the teaching of wave-particle duality and superposition of states. We are particularly interested in studies motivated by the problem of interpretation in quantum physics.

3.1 Studies on Wave-Particle Duality

Cheong and Song (2014) provided three levels of the meaning of duality based on a distinction between *prediction rules* and *reality-related interpretation*. The first level consists of interpretations of particular experiments such as the photoelectric effect or the double-slit experiment without relating duality to other concepts of the quantum formalism. It is usually formulated in terms of quantum objects having both wave and particle properties. The second level relates the wave-particle duality to other concepts of the quantum formalism such as the wave function and its probabilistic interpretation. These concepts, however, are used merely as calculation tools without making any claim of reality. Reality claims or normative claims about the role of quantum theory constitute the third level of meaning of duality. This level involves diverse and mutually conflicting interpretations about the real behavior of quantum objects. Based on these three levels of duality, Cheong and Song suggest a *suspensive perspective* that takes a neutral stance without privileging a particular interpretation.

Lautesse et al. (2015) analyzed the epistemological position of secondary physics textbooks, in France, toward the referents of the quantum theory. Focusing on wave-particle duality, they classified the books into two analytical categories: the conservative and innovative positions. The conservative position refers to objects of quantum physics using familiar classical terms, in line with the Copenhagen Interpretation. The innovative position, which is associated with the works of Bunge and Lévy-Leblond, characterizes the quantum objects in their own terms, emphasizing that they are neither particles nor waves, but rather new physical entities — namely, quantons. The study showed a dominance of the conservative position, in which the words *particle* and *wave-particle duality* are present in all the textbooks, even in those that adopt an innovative conception. The result shows that the secondary physics textbooks in France are still reluctant to abandon classical terms.

In their analysis of how instructors' interpretations of quantum physics impact on student perspectives, Baily and Finkelstein (2015) identified three different instructional approaches: Realist/Statistical, Matter-Wave, and Copenhagen/Agnostic. These categories were based on classroom observations and interviews and were illustrated by examining how instructors discuss the double-slit experiment with single electrons. The instructor who adopted a Realist/Statistical approach taught students that each electron goes through one slit or the other, but it is impossible to determine it without destroying the interference pattern. The instructor who adopted a Matter-Wave approach told students that each electron is a delocalized wave packet that propagates through both slits and interferes with itself. The instructor who adhered to a Copenhagen/Agnostic view stated that the wave of probability passed through both slits, but asking which path an individual electron took without placing a detector at one of the slits is an ill-posed question. Pre- and post-tests suggest that different instructional approaches have distinct impacts on student perspectives.

Henriksen et al. (2018) investigated how Norwegian upper secondary students interpret the wave-particle duality of light before and after a teaching sequence. The analysis shows that most students adopted a dual perspective, which includes formulations such as light *is* both particle and/or wave, light *behaves* like a particle and/or a wave, and light *can be described* as particle and/or wave. While the first formulation expresses a realist view, the second and the third formulations suggest an instrumentalist approach as they stress the role of scientific models. In small-group discussions after the teaching sequence, most students showed a view aligned with the Copenhagen interpretation: particle o*r* wave depending on the experiment. They had the opportunity to reflect on the idea that light is both localized and extended or that a wavelength is being ascribed to a particle. Attempts to overcome these paradoxes led some students to express the misconception that light is made of particles that move in wave-shaped trajectories (see Olsen, 2002).

3.2 Studies on Superposition of States

Greca and Freire (2003) investigated how a didactic strategy based on a realist, but orthodox interpretation of quantum physics impacts on students' mental models. The didactic strategy emphasizes the concept of quantum state as representing the reality of a physical system independent of measurement processes. In a qualitative study involving three groups of engineering students in Brazil, they identified four categories: *quantum object nucleus* (students explain quantum phenomena from general principles), *incipient quantum object nucleus* (students understand uncertainty principle and probabilities, but show difficulties with linear superposition of states), *classical nucleus with some quantum ingredients* (students use classical nuclei to visualize quantum phenomena), and *undetermined* (impossible to find any pattern in students' responses). The authors concluded that the didactic consequences of different interpretations of quantum physics, and the conceptions students acquire from them, could be an interesting topic to be considered in debates about the problem of interpretation in quantum mechanics.

In a study involving upper secondary students in Norway, Myhrehagen and Bungum (2016) investigated how students interpret the Schrödinger's cat thought experiment. After reading a short description of the Schrödinger's cat and having a brief introduction to the concept of superposition, students were asked to formulate their reflections on the thought experiment. Analysis showed that some of the responses can be related to philosophical interpretations of quantum physics, including the Copenhagen interpretation (you cannot know whether the cat is dead or alive before you open the box), hidden variables interpretation (the cat is in "reality" either dead or alive, not both dead and alive), and even more literal interpretation (the cat can be dead and alive at the same time). The authors concluded that the lack of knowledge about the purpose and the historical context of the thought experiment limits students understanding.

In another, more recent study, Bungum et al. (2018) investigated how small-group discussions improve students' understanding of quantum physics dilemmas such as the wave-particle duality of light and the Schrödinger's cat thought experiment. The analysis shows that a majority of group discussions are productive in the sense that students build on each other's utterances and challenge what was said. In terms of the potentials for learning, the authors identified three broad functions of the group discussions: articulating conceptual difficulties, deepening understanding by exchange of views, and developing new questions. With respect to discussions about Schrödinger's cat thought experiment, some of the students' reflections can be related to interpretations of quantum theory, including the difference between hidden variables and the Copenhagen interpretation. The authors concluded that teachers can support students' understanding by pointing to how their reflections reflect various views that professional physicists hold about interpretations of quantum physics.

