VNIVERSITAT DE VALÈNCIA

Facultat d' Economia

Unit 3. The Capital Asset Pricing Model (CAPM)

Corporate Finance

Degree in International Business

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Agenda

1. Introduction
2. Assumptions behind the CAPM
3. Capital Market Line (CML)
4. Security Market Line (SML)

REFERENCES

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BODIE, Z., R.A., A. KANE, AND A. MARCUS. INVESTMENTS. ED. MCGRAW-HILL (2009)→ CH 9 (pp: 279-318)

ROSS, WESTERFIELD and JAFFE. Corporate Finance. Tenth edition. McGraw-Hill (2013) → Chapter 11

3.1. Introduction

- Efficient investors will need to divide their budgets between two funds or investment alternatives:
  - Investing in the Market Portfolio (M) representative of all risky securities.
  - Investing in the Risk-Free Asset (R_F).

Depending on their aversion to risk, these investors will choose a certain portfolio and position themselves along the line.

The Capital Asset Pricing Model (CAPM):

- This is the equilibrium model that underlies all modern financial theory.
- It is based on the principles of diversification with simplified assumptions.
- Markowitz, Sharpe, Lintner and Mossin are credited with the development of this model.

3.2. Assumptions behind the CAPM

- Three main assumptions underlie the CAPM:

1) Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free interest rate.

2) Investors hold only efficient portfolios of traded securities – portfolios that yield the maximum expected return for a given level of volatility.

3) Investors have homogeneous expectations regarding volatilities, correlations, and the expected returns of securities.

3.3. Capital Market Line (CML) (I)

– In line with the above:
  - Efficient Portfolios adopt the following formal ex-post structure:

$$\tilde{R}_P = \omega \tilde{R}_M + (1-\omega) R_F \qquad (3.1)$$

  - The ex-ante expected value of this portfolio is:

$$E(\tilde{R}_P) = \omega \cdot E(\tilde{R}_M) + (1-\omega) \cdot R_F \qquad (3.2)$$

$$\sigma_P = \omega \cdot \sigma_M \qquad (3.3)$$

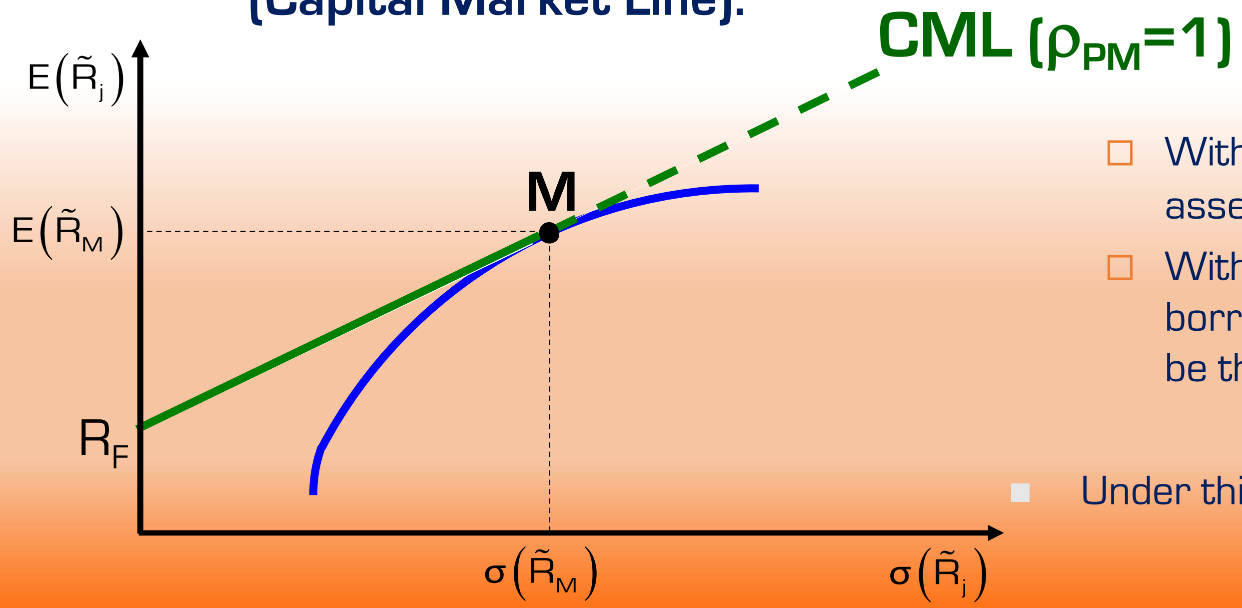
  - We assume that these portfolios are well diversified and therefore have only systematic or market risk.

