VNIVERSITAT DE VALÈNCIA
Facultat d' Economia

Unit 1. Risk and Return

Corporate Finance

Degree in International Business

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Agenda

1. Introduction to Risk and Return

2. Characteristics of Financial Assets

3. Portfolio and Diversification

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BERK, and DEMARZO. Corporate Finance. Ed. Pearson Education 2011 → Chapters 11 and 12.

GRINBLATT, AND TITMAN. Financial Markets and Corporate Strategy. MCGRAW-HILL 2002 → CHAPTER 4.

1.1. Introduction

Investors face risks when buying financial securities due to uncertainty in relation to:

- The future selling price of the security (Pt).
- The future cash flows that will be given to the investor.

Several sources of risk exist:

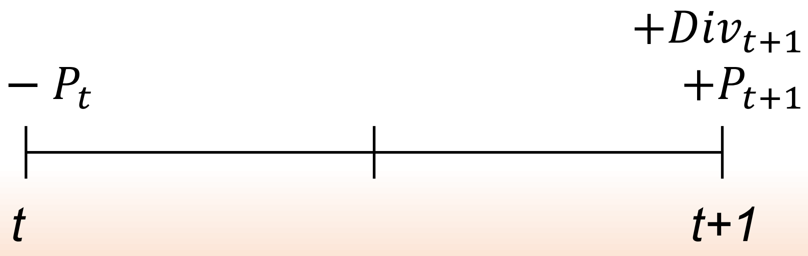
1. Liquidity risk
2. Solvency risk
3. Interest rate risk
4. Regulatory risk
5. Natural disaster risk
6. Exchange rate risk
7. Credit risk
8. Labor risk
9. Currency risk

1.2. Characteristics of financial assets

Financial assets are characterized by their risk and return.

The realized return from an investment in asset i for the period t to t+1 is derived from:

- The dividend yield
- The capital gain yield



$$R_i = \frac{d_{i,t+1} + P_{i,t+1} - P_{i,t}}{P_{i,t}} = \underbrace{\frac{d_{i,t+1}}{P_{i,t}}}_{\text{Dividend Yield}} + \underbrace{\frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}}_{\text{Capital Gain Yield}}$$

1.2. Characteristics of financial assets

Example 1.1.

Ocean Corporation paid a one-time special dividend of $4.25 on March 31, 2019. Suppose you bought Ocean Corporation stock for $33.12 on January 1st, 2019 and sold it immediately after the dividend was paid for $29.27. What was your realized return from holding the stock?

Return from January 1, 2019 until March 31, 2019 is equal to:

$$\tilde{R}_i = \frac{\tilde{d}_{i,t+1} + \tilde{P}_{i,t+1} - P_{i,t}}{P_{i,t}} = \frac{4.25 + 29.27 - 33.12}{33.12} = 0.0121 \text{ or } 1.21\%$$

1.2. Characteristics of financial assets

Example 1.1.

Ocean Corporation paid a one-time special dividend of $4.25 on March 31, 2019. Suppose you bought Ocean Corporation stock for $33.12 on January 1st, 2019 and sold it immediately after the dividend was paid for $29.27. What was your realized return from holding the stock?

The return (1.37%) can be broken down into the dividend yield and the capital gain yield:

$$\text{Dividend Yield} = \frac{\tilde{d}_{i,t+1}}{P_{i,t}} = \frac{4.25}{33.12} = 0.1283 \text{ or } 12.83\%$$

$$\text{Capital Gain Yield} = \frac{\tilde{P}_{i,t+1} - P_{i,t}}{P_{i,t}} = \frac{29.27 - 33.12}{33.12} = -0.1162 \text{ or } -11.62\%$$

1.2. Characteristics of financial assets

Example 1.1.

Ocean Corporation paid a one-time special dividend of $4.25 on March 31, 2019. Suppose you bought Ocean Corporation stock for $33.12 on January 1st, 2019 and sold it immediately after the dividend was paid for $29.27. What was your realized return from holding the stock?

- The return (1.21%) therefore includes:
  - Return generated from dividends (12.83%).
  - Capital loss (-11.62%).

Both dividends and capital gains (or losses) contribute to total realized return. Note that ignoring either of these contributions would create a highly misleading impression of Ocean Corporation's performance.

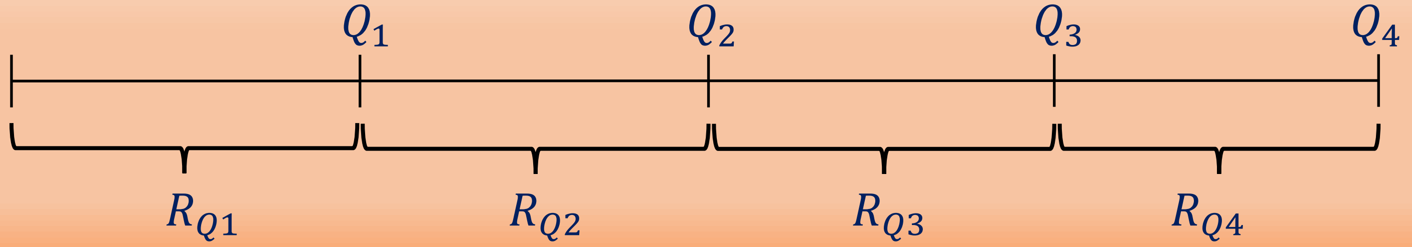
1.2. Characteristics of financial assets

Annual realized return

If a stock is held beyond the date of the first dividend and it is assumed that all dividends are immediately reinvested to purchase additional shares of the stock:

- Eq. 1.1. is used to compute a stock's return between dividend payments.
- The following equation is used to compound the returns from each dividend interval and calculate the return over a longer horizon (dividends paid quarterly).

$$(1 + R_{annual}) = (1 + R_{Q1})(1 + R_{Q2})(1 + R_{Q3})(1 + R_{Q4}) \quad (1.2)$$



1.2. Characteristics of financial assets

Example 1.1.