4 Method

4.1 Educational Context, Sample, and Data Collection

This study was carried out with a group of students from the Master's Degree in Secondary Education Teacher Training at the University of Valencia, Spain. It provides the pedagogical training required for the regulated profession of teacher in secondary education all over the country. The Master's in Secondary Education Teacher Training offers several specializations, including "Physics and Chemistry." The present study focuses on the Learning and Teaching Physics and Chemistry classes, which address issues related to teacher training and science education in general (e.g., the learning of scientific concepts, procedures, socio-scientific issues, and education for sustainability). This component also includes a proposal of didactic unit — a teaching sequence that may include, for example, a proposal of how to teach quantum physics in the final year of upper secondary education, in the subject Physics (optative).

The participants consisted of 36 white, mostly middle-class students (17 men and 19 women) with age ranging from 22 to 46 years old. Their academic background varies greatly, including people with Ph.D. in physics ($N=3$), Ph.D. in chemistry ($N=3$), undergraduate degree in physics ($N=3$), undergraduate degree in chemistry ($N=18$), undergraduate degree in biotechnology ($N=5$), undergraduate degree in chemical engineering ($N=3$), and undergraduate degree in geological engineering ($N=1$). Only students with Ph.D. or undergraduate degree in physics have an adequate training in quantum mechanics. Students with Ph.D. or undergraduate degree in chemistry have an adequate training in physical chemistry, which includes the study of quantum physics applied to atoms and molecules. The other students have no training in quantum physics at all.

The Learning and Teaching Physics and Chemistry classes usually take place in the old building of the Faculty of Teacher Training at the University of Valencia. The classroom is equipped with 1 regular desk, 1 desktop computer, 1 data show projector, and 8 hexagon activity tables around which the students are arranged in small groups. In this study, students organized themselves in six groups of five members and one group of six members. The dynamics of the classes consist of an alternation between oral expositions (involving PowerPoint presentation, virtual simulations, and experimental demonstrations) and small-group discussions. The quantum physics unit planned for this group of students consisted of three lessons of 2.5 h each.

In order to elicit students' participation in small-group discussions, the instructor posed some conceptual questions related to the topics of the lessons. These questions focused on wave-particle duality, the wave function, superposition of states for two-state systems, the collapse of the wave function, and calculations of probabilities in the Stern-Gerlach experiment. The small-group discussions were recorded by students themselves using their smartphones. The audio files were later sent to their instructor via e-mail. All discussions among students were carefully listened and transcribed for analysis. All files were transcribed in their original language (Spanish and/or Catalan), and only later they were translated to English. Students gave their consent for the audio files to be used in research.

4.2 Quantum Physics Unit for Teacher Education: a Proposal for Secondary School

The teaching sequence planned for this study was based on previous research (Sinarcas & Solbes, 2013) and aims at overcoming some learning difficulties reported in previous studies (Gil & Solbes, 1993; Kalkanis et al., 2003). It included the following topics: atomic spectra, the photoelectric effect, wave-particle duality, indeterminacy relations, the wave function and its probabilistic interpretation, and atomic orbitals, which are what the Spanish curriculum establishes for the teaching of quantum physics in high school. Atomic spectra and the photoelectric effect are presented as two of the main problems that originated the *crisis* in classical physics (Kuhn, 1962). Both episodes provide the context to discuss issues related to Nature of Science as they show how classical physics was unable to explain these phenomena as they required new hypotheses about light and matter that break with classical theories. The basic assumption is that modern physics can be introduced in ways that promote an adequate image of the nature of science (Gil & Solbes, 1993).

Discussions on the wave-particle duality emphasize how photons and electrons are neither particles nor waves, but objects of a new kind that exhibit a unique quantum behavior. Here, we overtly adopt a realist (but not classicist) interpretation to quantum theory based on the view put forward by Bunge and Lévy-Leblond (Bunge, 1965; Lévy-Leblond & Balibar, 1990). The emphasis on the concept of quantons (Bunge, 1968) stresses the need of a new formulation (Schrödinger's equation) to describe the state and evolution of physical systems, one that is inherently distinct from those used to describe classical entities like particles (Newton's second law) and waves (d'Alembert equation). At this point, he focused on the wave aspects of the wave function to introduce the indeterminacy relations.

In this particular study, we used the components of spin as a case study to discuss the wave function and its probabilistic interpretation for two-state systems (Pessoa, 2003; Sakurai, 1994). This reformulation of the original proposal (Solbes & Sinarcas, 2010) facilitates the presentation of the wave function collapse, the calculation of probabilities, and the indeterminacy relations for conjugated variables. The teaching sequence also emphasizes misconceptions associated with atomic orbitals, especially those in which the orbital is depicted as a region of space that the electrons may or may not occupy (Niaz & Fernández, 2008; Petri & Niedderer, 1998; Taber, 2005). Finally, we summarize the differences between classical physics and quantum mechanics, emphasizing the limits of validity of classical physics.

In terms of procedure, the quantum physics unit also included demonstrations of emission spectra using a pocket spectroscope, virtual simulations of the photoelectric effect and the Stern-Gerlach experiment (https://phet.colorado.edu/), and a short animation about electron interference in the double-slit experiment featured by Dr. Quantum (https://www.youtube.com/watch?v=htSIjIyF9bU). In particular, the end of this animation is used to warn students about the pseudoscientific use of quantum theory. The unit also includes the use of quantum physics concepts to discuss issues of social and technological relevance (Science-Technology-Society relations). In particular, it was highlighted the great scientific and technological developments made possible by quantum physics (e.g., electron and tunneling microscopes, photocells and solar cells, lasers), and implications of microelectronics to modern society. We also discussed the social and political context in which the theory of quantum mechanics was born.

