Validation of a set of compact and low-cost Stokes lenses aimed at refractive astigmatism measurement for optometric practice

Jose Ricardo Albero-Moreno,[a] César Albarrán-Diego,[b] and Vicente Micó[b,*]

[a]Clinica Baviera Castellón, Castelló, Spain

[b]Universitat de Valencia, Departamento de Óptica y Optometría y Ciencias de la Visión, Facultad de Física, Valencia, Spain

Abstract. We describe the construction and characterization of three Stokes lenses that can be used in optometric practice to measure different ranges of refractive astigmatism using subjective routines. The proposed set of Stokes lenses is assembled from Risley prism mounts coming from a Nidek phoropter in which the optical prisms are replaced by three different sets of cylindrical lenses (±0.75, ±1.25, and ±1.75 D) coming from a trial lens case to cover a different astigmatism range (1.5, 2.5, and 3.5 D in modulus, respectively). We present the experimental characterization of the three Stokes lenses with the power vector notation and use an automatic lensmeter (Topcon CL-300) for dioptric power measurement. The measured dioptric power profiles are in concordance with the theoretical predictions, and statistical testing reveals no differences between measured and theoretical values for the three lenses, neither in spherical nor in cylindrical components ($p \geq 0.05$ for all cases). Bland–Altman plots show very good agreement between measured and theoretical values with limits of agreement below 0.12 D for both M and J components for the three lenses. These Stokes lenses can be easily constructed from regular Risley prism mounts, so they can be used in almost any phoropter model for refractive error compensation. © *2022 Society of Photo-Optical Instrumentation Engineers (SPIE)* [DOI: 10.1117/1.OE.61.12.121803]

Keywords: Stokes lens; astigmatism; power vectors analysis; ophthalmic devices.

Paper 20220421SS received Apr. 22, 2022; accepted for publication Aug. 1, 2022; published online Sep. 21, 2022.

1 Introduction

The history of cross-cylinder lenses cannot be understood without considering the development of astigmatism. Astigmatism can be traced back to the beginning of the 19th century when Young discovered it while working with his own eyes.[1,2] Young's work concerning astigmatism was mainly descriptive, but he also made quantitative measurements of refraction and accommodation using his own optometer and dealt with many other aspects of visual and ophthalmic optics such as biometric parameters, peripheral refraction, longitudinal chromatic aberration, depth-of-focus, and instrument myopia. A quarter of century after this, Sir George Airy measured the astigmatism in his left eye and ordered the manufacturing of a cylindrical lens for its correction.[3,4] Because he was myopic as well, Airy conceived the combination of two cylindrical and concave lenses for correcting his own refractive error. But finally, it was more convenient from a practical point of view to combine one surface cylindrical and the other spherical, both being concave.

It is not our aim to review in depth the historical development of astigmatism, which can be consulted elsewhere,[5,6] but rather to provide its key points and milestones. The following one in this temporal line seems to be done by Stokes[7] in the middle of the 19th century.[8,9] Stokes envisioned a lens made from two cylindrical surfaces of equal radii, one concave and the other convex, with their axes crossed at right angles. He called such a lens an astigmatic lens. Moreover,

*Address all correspondence to Vicente Micó, vicente.mico@uv.es

0091-3286/2022/$28.00 © 2022 SPIE

Stokes described his astigmatic lens as a vector and showed that the combination of two astigmatic lenses is equivalent to a resulting astigmatic lens fully described from the addition of the individual vectors. This concept brought the birth of the power vector notation rounded by Thibos et al.[10] around 150 years after.

Maybe the most relevant feature of the Stokes lens was its capability for varying the astigmatic power because the two cylinders were arranged in such a way that they can be rotated in opposite directions. Thus, it resulted in zero dioptric power when the axes of the two cylinders were parallel and in maximal astigmatic power (the addition in modulus of the two cylinders) when the axes were perpendicular. In particular, the cylinders used by Stokes had a power of around 4 D, so the astigmatic lens was able to generate sphero-cylindrical powers ranging from 0 to (+4) and (−8) D of sphere and cylinder, respectively.[11]

However, the astigmatic lens was considered to be a curiosity rather than a real instrument for astigmatism determination, and it was Jackson[12] who realized the real potential of the Stokes lens for determining both cylindrical power and cylinder axis[13] in an astigmatic eye. Jackson proposed a modification of the Stokes lens in which the cylinder's axes were fixed at orthogonal directions to each other and the cylindrical powers were restricted to ±0.25 D (or ±0.50 D as maximum). This modification is known as Jackson crossed cylinder lens (or simply as cross cylinder), and it is currently widely used in clinical subjective routines mainly for refining astigmatism (both axis and power) as well as spherical power.[14] Nonetheless, the general acceptance by the ophthalmological community did not come right after the Jackson findings but with the work developed by Crisp[15–20] (a much more prolific writer) some years later.