Given the following information for Ocean Corporation stock, determine the annual realized return:

Date	Price	Dividend	Return
January 1st, 2019	33.12		
March 31st, 2019	29.27	4.25	1.21%
June 30th, 2019	26.45	3.12	1.02%
September 30th, 2019	25.10	1.19	(0.60%)
December 31st, 2019	25.83	0.52	4.98%

$$(1 + R_{annual}) = (1 + R_{Q1})(1 + R_{Q2})(1 + R_{Q3})(1 + R_{Q4})$$

$$R_{annual} = (1.0121)(1.0102)(0.9940)(1.0498) - 1 =$$
$$= 0.0669 \text{ or } 6.69\%$$

1.2. Characteristics of financial assets

Compound Annual vs Arithmetic Returns

– **Compound annual return.** This is used to describe the long-term historical performance of an investment.

$$V_{TS} = V_{TB}(1 + R)^n \qquad (1.3)$$

where: V_{TS} is the price of the stock when sold, V_{TB} is the price when bought, and R is the annual compound return.

– **Arithmetic (average) annual return.** This is used when attempting to estimate the future expected return of an investment according to its past performance.

$$\bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots + R_T) \qquad (1.4)$$

1.2. Characteristics of financial assets

Example 1.2.

Suppose \$1,000 were invested in a stock market index in 2015 and by the end of 2019 the investment had grown to \$1,432.27. Compute the compound annual return.

$$V_{TS} = V_{TB}(1 + R)^n$$

$$1{,}432.27 = 1{,}000(1 + R)^5; R = 0.0745 \text{ or } 7.45\%$$

1.2. Characteristics of financial assets

Example 1.3.

The following table shows the realized annual returns (%) for a stock market index. Compute the arithmetic annual return.

Year	Return (%)
2015	7.45
2016	15.63
2017	-10.12
2018	5.46
2019	26.30

$$\bar{R} = \frac{1}{5}(0.0745 + 0.1563 - 0.1012 + 0.0546 + 0.2630) = 0.0894 \text{ or } 8.94\%$$

1.2. Characteristics of financial assets

Variance and volatility of realized returns

– **Variance.** This measures the variability in returns.

$$Var(R) = \frac{1}{T-1}[(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2] \quad (1.5)$$

– **Standard deviation.** This is the square root of the variance. Given that the variance is in units of percent-squared, as we square the returns, we take the square root to get the standard deviation in units of %.

$$SD(R) = \sqrt{Var(R)} \quad (1.6)$$

1.2. Characteristics of financial assets

Example 1.3.

Consider the information given in Example 1.2. as well as the calculated arithmetic annual return (6.94%). Compute the variance and the standard deviation.

Year	Return (%)
2015	7.45
2016	15.63
2017	-10.12

Year	Return (%)
2018	5.46
2019	26.30

$$Var(R) =$$
$$= \frac{1}{5-1}[(0.0745-0.0894)^2+(0.1563-0.0894)^2+(-0.1012-0.0894)^2+(0.0546-0.0894)^2$$
$$+(0.2630-0.0894)^2] = 0.0181$$

$$SD(R) = \sqrt{0.0181} = 0.1345 \text{ or } 13.45\%$$

1.2. Characteristics of financial assets

Summary of formulas for working with historical returns

	Formula
Historical return	$\tilde{R}_i = \frac{\tilde{d}_{i,t+1} + \tilde{P}_{i,t+1} - P_{i,t}}{P_{i,t}}$
Compound Annual Return	$V_{TS} = V_{TB}(1+R)^n$
Arithmetic Annual Return	$\bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots + R_T)$
Variance (Var)	$Var(R) = \frac{1}{T-1}[(R_1-\bar{R})^2 + (R_2-\bar{R})^2 + \cdots + (R_T-\bar{R})^2]$
Standard Deviation (SD)	$SD(R) = \sqrt{Var(R)}$

1.2. Characteristics of financial assets

Expected return of asset i

– **Return** is defined as a discrete random variable. Expected return is:

$$E(\tilde{R}_i) = \sum_{k=1}^{m} R_{ik} \cdot P_k \qquad (1.7)$$

where:

- R_{ik} is the return for asset i in state k of nature.
- P_k is the probability that state k occurs, $\forall k$=1,2,…,m.
- m are the different states of nature that may occur.