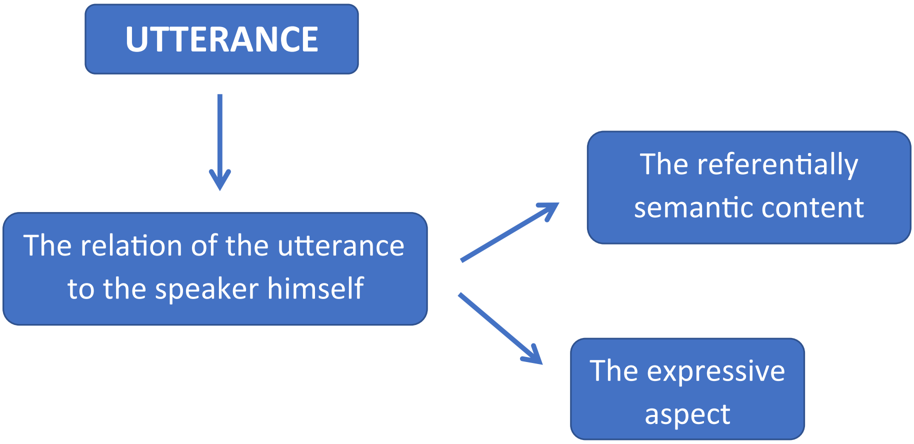

Fig. 1 Method of discourse analysis based on Bakhtin's account of the utterance

4.3 Sociocultural Discourse Analysis

The analytical framework that guides our study is based on sociocultural discourse analysis (Bakhtin, 1986; Wertsch, 1991). Previous studies have drawn on a sociocultural view of learning to examine small-group discussions and their potential for improving students' understanding of quantum concepts (Bungum et al., 2018; Henriksen et al., 2018). The method of research used here is grounded in the properties of the utterance as outlined by Bakhtin (1986). According to this author, the utterance is characterized by three properties: boundaries, finalization, and the generic form. The boundaries specify the limits of the utterance, which is marked by a change of speaking subject. The finalization, in turn, is an indicative of the conclusion of the utterance, which opens the possibility of responding to it. Finally, the generic form is determined by the relation of the utterance to the speaker himself and to other participants in the speech communication.

With respect to the relation of the utterance to the speaker himself (the focus of what follows), it is characterized by two subsidiary issues: the referentially semantic content of the utterance and its expressive aspect (Bakhtin, 1986). The referentially semantic content is the topic of the utterance. It may range from a very complex scientific theory such as quantum mechanics to a particular object visually present in the science classroom. The expressive aspect, on the other hand, is defined in terms of the speaker's emotional evaluation of the utterance, which belongs to the domain of stylistic. According to Wertsch (1991), the more general notion of perspective, or viewpoint, is more adequate for a sociocultural analysis. In what follows, we focus on the negotiation of perspective that occurs when groups of individuals work together to represent physical reality. An operationalization of the method of discourse analysis is summarized in the diagram below (Fig. 1).

There are several ways in which the referentially semantic content of the utterance can be classified. One possible distinction is that between non-linguistic and linguistic objects. While the former refers to objects existing in the extra-linguistic context, the latter includes cases in which "language is used to speak *about* language" (Wertsch, 1991, p. 109). Given the context of our study, which focuses on the use of scientific concepts, the categories of non-linguistic and linguistic objects are harnessed to highlight another important distinction — that between entities that are assumed to exist independent of language use (e.g., electrons), and the symbolic representations of such entities (e.g., mathematical objects).

With respect to the expressive aspect of the utterance, our study focuses on the notion of *referential perspective*, which is defined in terms of the "viewpoint utilized by the speaker in order to identify a referent" (Wertsch, 1985, p. 168). From this standpoint, the speaker always introduces a particular position when referring to an object, which is marked by the use of referring expressions. For example, in the present study, quantum objects such as photons and electrons can be referred to in a variety of ways: in terms of a particle category, a wave category, from a perspective that combines the two categories,[1] or even from a perspective that admits a third category: a view we shall call quantum perspective. These notions are used in this study to identify *modes of discourse* (Wertsch, 1987), which are generalized forms of discursive organization that frame thinking and speaking.

5 Results

In order to examine how this group of master students negotiate perspective as they progress during the learning activity, we focused on two specific moments of the didactic unit, which includes the following topics: (1) the wave-particle duality; (2) the wave function and its probabilistic interpretation applied to two-state systems. For each excerpt of discourse transcribed bellow, the students' utterances were analyzed in terms of the referentially semantic content and the expressive aspects (referential perspective). The goal of this study is twofold: (a) to investigate student perspectives and their dynamics during group discussions; (b) to evaluate how an emphasis on two-state systems impacts on student perspectives and whether they are consistent with our learning goals. In what follows, the names of students were replaced with pseudonyms in order to unsure their anonymity.

5.1 Wave-Particle Duality

In this part of the teaching sequence, students were taught about the de Broglie's postulate, the connections between the energy levels in Bohr's atomic model and standing waves in classical physics, calculations of de Broglie wavelength for subatomic and macroscopic objects, the diffraction of electrons, and applications of the electron's wave behavior. The students also watched the animation about the interference of electrons featured by Dr. Quantum. In particular, the instructor took the animation as an opportunity to discuss the misconception related to the role of the observer in measurement processes that implies that electrons have agency. In order to promote small-group discussions, the instructor posed the following questions to the whole class:

1) How can we interpret the wave behavior of objects like electrons?
2) What do these waves consist of?

[1] Particle *or* wave, according to the experiment (Copenhagen interpretation), or particle *and* wave, that is, a particle guided by a pilot-wave (de Broglie's interpretation).

