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Article

Comparing Mathematics Early Years Education in Spain, Portugal and Slovenia

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Abstract: This work aims to examine how the learning of mathematics in early childhood is developed in different policies, particularly within the processes of formal education and care in early years institutions. A comparative analysis of early mathematics education policies across countries must consider cultural differences, teaching practice, structural differences and institutional framework conditions, as well as the initial training and professional knowledge of teachers and educators. Extracted from the official country regulations, the following pages include some of the main characteristics of the national systems of early childhood education and care (ECEC) in Portugal, Slovenia and Spain, as well as a comparison of the ECEC guidelines concerning mathematics education between these three countries. There is an international consensus on an approach to early mathematics education inspired by realistic mathematics education (RME), i.e., on the importance of working mathematically in context, as well as on the idea of doing so through play, developing the language to communicate mathematical ideas. However, we found that these three aspects are reflected very differently in the official regulations of the three countries: while in Spain the development is very detailed and emphasizes the holistic approach and the role of mathematics in exploring the environment, the Portuguese curriculum emphasizes the role of mathematics as a form of language. The Slovenian curriculum, at last, focuses on the concepts and procedures associated with each mathematical sense. Furthermore, there are structural features concerning the regulation and type of ECEC system that have an influence in the implementation of the curriculum.

Keywords: early childhood education; mathematics learning; comparative study; realistic mathematics education; mathematics curriculum

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1. Introduction

The teaching and learning of mathematics in early childhood education is considered to require a greater reflection and deepening than has been devoted to it so far. There is little research production on early mathematics education and most of it is concentrated in the last twenty years [1,2]. It is, therefore, a recent area of research, but there are several points of agreement in the academic community. The first point is the importance of learning mathematical concepts at an early age to facilitate the transition to formal mathematical knowledge, as it lays the cognitive foundations for mastering mathematical concepts in later educational stages [3]. Secondly, the transition from intuitive mathematical concepts and basic cognitive processes to formal mathematical concepts must be done by using contexts that are meaningful, relevant to children and related to their experiences [4–6]. In the third place, it is crucial to assume that much of the early mathematical learning occurs in the context of children's play [7]. Fourth, the importance of language and communication as a critical tool to improve the mathematical competencies of children in early childhood education must be emphasized [8,9]. Finally, time dedicated to mathematization should be increased and additional support for children should be guaranteed in major terms more so than those indicated in policy documents. The features of a good mathematics pedagogy at an early age can be identified with reference to these principles [10]. Both the principles and the features of pedagogy are consistent with the aim of helping young children to develop mathematical proficiency.

This work is part of the MEYE (Mathematics in Early Years Education) project, an Erasmus+ consortium of higher education that includes vocational training centers, early childhood education and care settings and universities. The MEYE Project has developed a joint work between several European countries, different in their approaches, but similar in a common goal, the improvement of the teaching–learning processes of the mathematical field in early childhood education. Internationally, there is a consensus that mathematics is a fairly homogeneous discipline, so that differences in educational outcomes should be explained by cultural and social factors, by the curriculum, or by teaching practices [11]. For this purpose, one of the aims within the MEYE project is to analyze how mathematics is taught in early childhood education and care (ECEC) and how its professionals are trained to this respect. It delves into the context of Portugal, Spain and Slovenia, providing a global framework for the teaching and learning of mathematics and establishing specific aspects for the intervention in the field mathematics. The comparative analysis and proposals for improvement in this field on the curricula of ECEC, as well as the VET on early years education in Slovenia, Portugal and Spain are part the intellectual products of this project. Specifically, in this paper we present the comparative analysis between the educational policy documents related to the teaching of mathematics in ECEC of these three countries.

2. Mathematics in Early Years Education Framework

In recent years, interest in early childhood mathematics education has increased since it is considered a priority goal to achieve mathematical literacy [12]. There is general agreement on the importance of promoting the developing mathematical thinking from early ages. This agreement is supported by studies that show that children enrolled in ECEC perform better during compulsory schooling in international tests such as PISA and TIMSS [13]. Furthermore, it seems that mathematical thinking and associated competences begin to form during this stage [14]. Aspects of children's informal mathematical knowledge, such as subitizing, counting or the use of unconventional units of measurement, are prerequisites for acquiring common mathematical knowledge in school [12].

In fact, the term informal mathematics has been used to refer to the mathematical knowledge that children initially use to interpret their immediate environment [15]. The National Council of Teachers of Mathematics (NCTM) in the United States also refers to intuitive and informal mathematics, and it considers that they are a necessary link to access more formal mathematics [8]. As a result, curricula also begin to reflect the presence and relevance of children's mathematical activity [16]. Ref. [17] points out the need to orient curricula towards the use of content in contexts and towards the development of mathematical processes that lead to the acquisition of mathematical competence. In this sense, the development of mathematical processes should be worked on in a systematic way in ECEC [18]. The NCTM process standards are defined as: problem solving; reasoning and demonstration; communication; representations; and connections [8]. The main goal of the curriculum is the development of mathematical competence, made up of these processes. The National Research Council [19] emphasizes that mathematization plays a central role in the development of competence and must permeate all learning and teaching activities, as it mobilizes all these processes. As the National Council for Curriculum and Assessment [10] points out, bringing mathematical competence to the fore has the potential to change the kinds of math and math learning experienced by young children but requires changes in teaching, curriculum, skills and additional supports [20].

Although there are very few studies comparing results between countries, in terms of mathematical proficiency, for early ages, some comparative studies show that the type of instruction could be relevant for the different outcomes of children [11,21]. However, the influence of the curriculum on school results has not been demonstrated, neither in compulsory education [22], nor in early childhood education [23]. There are some studies, including this work, that analyze curricular materials, but [22] shows that it is not possible to conclude relationships with the differences among countries and academic performance. According to [24], although this type of comparisons should be used with caution, they allow useful transfers of good practices, informed decision-making and deepen the understanding of those interactions.