1.2. Characteristics of financial assets

Expected risk of asset i (Var and SD)

$$Var(\tilde{R}_i) = \sigma^2(\tilde{R}_i) = \sigma_i^2 = \sum_{k=1}^{m} P_k \cdot \left[R_{ik} - E(\tilde{R}_i)\right]^2 \quad (1.8)$$

where:

R_{ik} is the return for asset *i* in state *k* of nature.

P_k is the probability that state *k* occurs, $\forall k$=1,2,...,m.

$E(\tilde{R}_i)$ is the expected return of asset *i*.

Note that a greater variance means further return dispersion, higher uncertainty, and a riskier investment.

$$SD(\tilde{R}_i) = \sigma(\tilde{R}_i) = \sigma_i = \sqrt{\sigma^2(\tilde{R}_i)} \quad (1.9)$$

1.2. Characteristics of financial assets

Summary of formulas for working with risk and return of asset i

	Formula
Expected return	$E(\tilde{R}_i) = \sum_{k=1}^{m} R_{ik} \cdot P_k$
Variance (Var)	$Var(\tilde{R}_i) = \sigma^2(\tilde{R}_i) = \sigma_i^2 = \sum_{k=1}^{m} P_k \cdot \left[R_{ik} - E(\tilde{R}_i)\right]^2$
Standard Deviation (SD)	$SD(\tilde{R}_i) = \sigma(\tilde{R}_i) = \sigma_i = \sqrt{\sigma^2(\tilde{R}_i)}$

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1.3. Portfolio and diversification

Portfolios

An **asset portfolio** (P) is a combination of financial assets. Investors must decide how much of the initial budget will be allocated to each individual asset in the portfolio. For a portfolio made up of N financial assets:

$$\omega_i = \frac{\text{Value of investment } i}{\text{Total Value of Portfolio}}; \ \omega_1 + \omega_2 + \ldots + \omega_N = \sum_{i=1}^{N} \omega_i = 1 \qquad (1.10)$$

where:

ω_i is the weight or budget share for asset *i*.

If $\omega_i > 0$, the investor buys asset *i* (long position).

If $\omega_i < 0$, the investor sells asset *i* (short position).

If $\omega_i = 0$, asset *i* is not included in the portfolio.

1.3. Portfolio and diversification

Expected return of a portfolio

The return on a portfolio is the weighted average of the returns on the investments in the portfolio:

$$\tilde{R}_P = \omega_1 \cdot \tilde{R}_1 + \omega_2 \cdot \tilde{R}_2 + \ldots + \omega_N \cdot \tilde{R}_N$$

(1.11)

$$\tilde{R}_P = \sum_{i=1}^{N} \omega_i \cdot \tilde{R}_i \qquad \sum_{i=1}^{N} \omega_i = 1$$

The expected return of asset portfolio P:

$$E(\tilde{R}_P) = \omega_1 \cdot E(\tilde{R}_1) + \omega_2 \cdot E(\tilde{R}_2) + \ldots + \omega_N \cdot E(\tilde{R}_N)$$

(1.12)

$$E(\tilde{R}_P) = \sum_{i=1}^{N} \omega_i \cdot E(\tilde{R}_i) \qquad \sum_{i=1}^{N} \omega_i = 1$$

1.3. Portfolio and diversification

Portfolio risk (Var and DS)

Variance of asset portfolio P made up of N assets:

$$Var(\tilde{R}_P) = \sigma^2(\tilde{R}_P) = \sigma_P^2 = \omega_1^2 \cdot \sigma_1^2 + \omega_2^2 \cdot \sigma_2^2 + \ldots + \omega_N^2 \cdot \sigma_N^2 + \ldots$$
$$\ldots + 2\omega_1\omega_2 \mathrm{Cov}(\tilde{R}_1, \tilde{R}_2) + 2\omega_1\omega_3 \mathrm{Cov}(\tilde{R}_1, \tilde{R}_3) + \cdots$$
$$+ 2\omega_{N-1}\omega_N \mathrm{Cov}(\tilde{R}_{N-1}, \tilde{R}_N) \quad (1.13)$$

Simplifying the above expression for $\mathrm{Cov}(\tilde{R}_i, \tilde{R}_j) = \sigma_{ij}$:

$$Var(\tilde{R}_P) = \sigma^2(\tilde{R}_P) = \sigma_P^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij} \quad (1.14)$$

Note that the variance of the return of a portfolio is not a weighted average of individual variances.

1.3. Portfolio and diversification

Covariance and correlation

A term called **covariance** appears in the expression of variance. It measures the relationship between the variability of two securities.

$$\sigma_{ij} = \sum_{k=1}^{m} P_k \cdot \left[\left(R_i - E(\tilde{R}_i)\right) \cdot \left(R_j - E(\tilde{R}_j)\right)\right] \quad (1.15)$$

Another measure of the relationship between two variables is the **correlation**.

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \qquad -1 \leq \rho_{ij} \leq +1 \quad (1.16)$$

Accordingly, $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ (1.17)

1.3. Portfolio and diversification

Covariance and correlation

– The covariance of an asset with itself is the variance of the asset.

$$\sigma_{ii} = \sigma_i^2 \tag{1.18}$$

– Therefore, the variance (risk) of a portfolio is:

$$\sigma_P^2 = \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij} \tag{1.19}$$

– **Covariance** and **correlation** measure the degree to which a pair of returns tend to move together.