Coming back to the Stokes astigmatic lens and despite the Jackson cross cylinders probably having a greater prestige worldwide, the Stokes lens has shown an inestimable validity as part of many ophthalmic and optometric applications and instruments.[21–23] Moreover, some evolutions of the initial Stokes lens have been proposed in recent years such as, for instance, an astigmatic lens including two identical rotatable cylinders,[24] pure cross-cylinders,[25,26] a combination of rotatable cross-cylinders with an additional fixed cross-cylinder lens,[27] variable aberration generation systems based on two rotatable phase elements,[28,29] and variable prism compensator.[30] Also, instead of generating a variable amount of a certain aberration, it is possible to use a Stokes lens for compensating/measuring the astigmatism produced in a certain instrument. In that sense, oblique incidence astigmatism in the human eye was corrected, thus improving the peripheral fundus images,[31] and small residual astigmatism has been compensated for by adapting a low-cost customized Stokes lens to different instruments such as an eye fundus camera[25], a microscope eyepiece,[26] and a Hartmann–Shack sensor,[32] as well as in digital microscopy[33,34] and focimetry.[34,35]

Ferrer-Altabás and Micó[34] reported a low cost, compact, and variable Stokes lens for astigmatism compensation in optical instruments that is based on an old phoropter Risley prism mount in which the prisms are replaced by a couple of cylindrical lenses of equal power but opposite signs (±1.50 D). As a result, the Stokes lens is capable of generating/compensating for astigmatism up to a maximum value of 3D (in modulus). Performance of such a Stokes lens was evaluated with the power vector formalism[10] and studied using an automatic lensmeter (Topcon CL-300) and an aberrometer (Zeiss iProfiler plus) for low and high order aberrations measurement, respectively. As reported in Ref. 34, the proposed Stokes lens showed a residual small amount of equivalent sphere as well as a negligible presence of high order aberrations.

In this contribution, we present the dioptric power characterization and performance evaluation for three different Stokes lenses composed of different cylindrical lens pairs, thus arriving at a different astigmatic compensation range. Instead of using an old Risley prism mount as in the work reported by Ferrer-Altabás and Micó,[34] we employed more recent mounts coming from Nidek phoropters as well as included an angular dial to directly measure the angle variation between cylinder axes in degrees (not in prism diopters as in previous work), thus, making its usability/readability much more comfortable and easier to use. These lenses take part in a different study (not included in this manuscript) for refractive error compensation following alternative subjective routines[36] and because of that, they are labeled with QR codes to prevent easy identification of them and to avoid bias in the data (it is worth noting that this encryption is not relevant for this paper). Thus, this manuscript presents the technical validation of a set composed of three lab-made Stokes lenses in which the measured dioptric power profiles generated by each

Stokes lens are compared with the corresponding theoretical prediction (as in previous work[34]), but here we include a statistical analysis (paired *t*-test, boxplots, and Bland–Altman graphs) of the measurements, thus expanding upon and completing the characterization reported in previous work.[34] Measurements were taken with an automatic lensmeter at steps of 2.5 deg, and statistical analysis of the results included Bland–Altman plots comparing the produced power against the theoretical one for the three studied lenses.

2 Methods

2.1 *Stokes Lens Mounting*

A set of three stokes lenses with three different powers were assembled from three Risley prims mounts extracted from Nidek phoropters (model RT-600). The optical prisms included in the mounts were replaced by three different sets of cylindrical lenses coming from a trial lens case. For this comparison, we selected the following pairs: ± 0.75, ± 1.25, and ± 1.75 D, thus covering the astigmatism ranges (in modulus) of 1.50, 2.50, and 3.50 D, respectively. The lenses were cut down in diameter to exactly fit with the proper dimensions and were placed with their plane face in contact with each other. Figure 1 shows one of the three assembles Stokes lenses that (a) includes the original Risley mount and its explosion in the different elements, (b) shows the pair of cylinders of power ± 1.75 D before and after being reduced in diameter, and (c) includes an image of the final assembled Stokes lens including the angular scale attached to the mount. Finally, Fig. 1(d) shows a photo of the three generated Stokes lenses with the previously described pairs of cylindrical lenses.

The cylinders are carefully positioned in such a way that their axes are aligned and oriented to 45 deg in the angular scale when the mount marks 0 deg. This is the case of the picture included at Fig. 1(c) and involves 0 D of generated power because both axes are parallel. But the rotation of the cylinder's axes from 0 deg to ± 45 deg generates the pure J_0 component being positive or negative depending on the specific rotation of each cylinder. Thus, when the positive cylinder is rotated $+45$ deg (clockwise direction) and the negative one does the opposite, the generated J_0 component raises the combination of two cylindrical lenses in the form of $(+1.75 \times 0$ deg) with $(-1.75 \times 90$ deg), which in turns yields a J_0 value of $[-1.75 \times 0$ deg] according to the criterion defined in Ref. 10. The opposite occurs for the opposite rotation direction. Note that we use a different notation to avoid confusion and distinguish between polar Fourier and sphero-cylindrical formalisms: because both notations contain a spherical value and astigmatism in polar representation, the sphero-cylindrical notation is presented using conventional parenthesis ( ), and square brackets [ ] are used for the polar Fourier notation.

Figure 2 includes a representation of one of the assembled Stokes lenses that is adjusted to generate J_0 and J_{45} components in Fig. 2(a) and 2(b), respectively. The red arrow points the 0 deg direction of the angular dial from where the cylinder's axes (yellow dotted line) are initially aligned at $+45$ deg. Starting from these positions, the first one (a) generates the J_0 component when rotating the cylinder's axes using the rotating wheel of the Risley mount and the second (b) does the same for the J_{45} component. Alternating between the initial positions is extremely easy using the ± 45 deg click-stop positions of the mount. So, from now on, we do not care about the generated astigmatic axis because it can be fixed by adjusting the global orientation of the Risley mount.