These questions were motivated by two learning goals: (i) to argue that the referents of quantum theory are neither particles nor waves, but something else (quantons in Bunge's terminology) and (ii) to argue that the wave function is not an actual wave in the real world, but rather a state function, that is, a mathematical entity that represents the state of a physical system. When looking at the transcriptions, we identified three different ways of speaking of the wave-particle duality. These three categories were created after the preliminary exploratory analysis of the data material (Creswell, 2012).

The first way involves a *quasi-history mode of discourse*, which focuses on the transition from classical physics to quantum mechanics. The notion of quasi-history, as used here, refers to mythical accounts of scientific discoveries that are presented in science textbooks with the purpose of propagating certain methodological and didactical viewpoints (Kragh, 1992; Whitaker, 1979). In this study, this discourse mode sometimes takes the form of an oversimplified narrative that includes some of the semi-classical models that marked the beginning of modern physics in the twentieth century. As Greca and Freire (2014) argued, naïve pedagogical choices regarding quantum physics based on the chronological sequence of its production may be misleading as it may reinforce undesirable bridges between classical and quantum concepts. The second way of speaking of wave-particle duality involves a standpoint that contrasts the features of quantum physics with those of classical physics — a view we shall call *oppositional mode of discourse*. In both discourse modes, students' utterances are populated by multiple voices, reflecting a diversity of perspectives. Finally, the third way of referring to wave-particle duality involves what we term *descriptive mode of discourse*. This mode tends to account for quantum physics objects on their own terms, without making any reference to other perspectives and interpretations.

5.1.1 Quasi-History Mode of Discourse

An example of quasi-history mode of discourse can be found in the discussion provided by group 2. This group consists of five young women, all with undergraduate degree in chemistry. The following is our translation.

Excerpt 1 (Group 2)

(1) Michelle: [reading aloud] How can we interpret the wave behavior of objects like electrons and what do these waves consist of? First, the electron was considered as a point-like mass that has a trajectory. But later de Broglie would perform a series of experiments and would see that they produce diffraction, so that we can say then that the electron behaves like a wave.

(2) Pauline: But then, where did it come from? I mean, they also figured out that this wave… where did it come from?

(3) Michelle: Sure.

(4) Pauline: So, how do we explain this kind of movement?

(5) Hannah: Ok, let's see. On the other hand, another hypothesis emerges that, let's say, replaced de Broglie's, but this will be considered wrong too. This hypothesis consists of, let's say, changing from a particle aspect, like you just said, to another, wave one. But, of course, this is not true either. A wave packet is not compact since it spreads rapidly through space, disappearing any particle movement. I mean, it's a little bit… Chaos!

(6) Emily: Of course, because what they didn't explain is that if this movement arises for a large number of electrons, when making the electrons go through… Or, following the diffraction with a few electrons, it also shows a wave movement. Therefore, the effect was the same with a few electrons as with many electrons.
(7) Pauline: And this is the wave movement that no one could explain. In conclusion…
(8) Michelle: Well.
(9) Hannah: The electron is not… In short, the electrons, or say, photons…
(10) Instructor: [addressing to the whole class] Let's suppose that…

In this excerpt, we identified a variety of referential perspectives being utilized by students. They started by referring to the electron from a classical point of view. In fact, we can hear the voice of classical physics in expressions such as "a point-like mass that has a trajectory" (utterance 1). Then, this view is replaced with another perspective, one that reconciles particle properties with wave aspects (e.g., diffraction). This view is reflected in Michelle's assertion that "we can say then that the electron behaves like a wave". On the other hand, Hannah introduces a third referential perspective, one that reduces all the particle aspects to wave properties. Nevertheless, she dismisses this view herself when she claims that a "wave packet is not compact since it spreads rapidly through space, disappearing any particle movement" (utterance 5). Finally, a particle perspective is discarded by Emily when she argues that in the diffraction experiment, "the effect was the same with a few electrons as with many electrons" (utterance 6).

In order to better understand the dynamics of perspective involved in this episode, we shall examine not only the expressive aspects of the utterances, but also the subtle changes in the referentially semantic content. For example, when Michelle switched from a classical standpoint to a referential perspective that reconciles particle and wave properties (a view we shall call *dual perspective*), the referent of her discourse changed as well. Instead of referring to the electron itself, she focused on the electron's *behavior*. In this case, the dynamics of perspective involve two types of discourse transition: one from a particle to a dual perspective, and the other from an ontological to a phenomenological level of description. The dynamics of the whole excerpt are summarized as it follows:

Phase 1 (utterance 1): Particle Perspective. Electron as referentially semantic content.
Phase 2 (utterances 1–4): Dual Perspective. Electron's behavior as referentially semantic content.
Phase 3 (utterance 5): Wave perspective. Electron as referentially semantic content.
Phase 4 (utterances 6–7): Dual Perspective. Electron's behavior as referentially semantic content.

It is worth noting that Hannah's final statement (utterance 9) suggests she was trying to overcome, or at last avoid, the categories of particle and wave. Nevertheless, it is fair to say that this group did not succeed in constructing a proper representation of quantum objects, at least according to our learning goals.

5.1.2 Oppositional Mode of Discourse

With respect to the oppositional mode of discourse, an illustrative case can be found in the discussion provided by group 1. This group consists of one physicist, one Ph.D. in physics, two chemists, and one biotechnologist. The presence of two experts in physics suggests that, in theory, their discussion can provide some insights into what kind of perspective change is needed in order to build an adequate representation of quantum objects.