A **positive** covariance or correlation means that the two returns move together.

A **negative** covariance or correlation means that the returns tend to move in opposite directions.

1.3. Portfolio and diversification

Covariance and correlation

Remember that the correlation coefficient between two returns can be:

$$-1 \leq \rho_{ij} \leq +1$$

– If $\rho_{ij} = 1$, the correlation is **positive** and perfect.

The returns of both assets move in the same direction and maintain a linear relationship with a positive slope.

– If $\rho_{ij} = -1$, the correlation is **negative** and perfect.

The returns of both assets move in the opposite direction and maintain a linear relationship with a negative slope.

– If $\rho_{ij} = 0$, the returns of both assets are **non-dependent**.

1.3. Portfolio and diversification

Covariance and correlation

If $\rho_{ij} = 1$, the correlation is perfectly positive.

The returns of both assets move in the same direction and maintain a linear relationship with a positive slope.

The **expected return** of the portfolio is: $E(\tilde{R}_p) = w_1 \cdot E(\tilde{R}_1) + w_2 \cdot E(\tilde{R}_2)$

The **Variance** of the portfolio is: $\sigma_P^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2(1)\,\sigma_1\sigma_2$

(1.20)

$$\sigma_P^2 = (\omega_1\sigma_1 + \omega_2\sigma_2)^2$$

The **Standard deviation** of the portfolio is:

$$\sigma_P = (\omega_1\sigma_1 + \omega_2\sigma_2)$$

Therefore, return and risk are linear combinations. When the return changes, risk also changes accordingly.

1.3. Portfolio and diversification

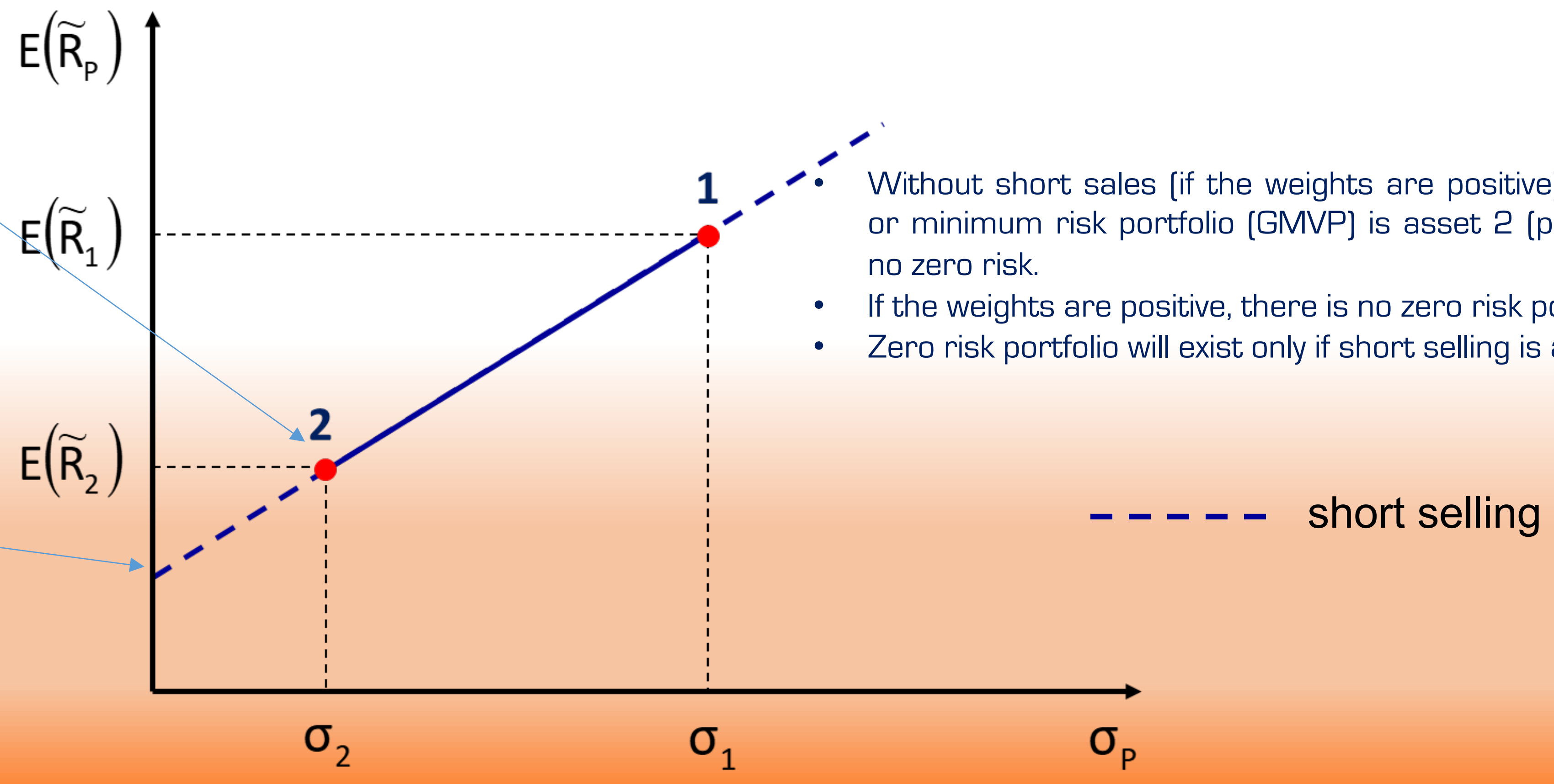
Covariance and correlation

If $\rho_{ij} = 1$, the correlation is perfectly positive.