2.2 *Measurement Procedure and Data Analysis*

Each of the three Stokes lenses was characterized using an automatic lensmeter (Topcon CL-300) for the measurement of the generated dioptric power when varying the inner cylinders. Measurement points were defined every 2.5 deg in the angular scale. Accuracy at the lensmeter was selected as 0.125 D, which is also the manufacturing tolerance of the cylinders integrating the Stokes lenses. The measurements are presented in the form of the dioptric power profile for both the sphere (S) and the cylinder (C) as well as the mean equivalent sphere (M) and Jacksonian astigmatic (J) components.

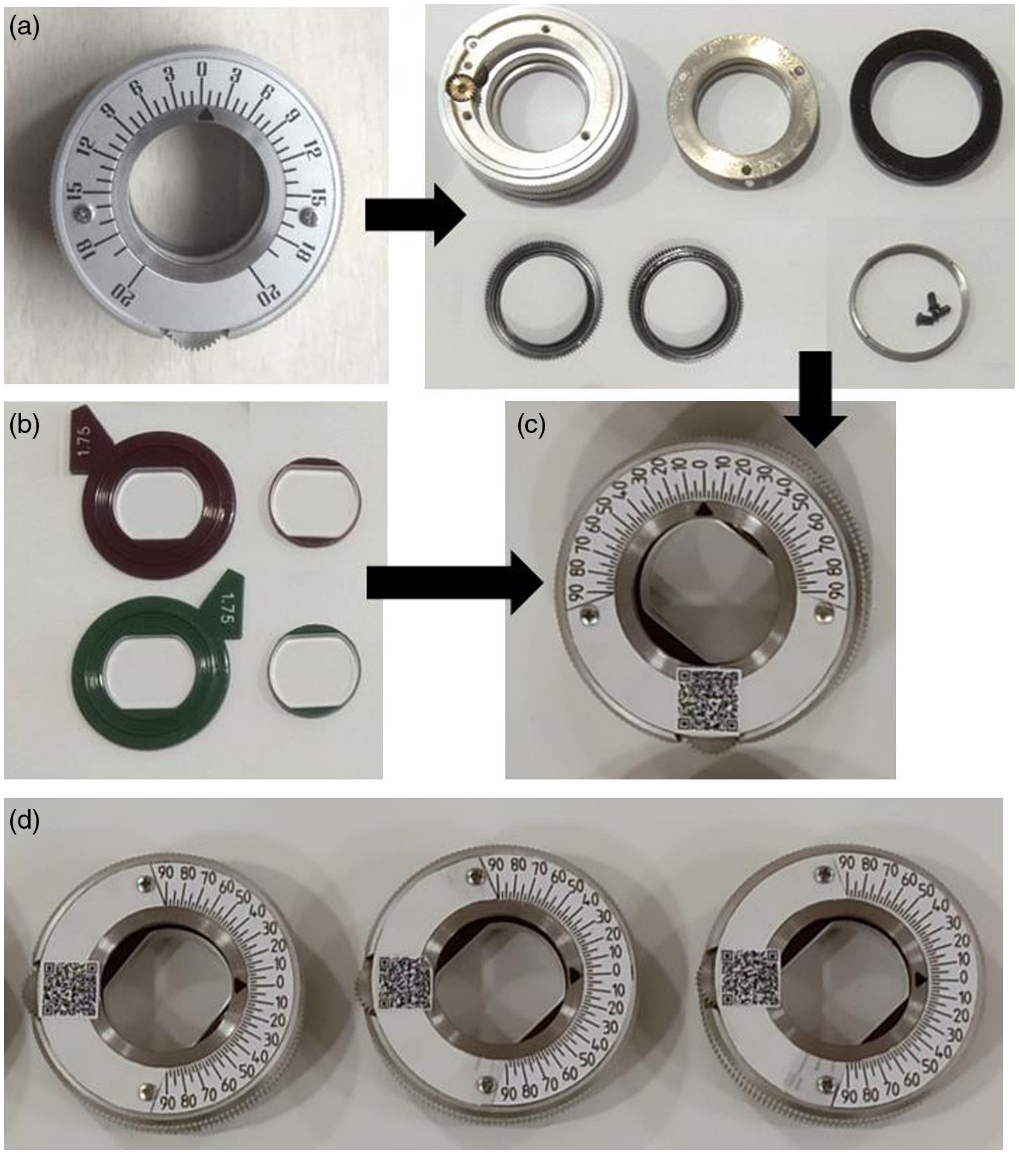

Fig. 1 Example of a Stokes lens assembled from (a) a Risley prism mount incorporating (b) two cylindrical lenses of equal but opposite signs powers (± 1.75 D). The final result is included in (c), and the three different Stokes lenses of different astigmatic power is presented in (d). It is worth noting that the angular dial attached to the mounts for direct determination of the rotation angle.