Excerpt 2 (Group 1)

(5) John: I'll start. The electrons… First, the interpretation as waves. First of all, they are not classical particles, but quantum particles.
(6) Paul: Uh hum.
(7) John: So, even if we are used to deal with mechanical waves, I mean…
(8) Paul: Yes.
(9) John: … those that need a medium to propagate…
(10) Paul: Yes.
(11) John: … the energy they transport. The quantum particles, what they transport are probability waves. I mean, they don't need any medium to propagate and we must emphasize that quantum theory is not, unlike many classical theories, deterministic. That is, if you have the initial conditions, you can't know…
(12) Mathews: The final conditions.
(13) John: … the final evolution of the system.
(14) David: Ok.
(15) John: That is why it is important to remind the probabilistic interpretation of quantum physics which, although it is not the only one that is currently in the sea of science, is the most standard or the most respected one.
(16) Paul: Very well. Someone wants to add something?

In this excerpt, John begins by making a clear-cut distinction between classical and quantum particles. Later, he outlined an opposition between mechanical waves, on the one hand, and quantum particles, on the other. In his account, the mechanical waves transport energy and require some "medium to propagate" (utterance 9). Quantum particles, in turn, require no medium because they transport "probability waves" (utterance 11). Finally, he adopts a probabilistic perspective when he asserts that "quantum theory is not, unlike many classical theories, deterministic" (utterance 11).

Once again, it is possible to hear more than one referential perspective being utilized by the students. But unlike the quasi-history mode of discourse, which adopts different perspectives when referring to one and the same entity, the oppositional mode of discourse assumes different perspectives for distinct objects. Thus, the wave perspective is used here to refer to mechanical waves whereas the referents of the quantum theory — quantum particles in John's terminology — are referred to from a quantum point of view. Finally, the introduction of a probabilistic perspective is followed by a change in the referentially semantic content, switching from non-linguistic to linguistic objects, that is, from *electrons* to *quantum theory*. Therefore, the dynamics of perspective in this excerpt are summarized as it follows:

Phase 1 (utterances 7–11): Wave perspective. Mechanical waves as referentially semantic content.
Phase 2 (utterance 11): Quantum perspective. Quantum particles as referentially semantic content.
Phase 3 (utterances 11–15): Probabilistic perspective. Quantum theory as referentially semantic content.

It is worth noting that, right from the beginning, this group went beyond the categories of particles and wave as they used the notion of quantum particles to refer to entities of quantum theory. This is consistent with our first learning goal, which asserts that the referents of the quantum theory are neither particles nor waves, but objects of a new kind. With respect to the second learning goal, students did not mention the concept of wave function in this particular excerpt. Instead, they focused on the probabilistic features of the quantum theory.

5.1.3 Descriptive Mode of Discourse

An example of descriptive mode of discourse can be found in the discussion provided by group 7. This group is composed by four chemists and one Ph.D. in chemistry. The following is our translation.

Excerpt 3 (Group 7)

(1) Peter: [reading aloud] How can we interpret the wave behavior of objects like electrons and what do these waves consist of?
(2) Amber: We are talking about a subject that is still a general controversy because the wave-particle duality still surrounds us and is difficult to interpret. On the one hand, the electron has mass and therefore it is a particle, but this mass has a wavelength associated… Right?
(A few exchanges later…)
(3) Peter: [returning to question 2] What do these waves consist of?
(4) Amber: The waves always have associated a wavelength that characterizes the wave and a wave function associated…
(5) Peter: Yes.
(6) Amber: … that… In the end, it… it wasn't like this.
(7) Peter: Ok, but…
(8) Amber: This wave function can be interpreted as the probability of finding the electron or whatever in a place.
(9) Peter: But we are talking about any sub-particle, right?
(10) Amber: Yeah, of any sub-particle. And if you have its wave function, it gives you the probability of finding it in a certain place.

As we mentioned above, the descriptive mode of discourse tends to account for quantum objects on their own terms, without making any reference to other perspectives or interpretations. In this excerpt, Amber describes the electron in terms of its properties: "the electron has mass and therefore it is a particle, but this mass has a wave length associated" (utterance 2). This is a clear case of perspective that reconciles wave and particle properties, but this time without changing the referentially semantic content. As a result, the conflict between the concepts of wave and particle arises and that is why the wave-particle duality seems to Amber (as well as to many other students) a subject that "is difficult to interpret" (utterance 2).

In the second part of this excerpt, however, Amber's discourse is marked by a change in the referentially semantic content, switching from non-linguistic to linguistic objects, that is, from *waves* to *wave function*. This allowed Amber to adopt a probabilistic perspective, which is consistent with the quantum point of view. The use of the term *sub-particle* (utterances 21–22) suggests a particle perspective as well as another subtle change in the referentially semantic content. The dynamics of perspective in this excerpt are summarized as it follows:

Phase 1 (utterance 2): Dual Perspective. Electron as referentially semantic content.
Phase 2 (utterances 16–20): Probabilistic perspective. Wave function as referentially semantic content.
Phase 3 (utterances 21–22): Particle perspective. Sub-particles as referentially semantic content.
Phase 4 (utterance 22): Probabilistic perspective. Wave function as referentially semantic content.

It is important to note that, although this group was able to achieve the second learning goal defined for this part of the teaching sequence (namely, to understand that the wave function is not an wave in the real world, but a mathematical entity that describes the state of a physical system), it is fair to say that they did not succeed in overcoming the categories of particle and wave in favor of a new ontology. The use of the terms *wave* and *sub-particle* is noteworthy in this respect. In the next section, we shall examine students' discussion about superposition of states and calculations of probabilities for two-state systems. This part of the teaching sequence was supported by the use of the PhET simulation of the Stern-Gerlach experiment.