As the expected return and standard deviation of the portfolio are linear combinations, all portfolios are located on a straight line that joins the two assets in the average space - standard deviation (as the return varies, the risk varies to the same extent). Always: w1 + w2= 1.

If we do not consider short selling (w1 ≥ 0 w2 ≥ 0) GMVP (Global Minimum Variance Portfolio): low risk portfolio has no zero risk.

If short selling is allowed GMVP: low risk portfolio has zero risk.



- Without short sales (if the weights are positive), the combination or minimum risk portfolio (GMVP) is asset 2 (point 2), which has no zero risk.
- If the weights are positive, there is no zero risk portfolio.
- Zero risk portfolio will exist only if short selling is allowed.

1.3. Portfolio and diversification

Covariance and correlation

If $-1<\rho_{ij}<1$, the correlation is not perfect.

Now the relationship between both assets can be either positive or negative:

The **expected return** of the portfolio is:

$$E(\widetilde{R}_p) = w_1 \cdot E(\widetilde{R}_1) + w_2 \cdot E(\widetilde{R}_2)$$

The **Variance** of the portfolio is:

$$\sigma_P^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2 \; < \; (\omega_1\sigma_1 + \omega_2\sigma_2)^2 \qquad (1.21)$$

The **Standard deviation** of the portfolio is:

$$\sigma_P \; < \; (\omega_1\sigma_1 + \omega_2\sigma_2)$$

ρ_{12} will determine the level of portfolio risk:

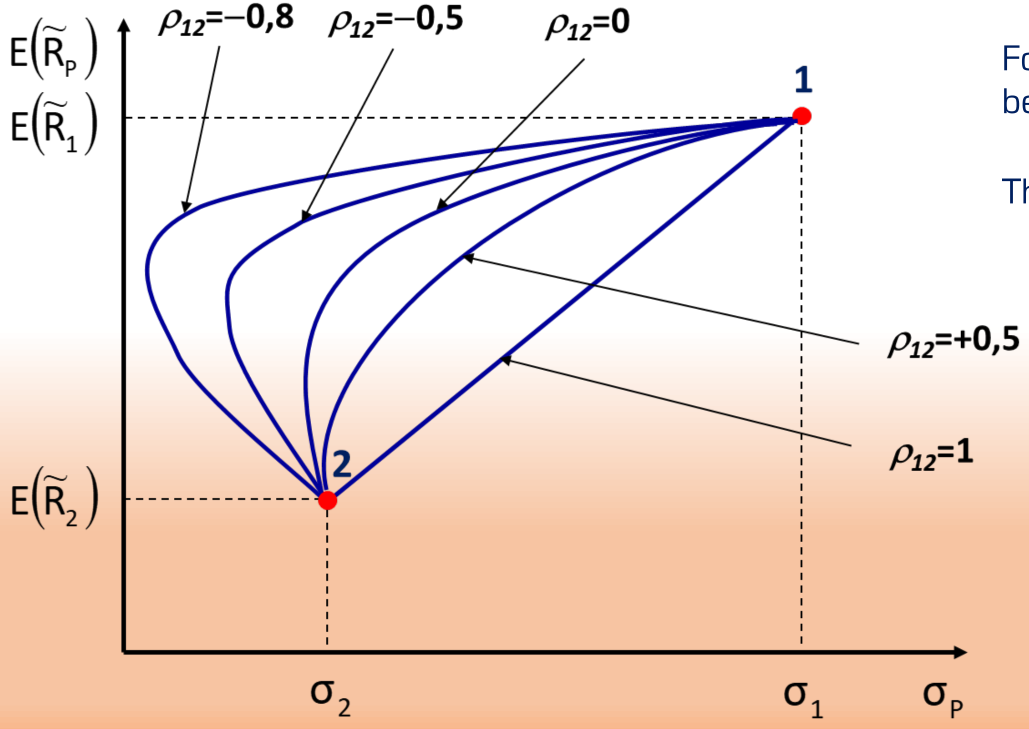
- If ρ_{12} is close to +1, there is more risk.
- If ρ_{12} is close to -1, there is less risk.

The standard deviation of the portfolio is less than the weighted average of the standard deviations of the securities.
There is a combination of minimal risk but no zero risk.

1.3. Portfolio and diversification

Covariance and correlation

If $-1<\rho_{ij}<1$, the correlation is not perfect.



For a certain expected return, if the correlation is lower, the risk will be lower and the portfolio will be further to the left.

There is no zero risk portfolio.

1.3. Portfolio and diversification

Covariance and correlation

If $\rho_{ij}=-1$, the correlation is perfectly negative.

The returns of both assets move in the opposite direction and maintain a linear relationship with a negative slope.