3.3. Capital Market Line (CML) (II)

- The correlation between any efficient portfolio and the market portfolio must be 1, since the portfolio is located on the line:

$$\rho_{PM} = \frac{\sigma_{PM}}{\sigma_P \sigma_M} = 1 \qquad (3.4)$$

- This means that on the "new efficient frontier" we will have only efficient portfolios → **CML (Capital Market Line).**



- With *lending portfolios*, part of the budget is invested into risk free assets and will give risk free return (Rf)
- With *borrowing portfolios*, we can finance (leverage) our investments by borrowing money from the market. The cost of borrowing is assumed to be the same as the lending rate (Rf).

- Under this new assumptions the Efficient Frontier is modified.

3.3. Capital Market Line (CML) (III)

- A limitation of the **CML** is that **it only considers Efficient Portfolios**.
- But in the market we can find both efficient and inefficient portfolios.
- We need a model that relates return and risk but for any type of portfolio.
- **Fundamental Equation of the CAPM Model (Capital Asset Pricing Model):** is materialized through **the SML (Security Market Line).**

3.4. Security Market Line (SML) (I)

- The SML implies that:
  - Investors will be able to invest in efficient portfolios that have only systematic risk (it is correctly diversified).
  - If they invest in an inefficient (not diversified) portfolio, they take on more risk but the market only pays them to bear systematic risk.

- The expression of the Securities Market Line (SML) is generated through **the Fundamental Equation of the CAPM Model:**
  - It measures how changes in the returns of the market portfolio affect the returns of an investment (title or portfolio).
  - This sensitivity is measured using the beta coefficient (β):

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2} \qquad (3.5)$$

3.4. Security Market Line (SML) (II)

– The β of a portfolio/asset j implies:

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2} = \frac{\rho_{jM} \cdot \sigma_j \cdot \sigma_M}{\sigma_M^2} = \frac{\rho_{jM} \cdot \sigma_j}{\sigma_M} \qquad (3.6)$$

– According to expression (3.5), the value of β, assuming efficiency, will be:

$$\beta_j = \frac{\overset{1}{\cancel{\rho_{jM}}} \cdot \sigma_j}{\sigma_M} = \frac{\sigma_j}{\sigma_M} \qquad (3.7)$$

– The β of the Market Portfolio implies:

$$\beta_M = \frac{\sigma_{MM}}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1 \qquad (3.8)$$

3.4. Security Market Line (SML) (III)

– Finally, the **CAPM Model** through the SML:

CAPM Model

$$E\left(\tilde{R}_j\right) = R_F + \left(E\left(\tilde{R}_M\right) - R_F\right)\ \beta j \qquad (3.9)$$

Where:

R_F : risk free rate, return of asset risk free (Treasury Bills)

$\left(E\left(\tilde{R}_M\right) - R_F\right)$ is the market risk premium. It is the premium for the systematic risk borne. Market returns are above the risk-free asset returns.

β_j is the beta of the investment (market risk). It is the sensitivity of the return of portfolio/asset j to changes in the performance of the market portfolio.

This equation is satisfied for both Efficient and Non-Efficient Portfolios.

3.4. Security Market Line (SML) (IV)

If we accept a linear relationship:

- The model can be represented with a line: **The Securities Market Line (SML)**.

- All securities should comply with the model, even the Market Portfolio (M).