5.2 Superposition of States and Calculations of Probabilities

After small-group discussions about wave-particle duality, the instructor moved to the next topic: *the wave function and its probabilistic interpretation*. He compared the time evolution of particles (governed by Newton's second law) and waves (governed by d'Alembert's equation) with that of quantum systems (governed by Schrödinger's equation). Finally, he focused on two-state systems using the components of spin in the z-direction as a case study. At this point, he introduced the PhET simulation of the Stern-Gerlach experiment. In order to elicit students' participation in the small-group discussions, the instructor posed the following questions:

3) What form does the wave function take for two-state systems such as the component of spin in the z-direction?
4) If we add a new Stern-Gerlach apparatus oriented in the x-axis, what is the probability of measuring, in this second apparatus, spin in the + x direction?
5) If we add a third Stern-Gerlach apparatus oriented in the z-axis, what is the probability of measuring, in this third apparatus, spin in the –z direction?

Question 3 was motivated by a particular learning goal: (iii) to emphasize that the wave function does not always look like a wave; in some cases (such as the components of spin), it may take the simple form of a linear combination of two eigenstates. Questions 4 and 5 were motivated by another learning goal: (iv) to emphasize the probabilistic feature of the quantum theory. While question 3 deals with the notion of superposition of states, questions 4 and 5 address the collapse of the wave function and the probabilities associated with each eigenstate.

When looking at the transcriptions, we note that this part of the teaching sequence involves a much less diverse set of perspectives. Throughout the activity, students basically used a particular way of speaking of the quantum state based on the principle of superposition of states; we shall call this view *superposition mode of discourse*. In some particular cases, however, the quantum state was referred to in terms of the projection postulate and, in other cases, in terms of the indeterminacy relations. In such particular cases, we characterize students' speech using the labels *reduction modes of discourse* and *indeterminacy mode of discourse*, respectively. These categories were also created after the preliminary exploratory analysis of the data material.

5.2.1 Superposition Mode of Discourse

An example of the superposition mode of discourse can be found in the discussion provided by group 5. This group is composed by two chemists, one geological engineer, one physicist, and one Ph.D. in physics. In the discussion below, the students were addressing question 3. The following is our translation.

Excerpt 4 (Group 5)

(65) Bill: What form does the wave function take? What form does a degree 2 polynomial take?

(66) Joseph: Let's see. I mean, for two states what happens is… This is like quantum computers.

(67) Bill: Come on, man! I don't know anything about this. Do you really think I know about computer? [laugh]

(68) Joseph: Well, I'm not sure about what is meant by the form it has to take. This I don't know, but what happens with a two-state system is that you have a wave function that is the sum of 1 and 2.

(69) Bill: Uh hum.

(70) Joseph: Ok. Then, when you measure, you apply an operator. This means that when you measure, it collapses to one of the two. It collapses to one or to the other. There is a probability of this getting here. Now, what form does it take? Exponential, I suppose. But I don't know if it has to do with that or…

(71) Richard: But maybe it can be that. One operator makes it to take a positive form and another operator makes it to be negative.

In this excerpt, it is quite clear the students were confused about what their instructor meant by the form the wave function may take. However, they were able to provide an adequate representation of the wave function as they adopted a *linear combination perspective*. This view is grounded in the superposition principle, which states that before a measurement, a system is assumed to be in a linear combination of its eigenstates. The linear combination perspective opposes to a *wavy* or *sinusoid* perspective that are often used to represent waves in the physical world. This view is apparent in Joseph's claim that "what happens with a two-state system is that you have a wave function that is the sum of 1 and 2" (utterance 68). In terms of the topic of discussion, it is clear that the referentially semantic content involved here (the wave function) is a linguistic rather than a non-linguistic object, which is consistent with our leaning goals.

In the second part of the excerpt, however, the students used a probabilistic perspective, which is reflected in the assertion that "when you measure, it collapses to one of the two. It collapses to one or to the other. There is a probability of this getting here" (utterance 70). This change of perspective was followed by a subtle change in the referentially semantic content, switching from the wave function to the *wave function collapse*. The dynamics of perspective of this excerpt are quite straightforward and are summarized as it follows:

Phase 1 (utterances 68–69): Linear combination perspective. Wave function as referentially semantic content.

Phase 2 (utterances 70–71): Probabilistic perspective. Wave function collapse as referentially semantic content.

It is worth noting that the students of this group went beyond the learning goals defined for this part of the teaching sequence. In addition to representing the quantum state of spin in terms of a linear combination perspective, they also addressed issues related to measurement processes and the probabilities associated with each possible outcome. The students' use of the concept *operator* (utterances 70–71) is noteworthy in this respect.

5.2.2 Reduction Mode of Discourse

An example of the reduction mode of discourse can be found in the discussion provided by group 2. In the discussion below, the students were addressing question 4. The following is our translation.

Excerpt 5 (Group 2)

(74) Emily: What happens if we add a second Stern-Gerlach and flip it?

(75) Michelle: For example, if they were in the same direction, both magnetic fields would be in z, so they would go through the same path. The probability would be 100%. It would be the same. They would get out… the same side they get in, they would get out.

(76) Emily: Sure.

(77) Hannah: Of course. But if you rotate 90°, the probabilities would change.

(78) Michelle: Yeah.

(79) Pauline: Yes. Let's say, the magnetic field would also flip and it would have nothing to do with the former. I mean, if they come out through the top, for example, the magnetic field in z would change and it would be in x, and they could come out through both the top and the bottom.

(80) Hannah: And we would have another linear combination.

(81) Pauline: Right, so it would affect.

In this excerpt, we identified two different perspectives being utilized by students. First, they examined the case in which the two magnets are oriented in the same direction and only later they analyzed the case in which the second magnet is reoriented along the x-axis. With respect to the former, students argued in terms of the projection postulate (also called the reduction of the state vector), which states that repeated measurements of the same observable in succession yield the same result. The quantum state of spin, in this case, is referred to in terms of what we call a *well-defined state perspective*, which is implied in Michelle's assertion that "if they were in the same direction, both magnetic fields would be in z, so they would go through the same path. The probability would be 100%." (utterance 75).