Therefore, the comparative analysis presented here does not aim to compare results between countries. In the comparative analysis, we address the didactic foundations that should inform any policy document on mathematics in early years education. The importance of instruction and the curriculum in mathematics literacy is highlighted. Particularly, we consider three didactic foundations on which there is consensus in the field of mathematics education: realistic mathematics education (RME); play as a context for learning mathematics; and language as a tool for developing mathematical thinking.

2.1. Realistic Mathematics Education

Mathematization is the central idea of realistic mathematics education (RME). Realistic mathematics education (RME) was the Dutch response to the need to reform its mathematics, as a reaction both to the so-called modern mathematics and to the mechanical and algorithmic teaching of mathematics. It was configured in the seventies of the last century, from the vision of the discipline of its most relevant figure, Hans Freudenthal, who, rather than formulating a theory about mathematics Education, developed an idea of mathematics as a human activity [25]. For Freudenthal, the mathematics taught and learned in school must relate to reality, be close to the experience of the students and be relevant to society [26]. Mathematics education must offer students the "guided" opportunity to "reinvent" mathematics through their practice [27], which implies an approach to the discipline not as a closed system, but as a mathematization process [26].

Although the central role of RME is recognized even in preprimary education, there is little research on the teaching of realistic mathematics in preschool-age children [2]. This process from the informal to the abstract based on the mathematization of the context [26] is the basis of realistic mathematical education (RME). According to [28], RME is based on the following characteristics, connected with socio-constructivist theories:

- The contexts as vehicles for the transition from the concrete to the abstract.
- Modeling as the axis of the mathematization process.
- The promotion of free productions by students in the teaching and learning processes of mathematics.
- The integrated work of the different content blocks in the mathematics curriculum.

Subsequently, some works [29] have delved into these characteristics to systematize a foundation of EMR in six principles, which we take from the synthesis of [30]:

1. Principle of activity: mathematics is conceived as a form of human activity, whose purpose is to mathematize—organize, generalize, and formalize—the world.
2. Principle of reality: the learning of mathematics occurs from work in real contexts, that is, solving problematic situations related to the life and/or the student's experience.
3. Principle of levels: through the mathematization of real situations, students travel a path from the concrete to the abstract at various levels of understanding: situational (understanding the context), referential (schematizing, simplifying reality), general (explore, geometrize) and formal (connect with mathematical concepts, solve symbolically).
4. Principle of guided reinvention: the mathematization process, guided by the teacher, entails an active reconstruction of the students of formal mathematical knowledge.
5. Principle of interaction: the teaching of mathematics is a social activity that produces learning through interactions—and subsequent individual reflection—between students and between students and the teacher.

6. Principle of interconnection: mathematical work from real contexts implies that the blocks of mathematical content cannot be treated separately, since problem situations require the development of interrelated mathematical content.

In this sense, the educational approach to children's mathematics education must open paths that allow children to develop spontaneous strategies when facing various situations, always aiming for generalization [31].

2.2. The Context of Children's Play

On the context of play for the teaching and learning of mathematics in early childhood education, [32] highlights the following two points:

- Children's learning should be embedded in a play-based curriculum, with teacher-led actions and imbued with mathematical meaning.
- In the role-play activities, children imitate cultural practices (such as supermarket, restaurant, laboratory), which form contexts for mathematical learning.

Educators need to understand how mathematics learning is promoted by young children's engagement in play, and how best they can support that learning [10]. Children's mathematical activity often develops without adult help, through a spontaneous interest in mathematical ideas, especially during play [7,33]. Therefore, through play, children explore and engage in activities that can be productive for learning mathematics [34]. However, sometimes play is not enough to mathematize the environment, and educators must carefully observe children's play to identify whether a mathematical learning opportunity has spontaneously emerged [35]. That opportunity should be exploited through the guidance of the educator, who should seize the moment to teach and fix the mathematical concepts involved [36].

At this stage, therefore, mathematics is embedded in the social and everyday environment, and play has a central place in that environment [34]. Therefore, play should be central to the mathematics that the educator teaches, and it is therefore an aspect that should be reflected in their training and in the ECEC curriculum.

2.3. Language as a Tool for Mathematics

Mathematical communication is an indispensable tool for children to develop mathematical thinking. The National Council of Teachers of Mathematics highlights the importance of mathematical communication as a process of mathematical thinking [8]. Children talking about their mathematical thinking and engaging in mathematization are identified as important ways for them to make their thinking visible [16].

In fact, learning is closely related to learning to communicate, that is, to the appropriation of media on specific objects in particular practices [32]. In mathematics, the media involves the use of representations. In the early stages of mathematical development, it is important to recognize that mathematical objects are accessible through symbolic representations [37], although these are linked to the concrete. Young children are not born with these symbolic tools and their capacity for reflection is based on mastering language and communication, developing through intensive communications with other people [9]. Teachers, when engaging children in active dialogue and supporting them in the cognitive process, build a kind of scaffolding and create the social context through which a child can cooperate through communication to develop mathematization [38]. It is important that during early childhood education children distinguish different forms of oral and graphic representations (concrete, pictorial, and the beginning of conventional notation) as means of communicating [39]. In the communicative process, children must progressively acquire the appropriate vocabulary to communicate mathematical ideas, as well as develop oral expression and acquire the habit of listening to others and interpreting their messages. In ECEC, teachers must provide opportunities for math talk and questioning. Much of this evidence comes from studies in which teachers promoted mathematical discussion [40,41].