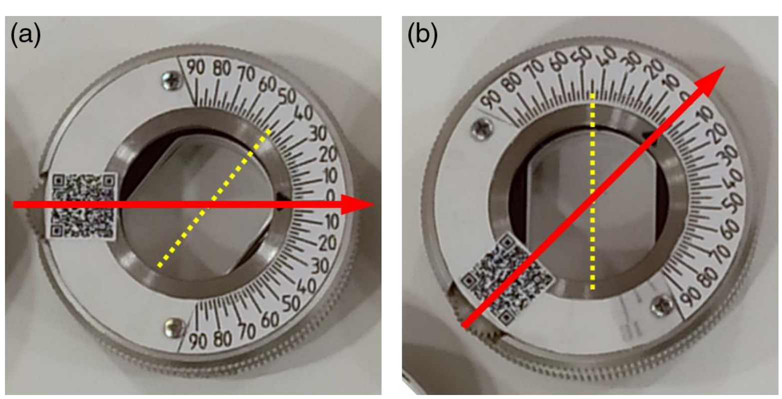

Fig. 2 Pictures of the Stokes lens for the generation of (a) J_0 and (b) J_{45} components.

Both the theoretical and the measured power values were converted to power vector notation (M and J) for statistical purposes. For each of the three Stokes lenses, differences between measured and expected or theoretical power values were studied for both M and J components by means of paired t-test (or the corresponding Wilcoxon signed rank test if the normality testing failed). Bland-Altman graphics were depicted to show the agreement between the theoretical and measured values for each of the three Stokes lenses. Sigmaplot v14.0 for windows was used for statistical and graphical purposes and the significance level was set to $\alpha = 0.05$.

3 Experimental Results

Dioptric power profiles of the generated Stokes lenses are first represented in the conventional sphero-cylindrical form. Figure 3 shows the comparison between the generated power profiles from the measurements for the three Stokes lenses (RGB color-coded plots) and the theoretical ones (gray dotted lines). The RGB graphs correspond to the power profiles generated from the combination of, respectively, ± 1.75, ± 1.25, and ± 0.75 D cylindrical pair of lenses. For each RGB plot case, the darker colors correspond to the sphere component, and the more luminous ones represent the cylinder value. As expected, Fig. 3 shows a sinusoidal variation of the dioptric power profile in which the sphere value is roughly half of the cylinder one with the opposite sign. This fact suggests that the generated dioptric power is Jacksonian with variable value depending on the total angle between the cylinder's axes, i.e., a pure variable astigmatic power in the form of a Jackson cross-cylinder lens having null (or close to) mean equivalent sphere.

To prove this last statement, Fig. 4 includes the generated power profiles using Fourier polar notation. Thus, Fig. 4(a) includes the mean equivalent sphere ($M = S + C/2$) component for the three Stokes lenses showing a value close to 0 D for all measurement points. We computed the mean value and its standard deviation ($\pm$SD) of the residual mean equivalent sphere component for the three Stokes lenses showing the following results: 0.0003 (± 0.048 D), -0.0028 (± 0.067 D), and -0.0026 D (± 0.054 D). Figure 4(b) shows the sinusoidal curve of the variable Jacksonian astigmatic ($J = -C/2$) component for the three Stokes lenses (RGB color coded

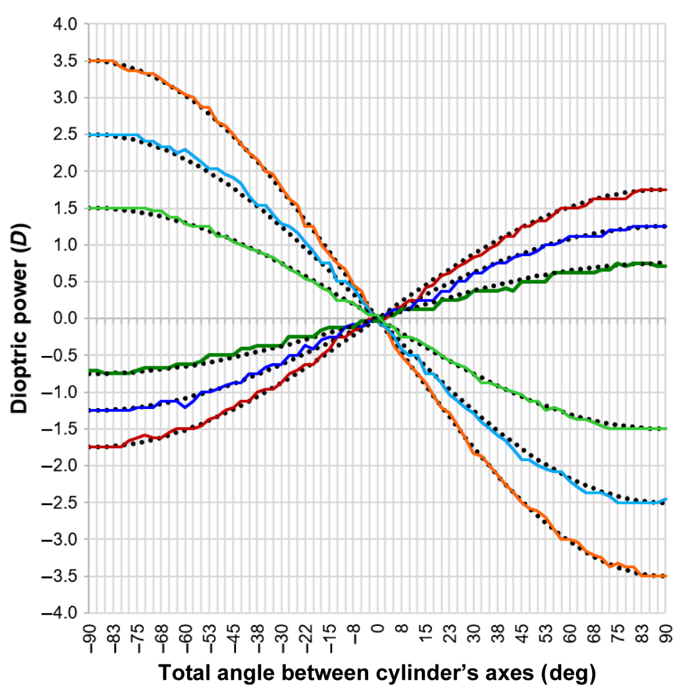

Fig. 3 Dioptric power profile comparison in sphere (darker colors) and cylinder (clearer colors) for the three generated Stokes lenses (RGB color coded plots) coming from the combination of ± 1.75, ± 1.25, and ± 0.75 D cylindrical lenses.