With respect to the latter, however, the students adopted a linear combination perspective, which is apparent in Pauline's assertion that "it would have nothing to do with the former. I mean, if they come out through the top, for example, the magnetic field in z would change and it would be in x, and they could come out through both the top and the bottom" (utterance 79). This is quite explicit in Hannah's assertion that "we would have another linear combination" (utterance 80). In this case, the change of perspective was not a result of a change in the referentially semantic content, but rather a change in the discourse mode, switching from reduction to superposition mode of discourse. The dynamics of perspective of this excerpt are summarized as it follows:

Phase 1 (utterances 75–76): Well-defined state perspective. Quantum state of spin as referentially semantic content.

Phase 2 (utterances 77–81): Liner combination perspective. Quantum state of spin as referentially semantic content.

Once again, it is worth noting that this group of students went beyond the learning goal defined for this part of the teaching sequence. In addition to relating the probabilistic feature of the quantum theory to the measurement process, they correctly used a well-defined state perspective in the calculation of probabilities, whenever it was necessary. This suggests a sort of complementarity between superposition and reduction modes of discourses in some particular situations.

5.2.3 Indeterminacy Modes of Discourse

An example of the indeterminacy mode of discourse can be found in the discussion provided by group 8. This group consists of three chemists, one physicist, and one biotechnologist. In this excerpt, the students were addressing question 5. The following is our translation.

Excerpt 6 (Group 8)

(81) Sophia: So, if we add a third one…

(82) Lucy: If it goes out positive, we have another 50–50 that it will be positive or negative. And if you have it negative, then it is another 50–50 that it will be positive or negative.

(83) Patricia: No, because… Can it go out negative?

(84) Lucy: No. Right. Negative doesn't go through.

(85) Patricia: So, we have just 50. The values vary 50–50 that it will be positive or negative in the third one.
(86) Lucy: The third one.
(87) Patricia: That is, it is as if it lost its memory.

In this excerpt, the students are discussing a setup involving three magnets. Lucy was calculating the probabilities of measuring the atomic spin in the third magnet when Patricia reminded her that the –x component of the atomic beam that enters the second magnet is blocked (see simulation on the website). In terms of referential perspective, the students adopted a linear combination point of view, which is implied in Lucy's assertion that "If it goes out positive, we have another 50–50 that it will be positive or negative" (utterance 82). It is also apparent in Patricia's assertion that "The values vary 50–50 that it will be positive or negative in the third one" (utterance 85).

In order to justify her position, however, Patricia introduced another discourse mode, on that is based on the indeterminacy relations. This is quite apparent in Patricia's assertion that "it is as if it lost its memory" (utterance 87). This form of discourse emphasizes how a sharp definition of the quantum state of spin in the x-direction destroys any previous information about the quantum state of spin in the z-direction. A sharp definition of the observable being measured implies a well-defined state perspective, which is implicit in Lucy's assertion that "Negative doesn't go through" (utterance 84). This change of referential perspective is followed by a subtle change in the referentially semantic content, switching from the quantum state of spin in the *z-direction* to the quantum state of spin in *x-direction*. The dynamics of perspective of this excerpt are summarized as it follows:

Phase 1 (utterances 82): Linear combination perspective. Quantum state of spin in the z-direction as referentially semantic content.
Phase 2 (utterances 83–85) Well-defined state perspective. Quantum state of spin in the x-direction as referentially semantic content.
Phase 2 (utterances 85–87): Linear combination perspective. Quantum state of spin in the z-direction as referentially semantic content.

Once again, it worth noting that the students of this group went beyond the learning goals previously defined for this part of the teaching sequence. Just like the previous group, they were able to use a well-defined state perspective whenever it was necessary. In this particular case, however, the use of a well-defined state perspective was motivated not by the projection postulate, but by the correct use of the indeterminacy relations. This suggests another complementarity between discourse modes — superposition and indeterminacy.

6 Discussion

In the present study, we focus on the notion of referential perspective (Wertsch, 1985) to examine student perspectives in quantum physics in the context of science teacher education. We identified a diversity of ways in which students speak of quantum concepts. Questions 1 and 2 were intended to provide a conceptual distinction between electrons and the wave function. With respect to the former, students adopted a diversity of perspectives to represent the electron, including a particle perspective, a wave perspective, a dual perspective (combining wave and particle properties), and a quantum perspective. Some of these perspectives resemble some of the interpretations of quantum mechanics advocated by physicists and philosophers of science in the past and nowadays. Hannah (excerpt 1), for example, provided a wave perspective that seems to be grounded in Schrödinger's undulatory mechanics (Schrödinger, 1926). Emily (excerpt 1) provided a particle perspective that seems consistent with the statistical interpretation put forward by Ballentine (1970). John (excerpt 2) adopted a quantum perspective that seems aligned with the realist, but not classicist interpretation advocated by Bunge (2003).

A careful analysis of students' account of wave-particle duality reveals that the dynamics of perspective often involve a subtle change in the referentially semantic content of the utterance. For example, when Michelle (excerpt 1) switched from a classical point of view (particle perspective) to a view that combines wave and particle properties (dual perspective), the referent of the utterance changed too, switching from the electron to the electron's behavior. This subtle change in the referentially semantic content amounts a transition from an ontological to a phenomenological level of description. This is somewhat consistent with Cheong and Song's (2014) distinction between prediction rules and reality-related interpretations and the distinct levels of meaning of wave-particle duality. While the ontological level of description involves reality-related interpretations (the electron is a point-like mass that has a trajectory), the phenomenological level involves interpretations of experiments, which correspond to the first level of the meaning of duality (electrons behaves like a wave).