3. Comparative Study Methodology

3.1. Research Aims and Methodological Procedures

Evidence from previous research indicate that comparing early mathematics education referring to different relevant aspects of the ECEC such as institutional frame conditions and teachers' education and curricula allow one to get a foundation for further assumptions and studies analyzing the differences [11]. To this respect, even this type of comparisons should be used with caution, they allow useful transfers of good practices, informed decision-making and deepen the understanding of the interactions between ECEC and its contexts [22,24]. Early mathematics education is comparable between Spain, Portugal and Slovenia as they share the same western heritage culture, and many western countries are suitable to analyze [42,43]. However, we ask whether ECEC background or basic frame conditions for early years mathematics education could make the difference in disparities between countries:

(1) Do these countries show significant differences in structural aspects of ECEC institutional frame conditions and early childhood teachers' preservice training?
(2) Can these differences be interpretated under complementary aspects that allow the definition of indicators to analyze curricula based on research in mathematics education aspects?

To address these questions, we conducted a comparison of relevant documents (e.g., official documents of education policy, national curriculum, content standards) of Spain, Portugal and Slovenia to get insights in the basic conditions of early mathematics education in the different countries. To carry out this comparative study we only used primary sources, which correspond to the different curricula of the current legislation of the national comparative units that are referenced in the last section of the article. Then, different variables and indicators were established and selected to construct the comparative study by using the extracted data: structure and regulation of the early childhood education system, early years mathematics curriculum, general early years educators initial training and, specifically, in mathematics.

The comparative study was based on two broad domains. First, the characteristics of the national curricula were analyzed in relation to three central aspects of early years mathematics education: the principles of realistic mathematics education, the context of play for learning mathematics, and the use of language as a tool for developing mathematical thinking. Secondly, the structural and institutional conditions in Slovenia, Portugal and Spain were compared. The aim was to show, by focusing on these aspects, the differences in approaches to early years mathematics education in the curricula of Slovenia, Portugal and Spain, and to suggest possible relationships with the material conditions—ratios, teacher training—reflected in the regulations of each country. This comparative research allowed us also to identify the structural features concerning the regulation and type of ECEC system that have an influence in the implementation of the curriculum. As a matter of fact, the analysis and proposals for improvement in the field of mathematics in the curricula of early childhood education and VET on early years education in Slovenia, Portugal and Spain are based on the comparative analysis presented here, in order to contribute to the elaboration of the MEYE intellectual products.

Finally, to develop the comparative study, a methodological process of multiple triangulations was established [44]. The researchers triangulated several data sources (legislation and curricula of the different countries involved). A theoretical triangulation was also established in search of complementary aspects that allow the definition of indicators to analyze curricula based on research in mathematics education, as explained in the theoretical framework. The three indicators were the following:

- Principles of realistic mathematics education.
- Role of play in mathematics learning.
- Language as a tool for the development of mathematical thinking.

3.2. Data Sources: Institutional, Curricular and Instructional

As explained in Section 3.1, multiple primary sources corresponding to the different curricula of the current legislation of the comparative national units (Portugal, Slovenia and Spain) were used to carry out this comparative study. The comparative tables with the extracted data are presented below. In Table 1, we find the differences in the structure of early childhood education. The indicators show differences both in the age range that makes up this educational stage and in the type of system and its governance; however, in the three countries analyzed, the reference institution is the Ministry of Education.

Table 1. General structure of ECEC in Portugal, Slovenia and Spain.

	Age Group	System Type	Core Minister	Governance Type
Portugal	3–6 years old	Split	Education	Decentralized
Slovenia	1–6 years old	Integrated	Education	Centralized
Spain	0–6 years old	Integrated	Education	Decentralized

Table 2 contains information from other institutional sources that complements the above data: years in which the current curriculum was approved; ages covered by the curriculum; the professional figure responsible for implementing it in the classroom; and the ratios of children per professional at this stage.

Table 2. ECEC regulations and guidelines in Portugal, Slovenia and Spain.

	Portugal	Slovenia	Spain
Official curriculum	2008	1998	2007 (0–3) 2006 (3–6)
Scope	1–6	1–6	0–6
Responsible professionals	Teachers	Teachers and assistants [1]	Teachers
Ratios by age (pupils/staff)	Under 1 year: 5 children per staff member (maximum group size 10) 1 year old: 7 children per staff member (maximum group size 14) 2 year old: 9 children per staff member (maximum group size 18) From 3- to 5-year-old: 12.5 children per staff member (maximum group size 25)	From 1 to 3 years old up to 7 (14 per group)	Under 1 year of age: 8 children per one qualified member of staff From 1- to 2-year old: 12 to 14 children per qualified member of staff From 2- to 3-year old: 16 to 20 per qualified member of staff. From 3- to 5-year-old: maximum group size for children is 25, and at least one member of staff must have a relevant qualification at university level

[1] Assistants are not responsible, but they can be.

Table 3 provides the central information to achieve the aim of this work, as it gathers all the references to the teaching and learning of mathematics—or related concepts—that appear in the curriculum. We performed the triangulation of sources [44] by classifying the data into three categories: whether mathematics has a defined area in the curriculum (and if not, in which area it is found); the global goals of the curriculum regarding mathematics; and the mathematics specific aims and contents.

Finally, in Table 4, we triangulate and classify the data related to early childhood education professionals, who oversee the transposition of the curriculum into teaching and learning practices. The first category of the table indicates whether there is a differentiated curriculum for teachers and educators, or whether it is integrated; the second category collects the main goals of the professionals of the stage.

Table 3. Official curriculum for mathematics childhood education related to children's learning in Portugal, Slovenia and Spain.