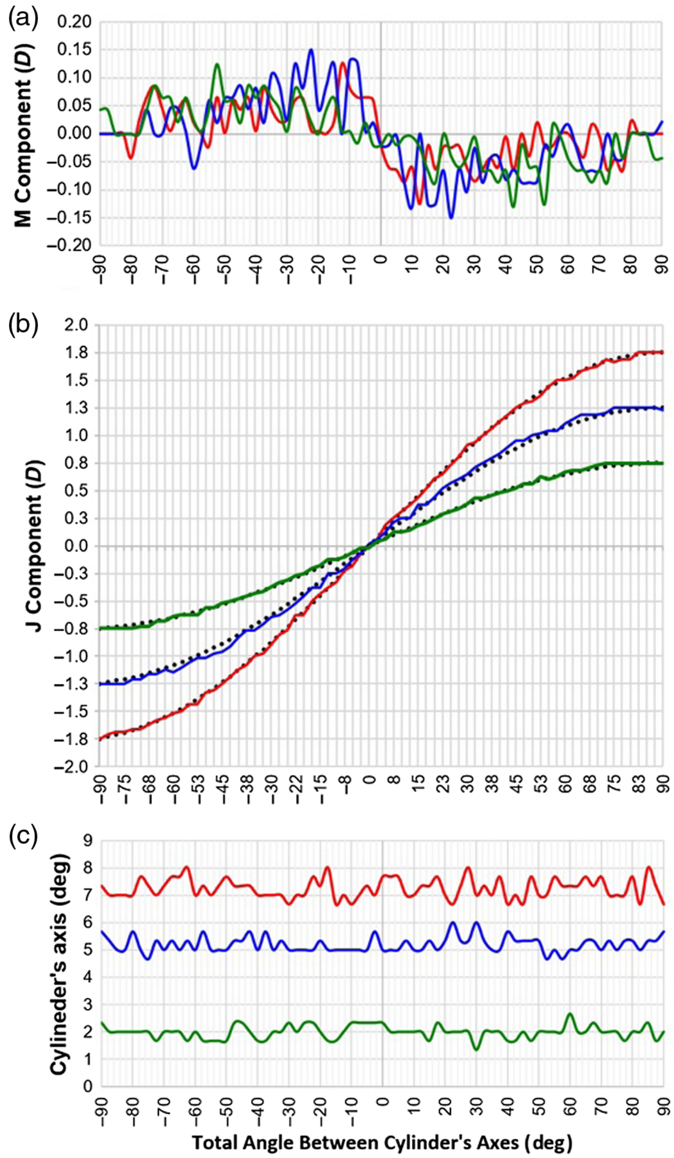

Fig. 4 Dioptric power profile comparison in Fourier polar notation: (a) residual near equivalent sphere value; (b) variable astigmatic power; and (c) the stability of the cylinder's axis.

graphs) in comparison with their theoretical plots (gray dotted lines). And Fig. 4(c) shows the stability of the astigmatic axis along the different measurement points.

Table 1 gives the statistical testing looking for significant differences between the measured and theoretical variables considering both refractive components for the three assembled Stokes lenses. All data sets passed the normality testing by means of the Shapiro–Wilk test, except J data for the ± 0.75 D lens, so the Wilcoxon signed rank test was used in this case to study differences between theoretical and measured values, and the t-test was used for the remaining comparisons (see Table 1). As can be seen, no significant differences were found ($p \geq 0.05$ in all cases), even the J component for the ± 1.75 D lens was close to statistical significance.

Figure 5 shows a boxplot of the differences between the measured and theoretical M values for each of the three Stokes lenses. Given that the M value is always close to 0 D, there is no need for a Bland-Altman plot. The boxplot in Fig. 5 also shows the 95% confidence interval around the mean difference values (0 D for the three Stokes lenses) for the three Stokes lenses. This interval for the ± 0.75 D Stokes lens is ($+0.11$ to -0.11 D), for the ± 1.25 D Stokes lens is ($+0.14$ to -0.13 D), and for the ± 1.75 D lens is ($+0.09$ to -0.10 D).

Table 1 Results of the statistical comparison between theoretical and measured values obtained for M and J refractive components with the three Stokes lenses. M: mean equivalent sphere; J: Jacksonian astigmatic component.

	Refractive component	
Lens (D)	M	J
±0.75	$t = 0.483$; $p = 0.630$	$w = 79.000$; $p = 0.713$
±1.25	$t = 0.329$; $p = 0.743$	$t = 0.796$; $p = 0.429$
±1.75	$t = -0.072$; $p = 0.943$	$t = 2.017$; $p = 0.05$

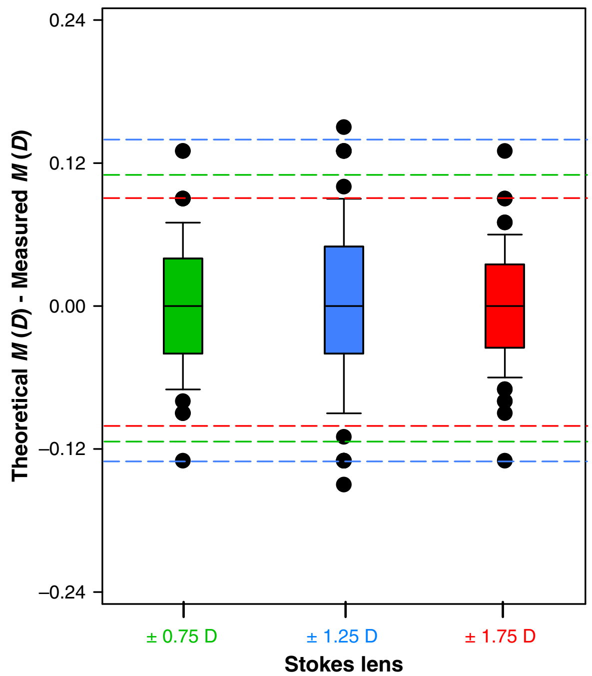

Fig. 5 Boxplot of the differences between measured and theoretical M values for each of the three Stokes lenses. A 95% confidence interval around the mean difference values were also drawn (dashed lines).