It is interesting to note what happens when students adhere to a dual perspective without changing the referentially semantic content. Amber (excerpt 3), for example, adopted a viewpoint that combines wave and particle properties when she stated that "the electron has mass and therefore it is a particle, but this mass has a wavelength associated." In keeping her focus on the ontological level of description, she came to the conclusion that it is "a subject that is still a general controversy" and "is difficult to interpret." In a recent study, Henriksen et al. (2018) reported that some upper secondary physics students in Norway display an uncritical duality in the sense that they accept the wave-particle duality description without reflecting on the fact that particles and waves are contradictory concepts. Our suspicion is that such a contradiction only arises when students fail to change the referentially semantic content. As long as they focus on a phenomenological level of description (the electron's behavior), the reconciliation between wave and particle properties is an empirical fact: in some experiments, it acts like a wave; in others, it acts like a particle.

With respect to the wave function, some of the students (excerpt 3) correctly associated it with "the probability of finding the electron or whatever in a place," adopting a probabilistic perspective. The same probabilistic perspective was used to refer to the quantum physics as a theory (excerpt 2). When applied to the components of spin in the Stern-Gerlach experiment (two-state system), the majority of students used a linear combination perspective. Although this part of the teaching sequence involves a much less diverse set of perspectives, this view contrasts with a wavy or sinusoid perspective that is usually used to represent actual waves in the real (classical) world. In a study with upper secondary physics students in Norway, Olsen (2002) reported that some students' responses displayed a clear misconception in which photons and electrons move in wave-shaped trajectories. We believe that an emphasis on two-state system way prevents this sort of perspective. In this sense, the teaching of wave-particle duality could benefit from the use of the Mach–Zehnder interferometer (Pereira et al. 2009; Müller & Wiesner, 2002).

Specifically with respect to the concept of superposition, Myhrehagen and Bungum (2016) reported that some upper secondary physics students in Norway display a literal interpretation of the word super-position, implying that a particle can be localized in several positions at the same time. In a study involving groups of undergraduate engineering students in Brazil, Greca and Freire (2003) found that some of the students interpret superposition of states as a combination of wave and particle behaviors of an object. None of these conceptions was observed in the present study. Basically, all students interpreted superposition for a two-state system as a linear combination of its possible measurement outcomes, adopting what we called a linear combination perspective. In particular cases involving repeated measurements of the same observable in succession, they correctly adopted a well-defined state perspective, sometimes guided by the projection postulate and other times guided by the indeterminacy relations.

7 Conclusion

In this paper, we examined small-group discussions in quantum physics in the context of science teacher education. Grounded in sociocultural discourse analysis (Bakhtin, 1986; Wertsch, 1991), we focused on the notions of referential perspective and referentially semantic content to investigate how students interpret the referents of the quantum theory. Analysis shows that students used multiple perspectives when referring to quantum physics concepts. The dynamics of perspective involved in this study is characterized not only by a change of referential perspective, but also by a change in the referentially semantic content. This kind of discursive transition allows us, for example, to switch from an ontological to a phenomenological level of description in accounts of the wave-particle duality. This, in turn, allows us to combine wave and particle properties without creating any direct contradiction.

In terms of the discursive organization, we identified a set of discourse modes employed by students. With respect to the wave-particle duality, these modes are the quasi-history, oppositional, and descriptive. The quasi-history mode of discourse focuses on the transition from classical physics to quantum mechanics. The oppositional mode of discourse contrasts the features of quantum theory with those of classical physics. While the former involves multiple perspectives for the same entity, the latter adopts different perspectives to distinct objects. Finally, the descriptive mode of discourse accounts for quantum systems in their own terms without making reference to other perspectives. In terms of the learning goals defined for this study, the oppositional mode of discourse seemed to be more adequate than the others since it emphasizes the main differences between classical and quantum physics. This is likely to avoid the tendency to overlap, or even mix-up, the conceptual frameworks of the two theories (Kalkanis et al., 2003).

With respect to wave function and its probabilistic interpretation, we identified the following discourse modes: superposition, reduction, and indeterminacy. When applied to two-state systems, the superposition mode of discourse is associated with a linear combination perspective while the others are associated with a well-defined state perspective. Unlike those that are associated with wave-particle duality, these modes of discourse complement one another rather than compete with each another. It is worth noting that the notion of discourse mode should not be confused with doctrines or the specific positions taken in a debate. According to Wertsch, "[m]odes of discourse are ways of framing thinking and speaking, and as such they constrain the position one is likely to accept" (Wertsch, 1987, p. 109). Obviously, the categories used here are not the only possible ones and other researchers might categorize the data material differently.

In terms of our learning goals, it is worth noting a limitation of the study — namely, the fact that some of the students had no previous familiarity with quantum theory. Still, it seems that the teaching sequence had a positive effect on this group of students. They had the opportunity to reflect on ontological issues, deviating from the more prevalent practice of *calculating quantum physics* (Johansson et al., 2018). With respect to discussions involving the Stern-Gerlach experiment, most students succeeded in predicting correctly the probability of measuring the atom's spin in a certain state, even when the situation involved more than one magnet. The focus on two-state systems seems to provide a context in which the concepts of quantum state, superposition, wave function collapse, and probability emerged more intuitively. The alternation between oral expositions and small-group discussions seems to provide a balance between authoritative and dialogic discourse since the students had the opportunity to explore and work on each other's view while the instructor introduced and developed the scientific viewpoint (Mortimer & Scott, 2003).

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Declarations

Ethics Approval Not applicable.

Consent to Participate All students consent to participate in the present study.

Consent for Publication All individuals involved in the present study consent for publication.

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