The **expected return** of the portfolio is:

$$E(\widetilde{R}_p) = w_1 \cdot E(\widetilde{R}_1) + w_2 \cdot E(\widetilde{R}_2)$$

The **Variance** of the portfolio is: (1.22)

$$\sigma_P^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2(-1)\,\sigma_1\sigma_2$$

$$\sigma_P^2 = (\omega_1\sigma_1 - \omega_2\sigma_2)^2$$

When we calculate the square root, we obtain two solutions (two symmetrical lines with opposite slopes).

(1.23)

The **Standard deviation** of the portfolio is:

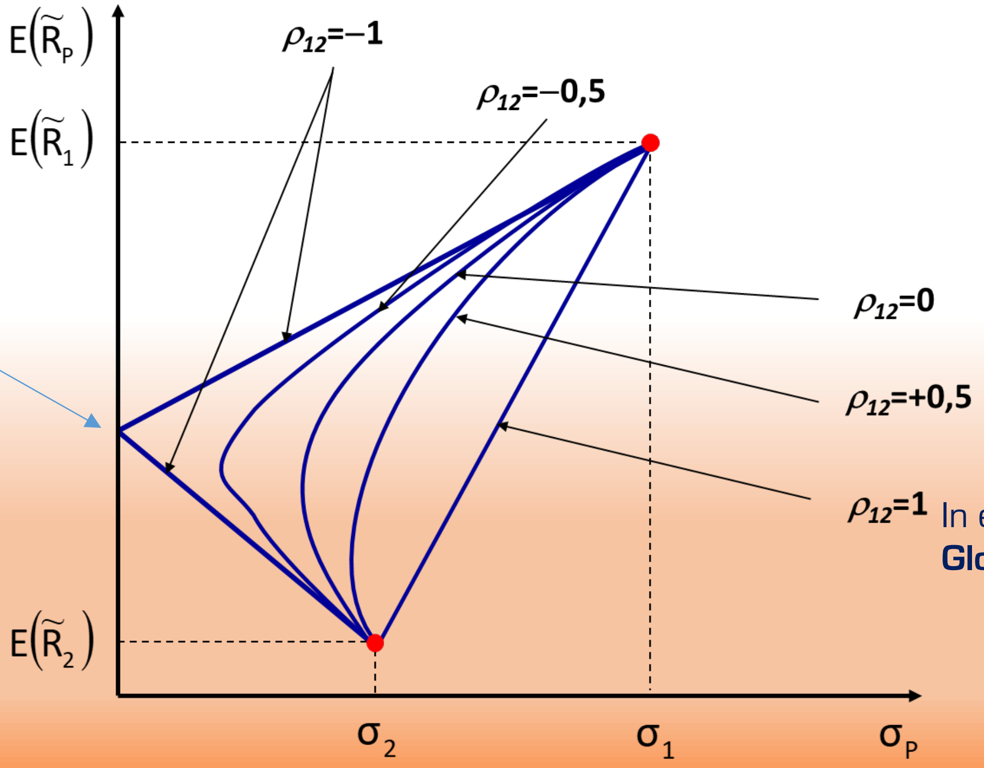
$$\sigma_P = \pm\,(\omega_1\sigma_1 - \omega_2\sigma_2)$$

1.3. Portfolio and diversification

Covariance and correlation

If $\rho_{ij}=-1$, the correlation is perfectly negative.

GMVP: Low risk portfolio. The minimum risk portfolio has zero risk and does not require short selling.



In each case, there is a portfolio that minimizes the risk: **Global Minimum Variance Portfolio**

1.3. Portfolio and diversification

Global Minimum Variance Portfolio

The **Global Minimum Variance Portfolio** is an optimal portfolio with lower risk.

To obtain the **GMVP** we calculate the partial derivative of the variance expression with respect to the weight of one of the tiles and equal to 0:

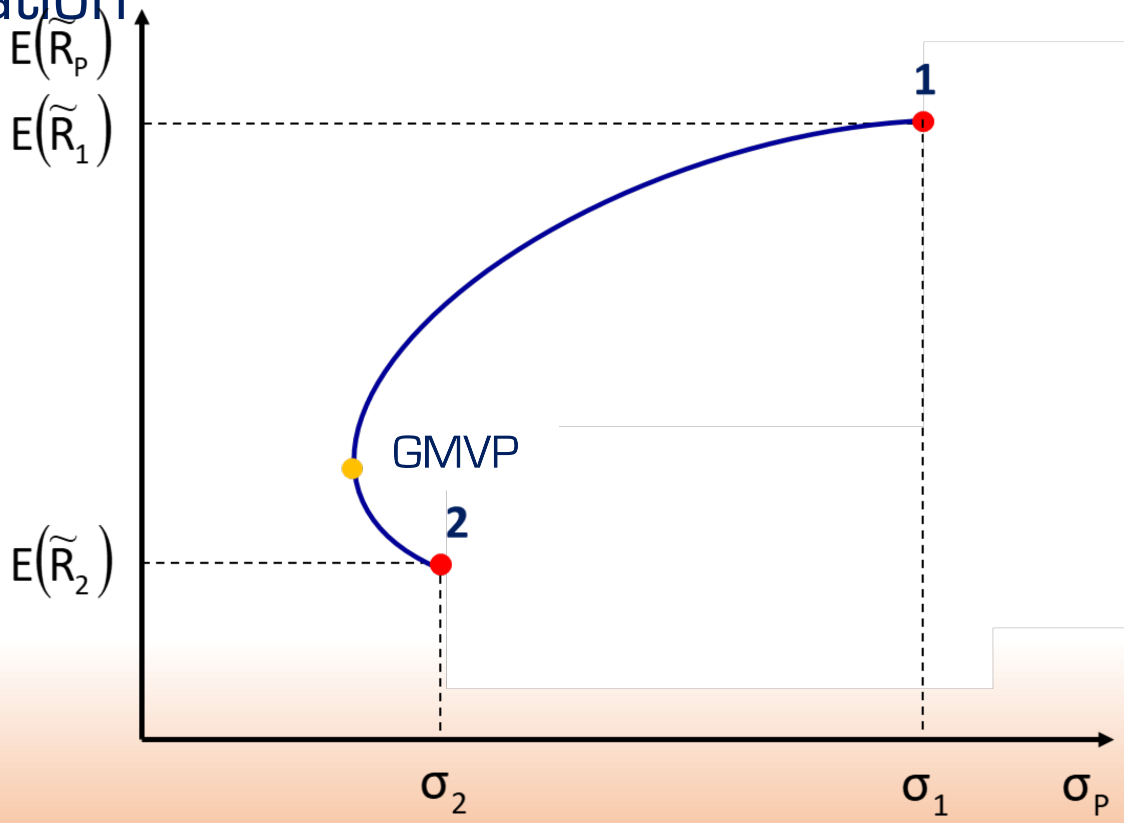
$$\frac{d\sigma_P^2}{d\omega_1} = 0$$

When doing so and isolating ω_1, we obtain the weight of ω_1 in the CMVG and, accordingly, ω_2 (i.e. $1 - \omega_1$).

$$\omega_1 = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \text{ or } \omega_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \quad (1.24)$$

1.3. Portfolio and diversification

Global Minimum Variance Portfolio



1.3. Portfolio and diversification

– Given a financial market with N = 2 financial assets: 1 and 2.

– **Portfolio characteristics in three special cases:**

– **a) Perfect and positive linear correlation (ρ_{12} = 1)**

$$\sigma_{12} = \sigma_1 \cdot \sigma_2$$

Asset returns are related according to a straight line with a positive slope: diversification HAS NO advantages.

b) Perfect and negative linear correlation (ρ_{12} = -1)

$$\sigma_{12} = -\sigma_1 \cdot \sigma_2$$

Asset returns are related according to a straight line with a negative slope: diversification DOES HAVE advantages.

c) Linear correlation ρ_{12} = 0

$$\sigma_{12} = 0$$

Asset returns are independent: diversification DOES HAVE advantages.

1.3. Portfolio and diversification

d) Imperfect linear correlation : $-1 < \rho_{12} < 1$

Total portfolio risk: Variance of a portfolio with two assets:

$$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + 2 w_1 w_2 \sigma_{1,2}$$

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

The lower the correlation coefficient (covariance), the lower the variance of the portfolio → Diversification has advantages. By properly combining financial assets, variance can be reduced.

But can DIVERSIFICATION reduce ALL risk?

In each case there is a portfolio that minimizes the risk: the MINIMUM VARIANCE PORTFOLIO.

By properly combining financial assets, risk can be reduced.

↓

Risk diversification by combining financial assets

1.3. Portfolio and diversification

Covariance and correlation

If we substitute expression (1.17) in equation (1.14), we obtain an alternative expression for the variance of a portfolio:

$$\sigma_P^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \rho_{ij} \sigma_i \sigma_j \qquad (1.25)$$

1.3. Portfolio and diversification

Summary of formulas for working with portfolios

	Formula
Portfolio weight	$\omega_i = \dfrac{\text{Value of investment } i}{\text{Total Value of Portfolio}}$
Portfolio Expected Return	$E(\tilde{R}_P) = \omega_1 \cdot E(\tilde{R}_1) + \omega_2 \cdot E(\tilde{R}_2) + \ldots + \omega_N \cdot E(\tilde{R}_N)$
Variance	$Var(\tilde{R}_P) = \sigma^2(\tilde{R}_P) = \sigma_P^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij}$
Standard Deviation (SD)	$SD(\tilde{R}_i) = \sigma(\tilde{R}_i) = \sigma_i = \sqrt{\sigma^2(\tilde{R}_i)}$

1.3. Portfolio and diversification

Summary of formulas for working with portfolios

	Formula
Covariance (I)	$\sigma_{ij} = \sum_{k=1}^{m} P_k \cdot \left[\left(R_i - E(\tilde{R}_i)\right) \cdot \left(R_j - E(\tilde{R}_j)\right)\right]$
Covariance (II)	$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$
Correlation	$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i\sigma_j}$

1.3. Portfolio and diversification

Market risk and specific risk

Risk (standard deviation) can be divided into:

- **Market risk** (also known as systematic or undiversifiable risk), which refers to the risk due to current trends in the economy, tax policy, interest rates, inflation, etc., that affect all securities.

  → Market risk cannot be diversified.

- **Specific risk** (also known as unsystematic, intrinsic or idiosyncratic risk), which is independent of market-wide phenomena. It refers to risk due to factors affecting a company, such as mismanagement, a factory fire, a product line becoming obsolete, the bankruptcy of a competitor, new regulations, etc.