	Defined Area	Global Goals	Aims and Contents
Portugal	No (the area 2—Area of Expression and Communica-tion, Domain of Mathematics)	2—Area of Expression and Communication—basic area of content that focuses on essential aspects of development and learning encompassing the learning related to symbolic activity and the progressive mastery of different forms of language	Numbers and operations - To identify quantities through different forms of representation. - To solve everyday problems, involving small amounts, using addition and subtraction). Data organization and processing - To collect relevant information to answer raised questions. - To use simple graphs and tables to organize the information collected and interpret them to answer the questions asked. Geometry and measurement - To find objects in a familiar environment, using guidance concepts. - To identify location recognition points and use simple maps. - To take the point of view of others, being able to say what can and cannot be seen from a certain position. - To recognize and operate with geometric shapes and figures, discovering and referring properties and identifying patterns and symmetries and projections. - To understand that objects have attributes and measurable data that allow comparing and ordering them. - To choose and use units of measurement to answer daily needs and questions. Interest and curiosity in mathematics - To show interest and curiosity in mathematics, understanding its importance and usefulness. - Feeling competent to deal with notions of math and solve small problems.
Slovenia	Yes (daily routine)	- To learn about mathematics in everyday life; - To develop a mathematical way of expression; - To develop mathematical thinking; - To develop mathematical skills; - To experience mathematics as a pleasant experience.	Children use names for numbers. ○ From naming individual objects children gradually move to counting and differentiating between numbers and numerals. ○ Children acknowledge one-to-one correspondence and practice it. ○ Children develop operations, which are the basis for addition and subtraction. ○ Children use symbols with which they record events and describe situations. ○ Children learn about graphical representations, which they make and read. ○ Children learn about the relationship between cause and effect. ○ Children become familiar with the probability of events and use expressions to describe it. ○ Children look for, perceive and use a variety of possible solutions. ○ Children check if the obtained solution is reasonable. ○ Children learn about symmetry, geometric bodies and shapes. Children learn about space and its boundaries, the exterior and interior. ○ Children use expressions to describe the position of objects (on, in, before, under, behind, at the front of, at the back of, above, beneath, left, right, etc.) and learn orientation in space. ○ Children classify and sort. ○ Children learn about the differences in measuring and counting as well as various common properties of materials and objects which are measured and individual objects which are counted. ○ Children become familiar with the strategies of measuring length, surface, and volume with scales and units.

Table 3. *Cont.*

	Defined Area	Global Goals	Aims and Contents
Spain	No (knowledge of the environment Block 1. Physical environment: Elements, relations and measurement)	3–6: Aims (art. 3) Early childhood education will contribute to the development of girls and boys in their capacities to: Begin in the logical-mathematical abilities, in the reading–writing and in the movement, the gesture and the rhythm.	Second Cycle (3–6): Area 2: knowledge of the environment. Goals: 1. Observe and actively explore their surroundings, generating interpretations of some situations and significant events and showing interest in their knowledge. 2. Relate to others, in an increasingly balanced and satisfactory way, progressively internalizing social behavior patterns and adjusting their behavior to them. 3. Know different social groups close to their experience, some of their characteristics, cultural productions, values and ways of life, generating attitudes of trust, respect and appreciation. 4. Begin in mathematical skills, functionally manipulating elements and collections, identifying their attributes and qualities and establishing relationships of groupings, classification, order and quantification. 5. Know and value the basic components of the natural environment and some of their relationships, changes and transformations, developing attitudes of care, respect and responsibility in their conservation. Contents: Block 1. Physical environment: Elements, relationships and measure The objects and materials present in the environment, their daily functions and uses. Interest in their exploration and attitude of respect and care towards their own and other people's objects. Perception of attributes and qualities of objects and subjects. Interest in the classification of elements and in exploring their qualities and grades. Contextual use of the first ordinal numbers. Approach to the quantification of collections. Use of counting as an estimation strategy and use of cardinal numbers referred to manageable quantities. Approach to the numerical series and its oral use to count. Observation and awareness of the functionality of numbers in everyday life. Exploration and identification of situations in which it is necessary to measure. Interest and curiosity about measuring instruments. Approach to its use. Intuitive estimation and time measurement. Temporary location of activities of daily life. Situation of himself and objects in space. Relative positions. Performing oriented displacements. Identification of flat and three-dimensional shapes in elements of the environment. Exploration of some elemental geometric bodies.

Table 3. *Cont.*

	Defined Area	Global Goals	Aims and Contents
Spain	No (knowledge of the environment Block 1. Physical environment: Elements, relations and measurement)	0–6: In relation to the area, the educational intervention will aim to develop the following capacities (goals) [6]: 1. Observe and actively explore your surroundings, physical, natural and social, develop a sense of belonging to them, showing interest in their knowledge and to develop in them a certain security and autonomy. 2. Relate to others, every time in a more balanced and satisfactory way, progressively internalizing the basic patterns of social behavior and adjusting their behavior to them. 3. Identify and approach the knowledge of different social groups close to their experience, to some characteristics of its members, cultural productions, values, and ways of life, generating attitudes of trust, respect and appreciation. 4. Investigate the physical environment by manipulating some of its elements, identifying its characteristics and developing the capacity to act and produce transformations in them. 5. Represent attributes of elements and collections and establish relationships of clusters, classification, order and quantification, starting in the mathematical skills. 6. Be interested in the natural environment, observe, and recognize animals, plants, elements and phenomena of nature; experiment, talk about them and develop attitudes of curiosity. 7. Know and value the basic components of the natural environment and some of its relationships, changes and transformations; developing caring attitudes, respect and responsibility in their conservation.	First cycle (0–3 years old). Contents: Block 1. Interaction with the physical and natural environment. Exploration and observation of objects and materials present in the middle through the performance of actions such as caressing, hitting, picking up, dragging, screwing, opening, blowing, etc., verbalizing the processes to discover sensations, characteristics and utilities. Anticipation of some effects of your actions on objects, animals or plants, showing interest in their care and avoiding risk situations. Interest in the investigation of elements and matters (water, sand, etc.), discovering some of their attributes and qualities such as cold, hot, dry, wet, large or small. Establishment of some similarities and differences. Classifications according to a criterion and ordinations of two or three elements per size. Performing actions on elements and collections such as gathering, distributing, mapping and counting elements, approaching quantification not numerically (many, few, some) and numerically (one, two and three), expressing satisfaction for the achievements. Anticipation of some daily routines or activities experiencing the first experiences of time (such as lunch or patio time) and intuitive estimation of their duration. Recognition and verbalization of some notions Basic spaces such as open, closed, up, down, inside and outside. Second cycle (3–6 years old). Contents: Block 1. Physical environment: elements, relationships, and measure. The objects and materials present in the environment, their daily functions and uses. Interest in their exploration and attitude of respect and care towards their own and other people's objects and care of them. Perception of similarities and differences between objects. Discrimination of some object's attributes and subjects. Interest in the classification of elements. Relationships of belonging and nonbelonging. Identification of qualities and their degrees. Ordination of gradual elements. Contextual use of the first ordinal numbers. Non-numerical quantification of collections (many, few). Quantitative comparison between collections of objects. Equality and inequality relations (same as, more than, less than). Exact quantitative estimation of collections and use of cardinal numbers referring to manageable quantities. Oral use of the numerical series to count. Observation and awareness of the functional value of the numbers and their usefulness in everyday life. Exploration and identification of situations in which it is necessary to measure. Some conventional and unconventional units and measuring instruments. Approach to its use. Interest and curiosity about measuring instruments. Intuitive estimation and time measurement. Location. Temporary activities of daily life. Detection of temporal regularities, such as cycle or frequency. Observation of some modifications caused by the passage of time in the elements of the environment. Situation of themselves and objects in space. Relative positions. Identification of flat shapes and three-dimensional elements of the environment. Exploration of some elementary geometric bodies. Notions. Basic topologies (open, closed, inside, outside, close, far, interior, exterior . . .) and realization of oriented displacements.