Figure 6 shows the Bland–Altman plots for the J component for the three Stokes lenses. Solid lines represent mean difference, and dashed lines represent the 95% confidence intervals. Mean J difference was very close to 0 diopters for the three lenses (−0.001 D for the ±0.75 D lens, 0 D for the ±1.25 D lens, and 0.005 D for the ±1.75 D lens). Despite the apparent variability of data, 95% confidence intervals are narrower in the three lenses than the ±0.125 D measurement error of the focimeter (from +0.032 to −0.034 D in the ±0.75 D lens, from +0.08 to −0.07 D in the ±1.25 D lens, and from +0.042 to −0.033 D in the ±1.75 D lens).

4 Discussion and Conclusions

We have presented the assembly and characterization of three different Stokes lenses coming from the combination of three different cylinder pairs having different powers. Therefore, the astigmatic generated power profile varies from 0 D to |1.50|D, |2.50|D, and |3.50|D for each Stokes lens. A couple of comments are appropriate at this point. The first one relates to the maximum generated astigmatic power (|3.50|D) that covers most of the cases in the population,

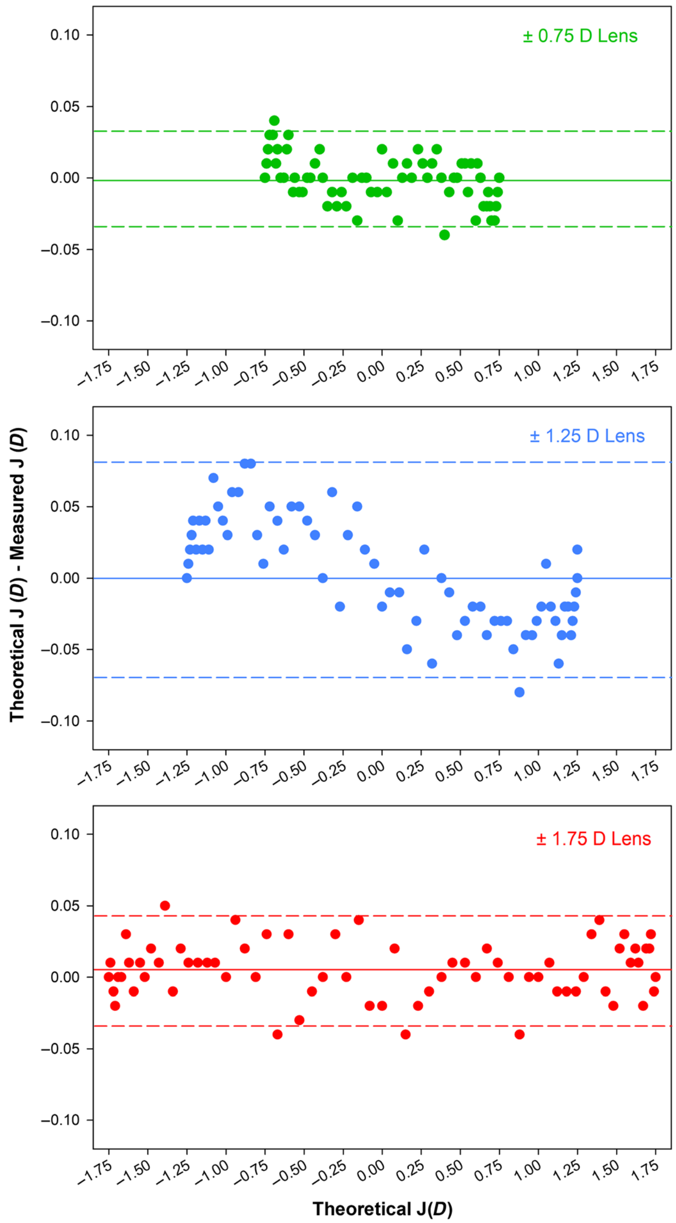

Fig. 6 Bland–Altman plots for the J component for the three Stokes lenses. Solid lines represent mean difference, and dashed lines represent the 95% confidence intervals.

for astigmatisms above 3.50 D (in module) are extremely rare.[37,38] Arriola et al found a prevalence of corneal astigmatism >3 D in just 3.1% of their sample composed of 6111 eyes.[37] A similar result was found by Nemeth et al.[38] with 2.47% of astigmatism over 2.99 D and only 1.46% of eyes having astigmatism >3.50 D.

The second comment relates to the need for a set of Stokes lenses with different generated astigmatic power. Because the power profile exhibits a steeper variation at the beginning [once starting to misalign the cylinder's axis from the initial parallel position—see Fig. 4(b)], the lower the cylindrical power is, the higher the sensitivity of the generated Stokes lens for astigmatism generation is. Thus, the minimum increment in the measured dioptric power corresponding to steps of 2.5 deg in the angular scale ranges from |0.125|D to |0.30|D for the Stokes lenses including ±0.75 D and the ±1.75 D cylinder's pairs, respectively. In that sense, the proposed set of Stokes lenses provides different sensitivities for astigmatic compensation and can be useful for future optometric studies with different astigmatic ranges. Even if a given phoropter can only include one Stokes lens, thus making it difficult to switch between different Stokes lenses in a clinical setting, it is possible to have different phoropters equipped with different Stokes lenses to be used in clinical subjective refraction and select the most appropriate one after knowing by objective examination (retinoscopy or autorefractometry) the tentative value of astigmatism in a given patient's eye. Also, the proposed set of Stokes lenses are suitable to be adapted to trial frames, thus enabling the option of Stokes lens selection from the beginning of the subjective routine if the amount of astigmatism is tentatively known a priori (retinoscopy or autorefractometry).