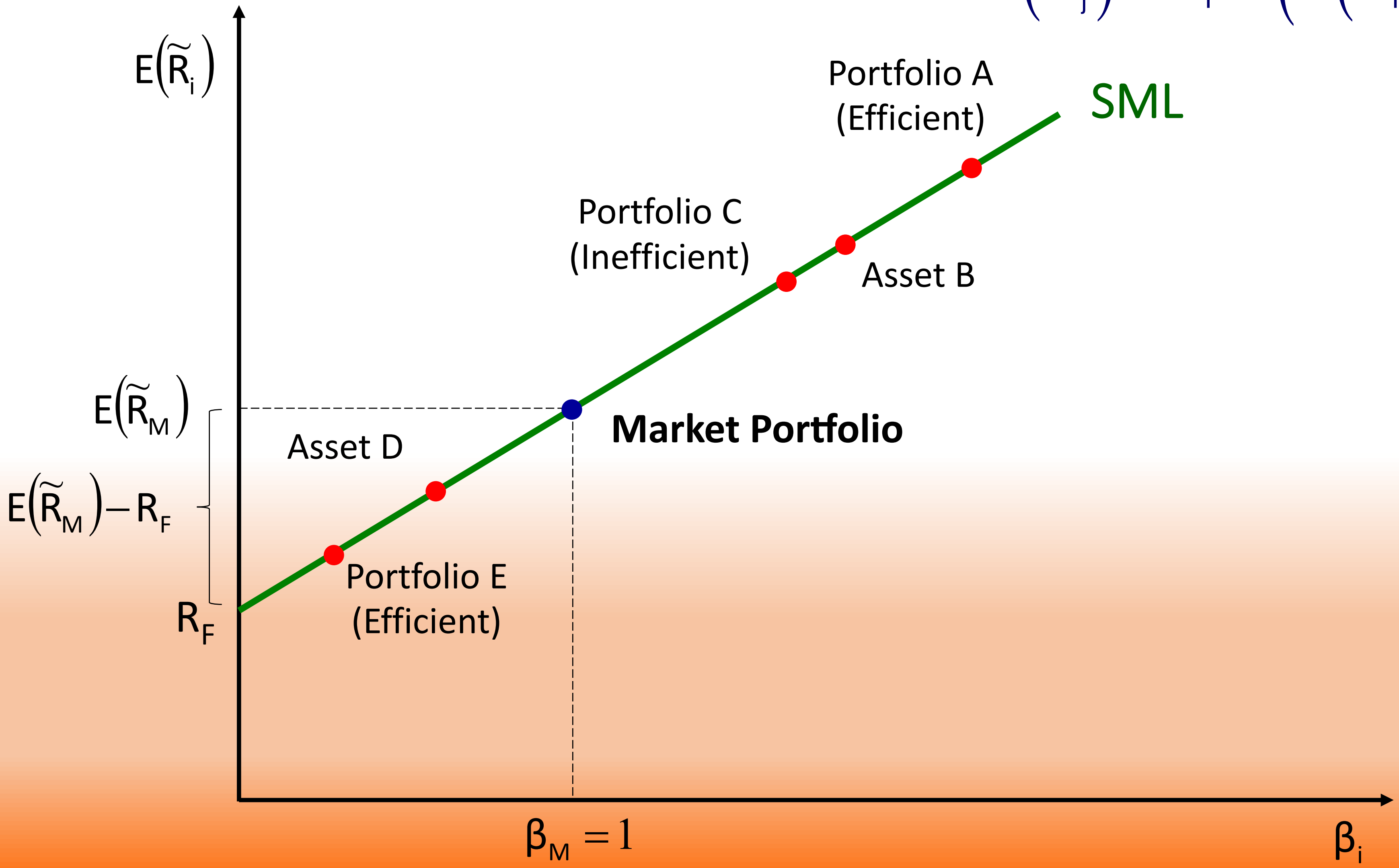
- Since we need only two points to draw a line, in our market we would know that:

  - the risk–free asset: $\beta_{rf} = 0$ and the expected return is R_F.
  - the Market Portfolio: $\beta_M = 1$ and the expected return is $E(R_M)$.

3.4. Security Market Line (SML) (V)

- The SML represented graphically:

$$E(\tilde{R}_j) = R_F + \left(E(\tilde{R}_M) - R_F\right) \beta j$$



3.4. Security Market Line (SML) (VI)

– Assets can be classified according to their β:

- **Aggressive Assets (βi> 1):** Securities whose risk is greater than that of the market itself.
  - If the market goes up, our portfolio will go up more.
  - If the market goes down, our portfolio will go down more.

- **Neutral Assets (βi = 1):** Securities whose risk is the same as that of the market.
  - If the market goes up, our portfolio will go up by the same proportion.
  - If the market goes down, our portfolio will go down by the same proportion.

- **Defensive Assets (βi <1):** Securities whose risk is lower than that of the market.
  - If the market goes up, our portfolio will go down to a lesser extent.
  - If the market goes down, our portfolio will rise to a lesser extent.

3.4. Security Market Line (SML) (VII)

- The relationship between SML and CML:
  - The SML for an asset or portfolio Z:

$$E\left(\tilde{R}_Z\right) = R_F + \left[E\left(\tilde{R}_M\right) - R_F\right] \cdot \beta_Z$$

  - Substituting the value of β by the expression (3.7), we obtain:

$$E\left(\tilde{R}_Z\right) = R_F + \left[E\left(\tilde{R}_M\right) - R_F\right] \cdot \frac{\rho_{ZM}\sigma_Z}{\sigma_M}$$

$$E\left(\tilde{R}_Z\right) = R_F + \left[\frac{E\left(\tilde{R}_M\right) - R_F}{\sigma_M}\right] \cdot \rho_{ZM}\sigma_Z$$

(3.10)

  - If portfolio Z is efficient:

$$\rho_{ZM} = 1 \quad \Rightarrow \quad SML = CML$$

3.4. Security Market Line (SML) (VIII)

- Therefore:
  - The SML for an asset or portfolio Z:

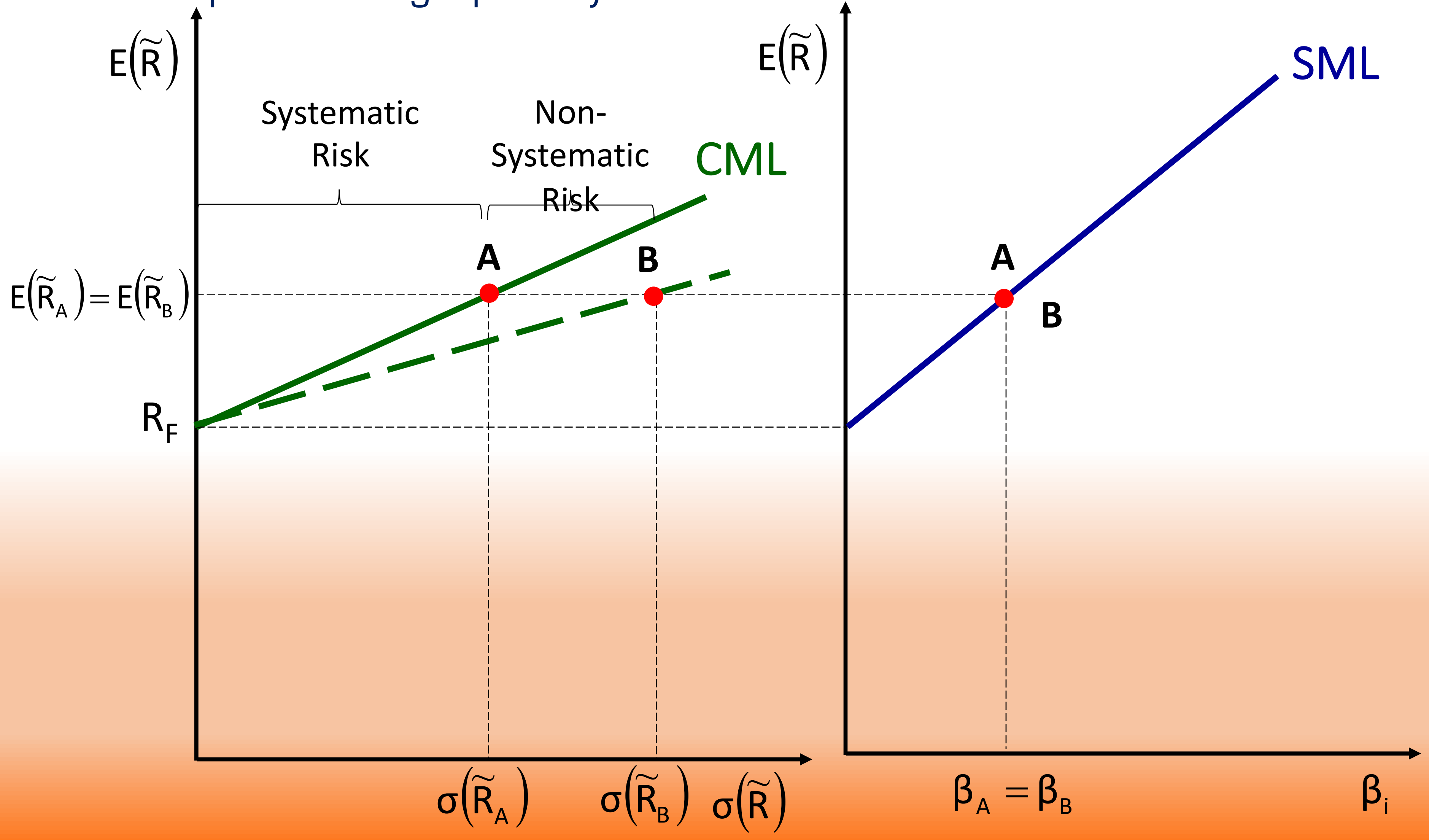
$$\text{SML} \Rightarrow E\left(\tilde{R}_Z\right) = R_F + \left[\frac{E\left(\tilde{R}_M\right) - R_F}{\sigma_M}\right] \cdot \rho_{ZM} \sigma_Z \qquad (3.11)$$

$$\text{CML} \Rightarrow E\left(\tilde{R}_Z\right) = R_F + \left[\frac{E\left(\tilde{R}_M\right) - R_F}{\sigma_M}\right] \cdot \sigma_Z \qquad (3.12)$$

  - The efficient portfolios are located in the CML and in the SML.
  - Individual securities and inefficient portfolios comply with the SML but not with the CML. They are therefore below the CML.

3.4. Security Market Line (SML) (IX)

– Represented graphically:



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