Table 4. Official curriculum for mathematics childhood education related to ECEC professionals in Portugal, Slovenia and Spain.

	Portugal	Slovenia	Spain	
Training Curriculum for ECEC professionals	Differentiated	Unitary	Differentiated (Educators)	Differentiated Teachers
Global goals	Purposes of the course are: - To develop the ability to use mathematics as an instrument of interpretation and intervention in reality. - To develop the ability to select mathematics relevant to each problem in reality. - To develop the skills to formulate and solve problems, to communicate, as well as memory, rigor, critical thinking and creativity. - To promote the deepening of a scientific, technical and humanistic culture that constitutes cognitive and methodological support both for the full insertion in professional life and for the continuation of studies. - To contribute to a positive attitude towards science. - To promoting personal fulfilment by developing attitudes of autonomy and solidarity. - To create capacities for social intervention by studying and understanding problems and situations in today's society, as well as by discussing systems and decision-making bodies that influence the lives of citizens, thus participating in the formation of active and participatory citizenship.	- To develop a deeper theoretical and practical knowledge that they have acquired in the context of the general educational subject of mathematics in the field of the elementary mathematics and basic geometry. - To upgrade their knowledge in the field of mathematical reasoning and logic to develop their mathematical thinking and expression. - To obtain knowledge to plan and implement the activities to develop children's mathematical thinking and expression. - To learn to direct/use/seize the ingenuity, creativity, autonomy and originality in mathituations. - To analyze and value their own work and present it in unique ways.	Article 5: General goals: (a) To identify and specify the programming elements, relating them to the characteristics of the group and the context to program the educational and social assistance intervention for children. (b) To identify and select teaching resources, describing their features and applications to organize them according to the activity and the recipients. (c) To select and apply resources and methodological strategies, relating them to the characteristics of the children and in the context to carry out the scheduled activities. (d) Select and apply evaluation techniques and instruments, relating them to the relevant variables and comparing the results with the established standard in the intervention process.	1. To know the goals, curricular contents and evaluation criteria for early childhood education. 2. To promote and facilitate learning in the early childhood, from a globalizing and integrative perspective of the different cognitive, emotional dimensions, psychomotor and volitional.

4. Discussion

Based on the data collected in the methodological framework, we compared on the one hand, the structural and institutional factors marked by the curriculum that establish the framework within which mathematics is taught and learned at an early age. On the other hand, the mathematics curricula in early childhood education in Spain, Portugal and Slovenia were compared using the categories established in the theoretical framework: principles of realistic mathematics education, role of play and language.

4.1. Structural and Institutional Differences between Spain, Portugal and Slovenia

There are three countries with three different contexts and three different educational trajectories regarding the type of ECEC system.

As we can see in Table 1, in Spain, early childhood education and care includes 0–6 years, with a decentralized curriculum while in Slovenia, it starts at one year (given the longest maternity leave) up to six years and presents a centralized curriculum. However, in Portugal, early childhood education covers 3–6 years old with a decentralized curriculum, and childcare is considered from 0 to 3 years old.

Moreover, as teacher training and knowledge differ between countries ([45], p. 28), it can be concluded that preservice education of the kindergarten teachers and in consequence kindergarten teachers' knowledge differ in different countries as well. These differences can be characterized as structural (university vs. vocational school), but also concern the content and methods, because instruction—and therefore instruction in teacher education as well—is a "culturally scripted activity" ([24], p. 10; [46,47]). Observing the initial training of teachers in these three countries, we find very different situations, because of differences in the context and conceptualization of education, as well as what is worked on with young children (see Table 4).

As can be seen in Table 3, significant differences can also be observed in the proportion of professionals per group, which have a direct implication in the quality of interventions with young children. In fact, the ratio in Spain, where there is one educator per so many children, does not allow developing (or observing) the exploration-based mathematics proposed in the curriculum. To this respect, the minimum statutory child-to-staff ratios in early years settings should respond not only to the age of children (as is common in OECD countries), but also the qualifications of the staff employed in the class should be considered to guarantee this requirement is responsive [48].