  → Specific risk can be diversified.

1.3. Portfolio and diversification

A large portfolio

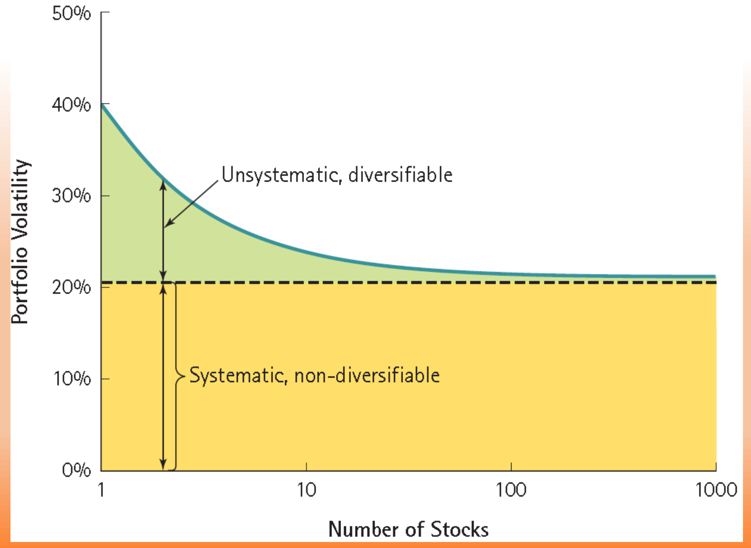
– If investors buy asset portfolios rather than single assets, they can reduce the overall risk of their entire portfolio because asset prices move independently (since they are influenced differently by macroeconomic conditions).

– Adding securities to a portfolio makes it possible to reduce the specific risk that single securities add onto the total return of the portfolio. Diversification occurs because:

  - each security has a lower relative weight in the portfolio since more securities are included.
  - a higher balance arises between favorable and unfavorable securities.

– Volatility decreases as the number of stocks in the portfolio grows.

– However, even for a very large portfolio, systematic risk cannot be diversified.



1.3. Portfolio and diversification

Naïve diversification

– By holding more than two stocks in their portfolio, investors can diversify their investment better. As more stocks are added to a portfolio, the specific risk (unsystematic, intrinsic or idiosyncratic risk) is reduced.

– Both components of portfolio risk can be identified in the variance formula:

$$\sigma_P^2 = \underbrace{\sum_{i=1}^{N} \omega_i^2 \sigma_i^2}_{\text{specific or diversifiable risk}} + \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij}}_{\text{systematic or non-diversifiable risk}} \qquad (1.14)$$

– A naive investor with no knowledge of financial markets can remove the specific risk from their portfolio by investing the same amount of money in many randomly chosen assets (N).

1.3. Portfolio and diversification

Naïve diversification: refers to the fact that some investors are completely unaware of the characteristics of the securities and the market and so diversify their portfolio randomly (at random).

They therefore reproduce efficient diversification by investing in a large number of titles ($N \to \infty$):

DEMONSTRATION

We form a portfolio with N titles and invest the same proportion in each (weighted portfolio).

(1.14)

i≠j

$$N \to \infty \qquad \omega_j = \frac{1}{N}$$

$$\sigma_P^2 = \sum_{i=1}^{N} \frac{1}{N^2} \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{1}{N} \frac{1}{N} \sigma_{ij}$$

$$\sigma_P^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N} \sigma_i^2 + \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{N} \sigma_{ij}$$

By multiplying and dividing the last term of the equation by (N-1), we obtain the variance of the portfolio in terms of the mean variance and the mean covariance ($\bar{\sigma}_i, \bar{\sigma}_{ij}$):

1.3. Portfolio and diversification

Naïve diversification

$$\sigma_P^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{N}\sigma_i^2 + \frac{N-1}{N}\sum_{\substack{i=1 \\ i\neq j}}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}\frac{1}{N(N-1)}\sigma_{ij} \quad (1.26)$$

Average variance $\bar{\sigma}_i$ Average covariance σ_{ij}

Rewriting the equation (1.25):

$$\sigma_P^2 = \frac{1}{N}\bar{\sigma}_i + \frac{N-1}{N}\bar{\sigma}_{ij} \quad (1.27)$$

1.3. Portfolio and diversification

Naïve diversification

– By rearranging equation (1.26), we get:

$$\sigma_P^2 = \frac{1}{N}\bar{\sigma}_i + \bar{\sigma}_{ij} - \frac{1}{N}\bar{\sigma}_{ij} \tag{1.28}$$

– If N→∞, the first and last term will be zero, so the portfolio risk will be the mean covariance.

$$\lim \sigma_P^2 = \bar{\sigma}_{ij}$$

– It can then be demonstrated that with a portfolio of at least 20 assets the specific risk can be eliminated to a greater extent.

A naive investor unaware of the characteristics of financial assets and financial markets can eliminate the risk of portfolio assets by investing the same amount of money (wi = 1 / N) in different randomly chosen assets.

Condition: the number of titles must be large enough (at least 15 or 20).

A portfolio with 15 or 20 securities eliminates (virtually all) the specific risk and the non-diversifiable risk.

The less correlated the yields of the securities, the smaller the non-diversifiable risk.