The presented paper falls in the research line established in Ref. 34 in which a Stokes lens is assembled from an old Risley prism mount and characterized in dioptric power profile using power vector formalism. Reference 34 focuses more on the mathematical conversion from prism diopters to dioptric power, high order aberration analysis, and the implementation of the reported Stokes lens as a device for astigmatism compensation in optical instruments (i.e., microscopes and lensmeters). In difference with Ref. 34, the proposed work is evaluated from an optometric perspective aimed at the ongoing development of a clinical methodology involving those lenses for refractive error evaluation. Although that optometric application is not included here, this manuscript completes and rounds the optical characterization reported in Ref. 34 from an optometric point of view. In addition, some improvements are achieved in comparison with the device reported in Ref. 34: (i) we have presented a more modern realization of the Stoke lens using more up to date Risley prims mounts, and (ii) we have included an angular dial for simplifying the conversion to dioptric power. As in Ref. 34, the experimental characterization reveals a mean equivalent sphere (M) component with a value close to 0 D for all measurement points and for the three Stokes lenses. Also, the stability of the astigmatic axis along the different measurement points is very high, showing 1 deg of maximum variation.

Statistical testing revealed no significant differences between theoretical and measured power values, both for M and J components, in the three assembled Stokes lenses. As can be seen from the Bland Altman plots, the agreement between the expected and measured power for both M and J components is very high. The 95% confidence intervals around the mean difference value are narrower than the precision error of the focimeter used to measure the Stokes lenses. Thus, it can be concluded that there are neither statistically significant differences, nor clinically relevant differences between the expected and the obtained values measured with the three Stokes lenses. All of these findings strongly support the use of these Stokes lenses for measuring astigmatism in monocular refractive error evaluation.

In summary, our work describes the construction and validation of three compact and cost-effective Stokes lenses from the combination of Risley prism mounts with three different (in dioptric power) cylinder pairs aimed at monocular evaluation of the astigmatism in refractive error subjective routines. Having in mind this application, maybe the main drawback is the loss of the prism function in the phoropter. Although rotary prisms are mainly used in the measurement of phorias and binocular balances and not in monocular refraction, it is possible to assemble both Risley-based devices in the interchangeable turret of the phoropter for those cases in which both functionalities (rotary prism unit and Stokes lens) are needed. Thus, the turret will contain the Stokes lens in addition to the fixed cross-cylinder and the rotary prisms.

Acknowledgments

Jose Ricardo Albero-Moreno acknowledges Clinica Baviera Castellón for the time dedicated to this research as well as for providing the material (Risley prism mounts and cylinders) to perform this research. All coauthors declare no conflicts of interest related to this article.

References

1. T. Young, "On the mechanism of the eye," *Philos. Trans. R. Soc.* **91**, 23–88 (1801).
2. D. A. Atchison and W. N. Charman, "Thomas Young's contribution to visual optics: the Bakerian Lecture on the mechanism of the eye," *J. Vis.* **10**(12), 1–16 (2010).
3. G. B. Airy, "On a peculiar defect in the eye and a mode of correcting it," *Trans. Cambridge Philos. Soc.* **2**, 267–271 (1827).