4.2. Instruction and Curricula Differences between Spain, Portugal and Slovenia

As can be seen in Table 3, while Portugal and Spain do not include mathematics as a defined area within the curriculum, Slovenia does, ensuring its own space within daily routines. In Portugal, mathematics makes up a domain of the area of Expression and Communication. In Spain, it is not even explicitly mentioned: the processes related to mathematical competence are formulated within the block: Knowledge of the environment, Physical environment: Elements, relations, and measurement. As we saw in the theorical framework, one of the characteristics that the teaching of mathematics in early childhood education must fulfill is to guarantee time and space to guarantee that all children develop mathematization processes [49]. Prescribing mathematics as an area of daily work with children, as Slovenia does, is a more effective way to work on mathematical processes in a systematic way, as recommended by experts [18,50]. Thus, the guidance report of the Education Endowment Foundation (EEF) states to "Make the most of moments throughout the day to highlight and use mathematics, for example, in daily routines, play activities, and other curriculum areas." ([41], p. 6), but only the Slovenian curriculum ensures this.

Regarding the global goals, Portugal focuses on essential aspects of development and learning, encompassing the learning related to symbolic activity and the progressive mastery of different forms of language. The emphasis is on two concrete processes of mathematical competence: communication and representations. However, the importance of the other three processes is neglected: problem solving, reasoning and connections [8].

The Spanish curriculum, on the other hand, includes as its global goal the development of logical–mathematical skills, shared with reading–writing and with movement, gesture and rhythm. Thus, it includes a series of very general aims, as can be seen in Table 3, which have a very tangential relationship with mathematical processes. Again, Slovenia's global goals are explicitly related to mathematical processes and are clear and synthetic. "To learn about mathematics in everyday life" refers to the use of relevant context and close to the children's experience [4–6]. "To develop a mathematical way of expression" is related to the processes of representations and communication, that is, to the dimension of mathematics as a language. "To develop mathematical thinking" and "to develop mathematical skills" may be two aims that are formulated too broadly, but they are part of the idea of mathematical competence and so are related to the processes of reasoning and problem-solving. Furthermore, "to experience mathematics as a pleasant experience" points to children's engagement during the teaching and learning of mathematics, highlighting the role of mathematics embedded in play.

In Table 5 we summarize which of the three relevant aspects of early childhood mathematics education—RME, play, language—appear in the curricula of Slovenia, Portugal and Spain.

Table 5. Presence of RME principles, the role of play in mathematics learning and mathematics as a form of language in the curricula of Slovenia, Portugal and Spain.

Country	Principles of Realistic Mathematics Education	Role of Play in Mathematics Learning	Language as a Tool for the Development of Mathematical Thinking
Slovenia	Do not appear in the curriculum.	Yes, it appears as part of the daily routine and pleasant activities for children.	Yes, mathematics appears as a way of expression.
Portugal	Do not appear in the curriculum.	Does not appear in the curriculum.	Yes, the focus is on mathematics as a form of language.
Spain	Yes, the principles—activity, reality, interaction, interconnectedness—do appear, although they are not explicitly formulated from the specificity of mathematics education.	It appears implicitly as part of environmental discovery activities and in reference to spontaneous interest but is formulated in a general way.	No, only the notion of verbalisation appears at some point.

Finally, the comparative analysis of the aims and contents also shows significant differences, although some similarities exist: in all three countries, the basic and initial aspects of mathematics are worked on: first, numbers and operations, and second, geometry and measurement. In Slovenia, they are considered in a general way, and in Portugal and Spain, the aspects to be achieved are specified. In the following lines, we analyze this block by block:

In numbers and operations, the goal is to identify quantities through different forms of representation, and to solve everyday problems, involving small quantities, using addition and subtraction. However, in the Spanish curriculum, essential phases of the representation of quantities are omitted. In the international guidelines [8,51], the learning of different ways of representing numbers is emphasized, but in the Spanish curriculum, it focuses on the use of writing to fulfil real purposes, without entering the phases of representation of numbers and quantities [18]. It alludes to the use in contexts of ordinal numbers and cardinal numbers, as well as of numerical series, and the usefulness of the number in everyday life. Again, the formulation is very general; the holistic approach prevents specifying all the mathematical processes involved. The Portuguese curriculum does include an explicit block of numbers and operations, in which representations and problem-solving appear. It would be necessary to specify the ordinal and cardinal approximations to the number sense, which do appear in Spain. Finally, in the Slovenian curriculum, the following items appear as aimss and contents related to this block: names of numbers (it is related to communication), differentiating number to count (ordinal) and number to express quantity, construction of the cardinal through correspondence, operations and use of symbols to describe situations (including different representations of the number). Although presented synthetically, the contents of the Slovenian curriculum are the most complete, although in this case it would be necessary to emphasize problem-solving, as Portugal does.

In ECEC, we can distinguish three large parts of geometry: the position, shapes and changes in position and shapes [52]. We already saw that the indicators about position in the Spanish curriculum were formulated in a holistic way, emphasizing the role of the geometric sense in the exploration of the environment rather than the mathematical process. The same happens with the following indicator, which would be framed in the shapes part: "Identification of flat and three-dimensional shapes in elements of the environment. Exploration of some elemental geometric bodies". The changes (movements) associated with the shape are not mentioned in the Spanish curriculum. In Portugal, we find a very complete and explicit description of the contents and processes associated with the spatial and geometric sense: "the location of objects in a familiar environment is intended, using orientation concepts; the identification of location recognition points and the use of simple maps; starting from the point of view of others, being able to say what can and cannot be seen from a certain position; the recognition and the success of operating with geometric figures and figures, discovering and referring properties and identifying patterns, symmetries and projections". It includes the three main parts: positions (orientation, reference points, points of view), shapes (properties of geometric figures, identification of patterns) and transformations (projections and symmetry). Finally, in the Slovenian curriculum, it is written that "Children learn about symmetry, geometric bodies and shapes", which is framed in terms of forms and change of form; "Children learn about space and its boundaries, the exterior and interior", which is related to position; "Children use expressions to describe the position of objects (on, in, before, under, behind, at the front of, at the back of, above, beneath, left, right, etc.) and learn orientation in space", which is also framed in terms of position and in displacements of position, but also emphasizes the acquisition of vocabulary and the communicative aspect.