4. J. R. Levene, “Sir George Biddell Airy, F.R.S. (1801–1892) and the discovery and correction of astigmatism,” *Notes Rec. R. Soc. London* **21**, 180–199 (1966).
5. T. Grosvenor, “How much do we know about astigmatism?” *Clin. Exp. Optom.* **90**, 3–4 (2007).
6. A. Grzybowski and P. Kanclerz, “Beginnings of astigmatism understanding and management in the 19th century,” *Eye Contact Lens* **44**(1), S22–S29 (2018).
7. G. G. Stokes, “On a mode of measuring the astigmatism of a defective eye,” *Rep. Br. Assoc.* **2**, 10–11 (1849).
8. M. Tree, “A rotating cross cylinder,” *Br. J. Ophthalmol.* **23**, 632–633 (1939).
9. W. F. Harris, “The Jackson cross-cylinder. Part 1: properties,” *S. Afr. Optom.* **66**, 41–55 (2007).
10. L. N. Thibos, W. Wheeler, and D. Horner, “Power vectors: an application of Fourier analysis to the description and statistical analysis of refractive error,” *Optom. Vis. Sci.* **74**, 367–375 (1997).
11. W. Tasman and E. A. Jaeger, *Duane's Clinical Ophthalmology*, Lippincott Williams & Wilkins, Philadelphia (2005).
12. E. Jackson, “A trial set of small lenses and a modified trial size frame,” *Trans. Am. Ophthalmol. Soc.* **4**, 595 (1887).
13. E. Jackson, “The astigmic lens (crossed cylinder) to determine the amount and principal meridians of astigmia,” *Ophthalmic Res.* **16**, 378–83 (1907).
14. W. Benjamin, *Borish’s Clinical Refraction*, 2nd ed., Elsevier, Butterworth-Heinemann, Missouri, (2006).
15. W. H. Crisp, “Combined use of cross cylinder and astigmatic dial,” *Ann. Ophthalmol.* **26**, 42–46 (1917).
16. W. H. Crisp, “Cylinder skiascopy and cross cylinders,” *Am. J. Ophthalmol.* **12**, 684–686 (1929).
17. W. H. Crisp, “The cross-cylinder tests, especially in relation to the astigmatic axis,” *Trans. Ophthalmol. Soc. UK* **51**, 495–514 (1931).
18. W. H. Crisp, “The crossed-cylinder tests, especially in relation to the astigmatic axis,” *Am. J. Ophthalmol.* **15**, 729–738 (1932).
19. W. H. Crisp, “Cross-cylinder misconceptions,” *Am. J. Ophthalmol.* **23**, 1403–1406 (1940).
20. W. H. Crisp, “A new cross-cylinder test for astigmatic axis, without use of test type,” *Am. J. Ophthalmol.* **26**, 571–576 (1943).
21. M. Lai et al., “Method and apparatus for measuring optical aberrations of an eye,” US Patent 6575572B2 (2003).
22. T. W. Raasch, “Clinical refractive instruments,” US Patent 6942342B2 (2005).
23. G. Marin, J. Perrin, and M. Hernandez-Castaneda, “Method for determining astigmatism of an eye of a person,” US Patent App. 16/609,629 (2020).
24. J. P. Foley and C. E. Campbell, “An optical device with variable astigmatic power,” *Optom. Vis. Sci.* **76**, 664–667 (1999).
25. J. Arines and E. Acosta, “Low-cost adaptive astigmatism compensator for improvement of eye fundus camera,” *Opt. Lett.* **36**, 4164–4166 (2011).
26. J. Arines and E. Acosta, “Adaptive astigmatism-correcting device for eyepieces,” *Optom. Vis. Sci.* **89**, 1524–1528 (2012).
27. S. Ferrer-Altabás, L. Thibos, and V. Micó, “Astigmatic Stokes lens revisited,” *Opt. Express* **30**, 8974–8990 (2022).
28. C. E. Campbell, “Variable power phase lenses,” *Appl. Opt.* **44**, 3438–3441 (2005).
29. E. Acosta and S. Bará, “Variable aberration generators using rotated Zernike plates,” *J. Opt. Soc. Am. A* **22**, 1993–1996 (2005).
30. S. D. Risley, “A new rotary prism,” *Trans. Am. Ophthalmol. Soc.* **5**, 412–413 (1889).
31. G. J. Wang, O. Pomerantzeff, and M. M. Pankratov, “Astigmatism of oblique incidence in the human model eye,” *Vis. Res.* **23**(10), 1079–1085 (1983).
32. E. A. Villegas, E. Alcon, and P. Artal, “Optical quality of the eye in subjects with normal and excellent visual acuity,” *Invest. Ophthalmol. Vis. Sci.* **49**, 4688–4696 (2008).
33. J. A. Picazo-Bueno, M. Trusiak, and V. Micó, “Single-shot slightly off-axis digital holographic microscopy with add-on module based on beamsplitter cube,” *Opt. Express* **27**, 5655–5669 (2019).

34. S. Ferrer-Altabás and V. Micó, "Characterization of a compact low-cost Stokes lens for astigmatism compensation in optical instruments," *Appl. Opt.* **59**, 3347–3352 (2020).
35. S. Ferrer-Altabás, M. Sanz-Sabater, and V. Micó, "Vectofocimetry: dioptric power measurement in manual focimeters using power vectors," *Opt. Lasers Eng.* **146**, 106703 (2021).
36. A. María-Revert et al., "An alternative clinical routine for subjective refraction based on power vectors with trial frames," *Ophthalmic Physiol. Opt.* **37**, 24–32 (2017).
37. P. Arriola-Villalobos et al., "Biometry data and prevalence of corneal astigmatism in caucasian spanish candidates for cataract surgery," *J. Fr. Ophtalmol.* **44**, 76–83 (2021).
38. G. Nemeth et al., "Astigmatism prevalence and biometric analysis in normal population," *Eur. J. Ophthalmol.* **23**, 779–783 (2013).

Jose Ricardo Albero-Moreno studied optics at the University of Alicante in 1996 and obtained his master's degree in optometry and research from the Camilo José Cela University in 2014. Currently, he is doing doctoral studies at the University of Valencia. He has been an external professor of optometry at CEU Cardenal Herrea University from 2014 and has developed his professional career in the field of refractive surgery for 23 years at Clínica-Baviera (Aier Eye Group).

César Albarrán-Diego obtained his degree in optics and optometry in 1996, completed his master's degree in advanced optometry and visual sciences in 2012, and his PhD in optometry and vision science in 2017, all from the Universitat de Valencia. He is currently an associate professor at the Department of Optics and Optometry and Vision Science at the UV, and his research interests include physiological optics and optometry applied to refractive surgery.

Vicente Micó received his MS degree in physics in 1999, his BS degree in optics and optometry in 2000, and his PhD in physics in 2008, all from the Universitat de Valencia. He is currently an associate professor at the Department of Optics and Optometry and Vision Science, where he is with the Optoelectronic Image Processing Group (www.uv.es/gpoei). As a physicist, he is actively working in optical metrology, digital holographic microscopy, optical superresolution, and lensless coherent imaging. As an optometrist, he is dedicated to theoretical aspects on physiological and visual optics as well as the development of novel optometric instruments for the measurement of ocular parameters.