Measurement is related to geometry as knowledge of space, with numbers and operations (by the process of obtaining, through comparison, a number as a result), and with knowledge of the natural environment [52]. Measurement is usually linked to geometry, but knowledge of the natural environment is related to measurements of time, capacity or mass, which are quantities that can be explored experimentally during early childhood education. Regarding measurement, in the Spanish curriculum, the holistic approach emphasizes the knowledge of the natural environment. Thus, it insists on "Exploration and identification of situations in which it is necessary to measure. Interest and curiosity about measuring instruments. Approach to its use. Intuitive estimation and time measurement. Temporary location of activities of daily life". It alludes to the concept of measuring instrument but not to the sense of measurement through the idea (and choice) of the unit of measurement and comparison. On the other hand, in the Portuguese curriculum the allusions to the natural environment are not sufficiently explicit (for example, the time), although the daily needs respond to this aspect. The process of choosing the unit of measurement and comparison to obtain a measure of magnitude does appear explicitly. In the case of the Slovenian curriculum, the measure appears in "Children learn about the differences in measuring and counting as well as various common properties of materials and objects which are measured and individual objects which are counted", which is an allusion to the meaning of number as a measure (the link between the measure with numbers and arithmetic is reinforced), and it also appears in "Children become familiar with the strategies of measuring length, surface and volume with scales and units", which refers to the process of measuring (choice of unit, comparison), but focuses only on spatial magnitudes. Therefore, the measure for the knowledge of the natural environment does not appear, exploring other magnitudes such as time.

In addition to the two main blocks, the curricula of Portugal and Slovenia dedicate a space to the organization, representation of data and probability. In Spain, there is no allusion to these contents. This is an important gap. Ref. [53] defines statistical literacy as the ability of people to interpret data, critically evaluate them and, when relevant, express their opinions regarding statistical information and arguments related to the data. Children intuitively handle and represent data. They should also incorporate notions about determination and uncertainty, especially at the communicative level: what can and cannot happen? What are the chances of something happening? With what reliability? This is the reason why statistics and probability have been incorporated into mathematics curricula since the age of three [8]. In the case of Spain, content organization proposals have been made [52], but it has not been incorporated into the curriculum. In the case of the Portuguese curriculum, it is formulated as "Collect relevant information to answer questions raised" and "Use simple graphs and tables to organize the information collected and interpret them in order to answer the questions asked", which includes the two main processes: data collection and organization and representation of data. However, there is no allusion to chance and uncertainty in the Portuguese curriculum. In Slovenia, this block appears as "Children learn about graphical representations, which they make and read", which refers to the representation of data, although the process of collecting and organizing data remains to be explained. It is also mentioned that "Children learn about the relationship between cause and effect", that is to say, causality and determination are explored, which is a first step to then address the third mention; "Children become familiar with the probability of events and use expressions to describe it", in which they already refer to chance and indeterminacy, emphasizing the communicative component.

Portugal and Slovenia also dedicate a mention to problem-solving, as we have seen, one of the central processes of mathematical competence [8]. In Portugal, it is written as "Feeling competent to deal with notions math and solve small problems", while in Slovenia it appears as "Children look for, perceive and use a variety of possible solutions", which emphasizes the importance of flexibility in problem-solving [54], and as "Children check if the obtained solution is reasonable". In this case, Portugal focuses more on self-regulation, since "to successfully complete a mathematical task, children must be able to self-regulate, and so the development of self-regulation is consistently linked to successful learning in early mathematics" ([41], p. 9), while Slovenia focuses on two relevant problem-solving processes: the knowledge and use of different resolution strategies (flexibility) and the validation and interpretation of solutions. In the Spanish curriculum, there is no mention of the problem-solving process.

In all cases, there are curricula guidelines for the ECEC professionals to be trained, with differentiated training and a better social appreciation and recognition based on dedication to children from 3 or 4 years old to compulsory school. Attention to the first years of ECEC corresponds to a more oriented vocational training (VET), while training addressed to the second part of the stage is more oriented toward higher education level. In the field of teaching and learning mathematics, the documents that we consulted showed significant differences. In Portugal, it is a more conceptual learning of the mathematical field (the following curricular modules are worked: numbers and geometry, including trigonometry; real functions and infinitesimal analysis; statistics and probabilities; and discrete mathematics) and less of didactic aspects. The essential and necessary theoretical bases of all the great systems of interpretation of reality that guarantee social intervention with responsibility and give meaning to the human condition are contemplated, to develop the capacities of: using mathematics as an instrument of interpretation and intervention in the reality; select the relevant mathematics for each problem in reality; develop the skills to formulate and solve problems, to communicate, as well as memory, rigor, critical thinking and creativity; promote the deepening of a scientific, technical and humanistic culture that constitutes a cognitive and methodological support both for the full insertion in a professional life and for the pursuit of studies; contribute to a positive attitude towards science; promote personal fulfilment by developing attitudes of autonomy and solidarity;

and create capacities for social intervention by studying and understanding the problems and situations in today's society, as well as by debating the systems and decision-making bodies that influence the lives of citizens, thus participating in the formation of an active and participatory citizenship.

In Slovenia, a more balanced weight is given to the aspects related to mathematics training, and the didactics of mathematics. The training of professionals aims to develop a theoretical and practical knowledge of elementary mathematics and basic geometry; update the knowledge in the field of mathematical reasoning and logic to develop mathematical thinking and expression; and in the most didactic aspects, planning and implementing activities to develop children's thinking and mathematical expression; take advantage of ingenuity, creativity, autonomy and originality in mathematical situations; and analyze and assess one's own work and present it globally. All this in order to participate in the planning and implementation of activities to develop children's mathematical thinking, expression and learning of basic mathematical concepts; detect the progress and development of the child in mathematical concepts, opinion and expression and relate it to the aims of the teaching of mathematics; detect the child's need to use mathematics and be able to take advantage of the situation in everyday life for the use of mathematics; and allow the child to experience mathematics as an enjoyable experience.

In Spain, two different professionals are trained in the stage 0–6 years, the educator in early childhood education, which is a training program in the field of vocational training, and the teacher of early childhood education. The first degree is a two-year training, which is accessed with the baccalaureate level or similar, and is the professional who is dedicated to training in the formal settings for 0–3 years, and nonformal education for 0–6 years old. It includes some minimum teachings at the national level and each autonomous community specifies the rest of the curriculum. The teaching of mathematics can be included in the early childhood education didactics modules in a very general way (not specific to mathematics), and in the expression and communication module, in which it is found as one more section within the teaching and learning general area. At the same time, in the practices (at the end of the second year), the students can observe and implement these types of learning in a real context, and with the practical stage, they would have the option of delving into this area. The early childhood education teacher is currently a university degree, and it is considered a regulated profession—that means with some common basic aspects in the university environment—so it is one of the few degrees that presents joint regulations at the university national level, which is contemplated in each university. There is a specific didactic and disciplinary module, called "Learning of Natural Sciences, Social Sciences and Mathematics" (60 European credits), which is usually restricted to specific didactic subjects, such as mathematics in early childhood education. In this module, very general aspects are intended, such as knowing the goals, curricular contents and evaluation criteria for early childhood education. To do so, it starts from the knowledge of the scientific, mathematical and technological foundations of the curriculum of this stage, as well as the theories about the acquisition and development of the corresponding learning, to gradually acknowledge the didactic strategies to develop numerical representations and notions of spatial, geometric and logical development, as well as to understand mathematics as sociocultural knowledge. We can conclude that in Spain, the training of these two professionals in the field of mathematics is didactic and differentiated when comparing with the two other training approaches. It is concluded that perhaps professional figures could have a unified or common training. In fact, the lack of training in mathematics didactics for educators coming from vocational training may limit the development of activities that begin to stimulate the mathematical sense of children from 0 to 3 years of age. In the same way, it would be advisable for all educators to be trained in mathematics didactics when forming pedagogical pairs of different professional profiles, as activities based on RME require a careful observation, and both professionals must be involved.

5. Conclusions

In Spain, there has been little tradition to incorporate the systematic work of mathematical processes in the early childhood education stage [18]. Mathematical contents of the curriculum appear diffuse and blurred; a holistic approach is used that prevents the precise formulation of the mathematical processes that lead children to acquire the bases of mathematical competence. Rather, it focuses on the role of mathematics as a tool for exploring the world. That is, it is in line with the purposes of realistic mathematics education (RME), but it does not guarantee a daily space in which children develop the mathematization of the environment. In fact, mathematics does not appear as a differentiated area and there are no global goals directly linked to mathematics. In addition, there are important shortcomings both in the inclusion of mathematical content (statistics and probability blocks do not appear, for example) and processes (there is no mention of problem-solving, nor of mathematical reasoning and the communicative aspect appears very little), since it basically focuses on the connections of mathematics with the environment.

In the Portuguese curriculum, mathematics is approached as part of the development of languages in early childhood education. This approach allows for a complete description of the mathematical concepts and processes that make up mathematical language, although there are some minor shortcomings in terms of the development of the idea of uncertainty. The symbolic approach—through language—also implies an insufficient development of intuitive mathematical representations, through the body or manipulative materials. In fact, in the Portuguese curriculum, little importance is given to context as a means of exploiting children's mathematical work. There is no mention, therefore, of the principles of realistic mathematics education and the connections of mathematics with the environment. Consequently, there is also no mention of the role of play and spontaneous interest in learning mathematics.

Regarding the Slovenian curriculum, we note that in the overall aims of the area of mathematics, reference is made both to the role of the environment and to the role of play, as well as to mathematics as a form of expression and communication. However, the contents are formulated without reference to the context, neither to play nor to the communication of mathematical thinking. Thus, for the concretization of the global goals, which do include these aspects, a technical description has been chosen that focuses on mathematical concepts and associated procedures. The contents are developed in a schematic but exhaustive way: of the three curricula, it is the one that covers the greatest number of concepts and procedures, dealing with logical thinking, number sense, the measurement of quantities, geometric and spatial sense, data representation and uncertainty.

Therefore, we can conclude that, from the point of view of mathematics education, the three curricula are complementary: the most exhaustive is the Spanish curriculum, and the one in which mathematization of the context, that is, mathematical activity as an exploration of the environment, acquires a greater importance. In the Portuguese curriculum, the role of language is more prominent than in the other two curricula. Finally, the Slovenian curriculum offers a more comprehensive account of mathematical contents, which are described from a more formal point of view than in the other two curricula: mathematical concepts and procedures specific to each content block are listed, without linking them to contexts or to forms of expression and communication.

In sum, the comparative conclusions facilitate the identification of the structural features concerning the regulation and type of ECEC system that have an influence in the implementation of the curriculum, and therefore an impact in the mathematization processes of young children in ECEC. Specifically, we addressed how the material and structural aspects that reflect the regulations of each country could affect the development in the classroom of the ideas expressed above: mathematization of the environment, mathematics within the game and mathematics as a language. Significant differences such as the observed one in the proportion of professionals per age group have a direct implication in the quality of interventions with young children and those mathematization processes. To this respect, solutions are often suggested to give additional support to the students to achieve mathematical competence, as well as to anticipate that a high ratio does not guarantee having space and time to co-construct ideas and mathematical objects with the children through a guide in the mathematization processes. However, in order to guarantee the suitability of young children learning environments, a pedagogical pair of ECEC professionals should work collaboratively in the implementation of the curriculum from the start of the stage to the transition to primary education.

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